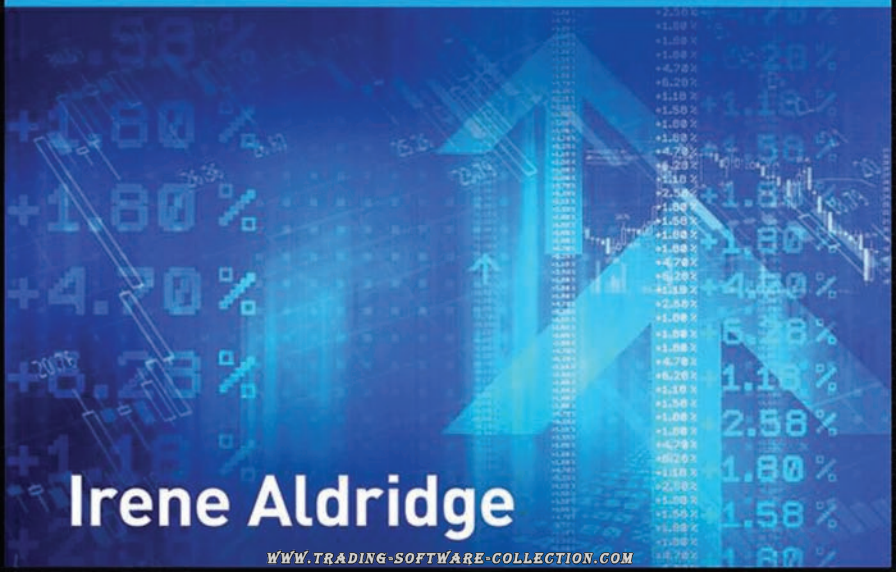
WILEY Trading

A Practical Guide to Algorithmic Strategies and Trading Systems

HIGH-FREQUENCY TRADING



Irene Aldridge

High-Frequency Trading

A Practical Guide to Algorithmic Strategies and Trading Systems

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To my family

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CHAPTER 1

Introduction

High-frequency trading has been taking Wall Street by storm, and for a good reason: its immense profitability. According to *Alpha* magazine, the highest earning investment manager of 2008 was Jim Simons of Renaissance Technologies Corp., a long-standing proponent of high-frequency strategies. Dr. Simons reportedly earned $2.5 billion in 2008 alone. While no institution was thoroughly tracking performance of high-frequency funds when this book was written, colloquial evidence suggests that the majority of high-frequency managers delivered positive returns in 2008, whereas 70 percent of low-frequency practitioners lost money, according to the *New York Times*. The profitability of high-frequency enterprises is further corroborated by the exponential growth of the industry. According to a February 2009 report from Aite Group, high-frequency trading now accounts for over 60 percent of trading volume coming through the financial exchanges. High-frequency trading professionals are increasingly in demand and reap top-dollar compensation. Even in the worst months of the 2008 crisis, 50 percent of all open positions in finance involved expertise in high-frequency trading (Aldridge, 2008). Despite the demand for information on this topic, little has been published to help investors understand and implement high-frequency trading systems.

So what is high-frequency trading, and what is its allure? The main innovation that separates high-frequency from low-frequency trading is a high turnover of capital in rapid computer-driven responses to changing market conditions. High-frequency trading strategies are characterized by a higher number of trades and a lower average gain per trade. Many traditional money managers hold their trading positions for weeks or even

months, generating a few percentage points in return per trade. By comparison, high-frequency money managers execute multiple trades each day, gaining a fraction of a percent return per trade, with few, if any, positions carried overnight. The absence of overnight positions is important to investors and portfolio managers for three reasons:

1. The continuing globalization of capital markets extends most of the trading activity to 24-hour cycles, and with the current volatility in the markets, overnight positions can become particularly risky. High-frequency strategies do away with overnight risk.
2. High-frequency strategies allow for full transparency of account holdings and eliminate the need for capital lock-ups.
3. Overnight positions taken out on margin have to be paid for at the interest rate referred to as an overnight carry rate. The overnight carry rate is typically slightly above LIBOR. With volatility in LIBOR and hyperinflation around the corner, however, overnight positions can become increasingly expensive and therefore unprofitable for many money managers. High-frequency strategies avoid the overnight carry, creating considerable savings for investors in tight lending conditions and in high-interest environments.

High-frequency trading has additional advantages. High-frequency strategies have little or no correlation with traditional long-term buy and hold strategies, making high-frequency strategies valuable diversification tools for long-term portfolios. High-frequency strategies also require shorter evaluation periods because of their statistical properties, which are discussed in depth further along in this book. If an average monthly strategy requires six months to two years of observation to establish the strategy's credibility, the performance of many high-frequency strategies can be statistically ascertained within a month.

In addition to the investment benefits already listed, high-frequency trading provides operational savings and numerous benefits to society. From the operational perspective, the automated nature of high-frequency trading delivers savings through reduced staff headcount as well as a lower incidence of errors due to human hesitation and emotion.

Among the top societal benefits of high-frequency strategies are the following:

- Increased market efficiency
- Added liquidity
- Innovation in computer technology
- Stabilization of market systems

High-frequency strategies identify and trade away temporary market inefficiencies and impound information into prices more quickly. Many high-frequency strategies provide significant liquidity to the markets, making the markets work more smoothly and with fewer frictional costs for all investors. High-frequency traders encourage innovation in computer technology and facilitate new solutions to relieve Internet communication bottlenecks. They also stimulate the invention of new processors that speed up computation and digital communication. Finally, high-frequency trading stabilizes market systems by flushing out toxic mispricing.

A fit analogy was developed by Richard Olsen, CEO of Oanda, Inc. At a March 2009 FXWeek conference, Dr. Olsen suggested that if financial markets can be compared to a human body, then high-frequency trading is analogous to human blood that circulates throughout the body several times a day flushing out toxins, healing wounds, and regulating temperature. Low-frequency investment decisions, on the other hand, can be thought of as actions that destabilize the circulatory system by reacting too slowly. Even a simple decision to take a walk in the park exposes the body to infection and other dangers, such as slips and falls. It is high-frequency trading that provides quick reactions, such as a person rebalancing his footing, that can stabilize markets' reactions to shocks.

Many successful high-frequency strategies run on foreign exchange, equities, futures, and derivatives. By its nature, high-frequency trading can be applied to any sufficiently liquid financial instrument. (A "liquid instrument" can be a financial security that has enough buyers and sellers to trade at any time of the trading day.)

High-frequency trading strategies can be executed around the clock. Electronic foreign exchange markets are open 24 hours, 5 days a week. U.S. equities can now be traded "outside regular trading hours," from 4 A.M. EST to midnight EST every business day. Twenty-four-hour trading is also being developed for selected futures and options.

Many high-frequency firms are based in New York, Connecticut, London, Singapore, and Chicago. Many Chicago firms use their proximity to the Chicago Mercantile Exchange to develop fast trading strategies for futures, options, and commodities. New York and Connecticut firms tend to be generalist, with a preference toward U.S. equities. European time zones give Londoners an advantage in trading currencies, and Singapore firms tend to specialize in Asian markets. While high-frequency strategies can be run from any corner of the world at any time of day, natural affiliations and talent clusters emerge at places most conducive to specific types of financial securities.

The largest high-frequency names worldwide include Millennium, DE Shaw, Worldquant, and Renaissance Technologies. Most of the high-frequency firms are hedge funds or other proprietary investment vehicles

TABLE 1.1 Classification of High-Frequency Strategies

Strategy	Description	Typical Holding Period
Automated liquidity provision	Quantitative algorithms for optimal pricing and execution of market-making positions	<1 minute
Market microstructure trading	Identifying trading party order flow through reverse engineering of observed quotes	<10 minutes
Event trading	Short-term trading on macro events	<1 hour
Deviations arbitrage	Statistical arbitrage of deviations from equilibrium: triangle trades, basis trades, and the like	<1 day

that fly under the radar of many market participants. Proprietary trading desks of major banks, too, dabble in high-frequency products, but often get spun out into hedge fund structures once they are successful.

Currently, four classes of trading strategies are most popular in the high-frequency category: automated liquidity provision, market microstructure trading, event trading, and deviations arbitrage. Table 1.1 summarizes key properties of each type.

Developing high-frequency trading presents a set of challenges previously unknown to most money managers. The first is dealing with large volumes of intra-day data. Unlike the daily data used in many traditional investment analyses, intra-day data is much more voluminous and can be irregularly spaced, requiring new tools and methodologies. As always, most prudent money managers require any trading system to have at least two years worth of back testing before they put money behind it. Working with two or more years of intra-day data can already be a great challenge for many. Credible systems usually require four or more years of data to allow for full examination of potential pitfalls.

The second challenge is the precision of signals. Since gains may quickly turn to losses if signals are misaligned, a signal must be precise enough to trigger trades in a fraction of a second.

Speed of execution is the third challenge. Traditional phone-in orders are not sustainable within the high-frequency framework. The only reliable way to achieve the required speed and precision is computer automation of order generation and execution. Programming high-frequency computer systems requires advanced skills in software development. Run-time mistakes can be very costly; therefore, human supervision of trading in production remains essential to ensure that the system is running within

prespecified risk boundaries. Such discretion is embedded in human supervision. However, the intervention of the trader is limited to one decision only: whether the system is performing within prespecified bounds, and if it is not, whether it is the right time to pull the plug.

From the operational perspective, the high speed and low transparency of computer-driven decisions requires a particular comfort level with computer-driven execution. This comfort level may be further tested by threats from Internet viruses and other computer security challenges that could leave a system paralyzed.

Finally, just staying in the high-frequency game requires ongoing maintenance and upgrades to keep up with the "arms race" of information technology (IT) expenditures by banks and other financial institutions that are allotted for developing the fastest computer hardware and execution engines in the world.

Overall, high-frequency trading is a difficult but profitable endeavor that can generate stable profits under various market conditions. Solid footing in both theory and practice of finance and computer science are the normal prerequisites for successful implementation of high-frequency environments. Although past performance is never a guarantee of future returns, solid investment management metrics delivered on auditable returns net of transaction costs are likely to give investors a good indication of a high-frequency manager's abilities.

This book offers the first applied "how to do it" manual for building high-frequency systems, covering the topic in sufficient depth to thoroughly pinpoint the issues at hand, yet leaving mathematical complexities to their original publications, referenced throughout the book.

The following professions will find the book useful:

- Senior management in investment and broker-dealer functions seeking to familiarize themselves with the business of high-frequency trading
- Institutional investors, such as pension funds and funds of funds, desiring to better understand high-frequency operations, returns, and risk
- Quantitative analysts looking for a synthesized guide to contemporary academic literature and its applications to high-frequency trading
- IT staff tasked with supporting a high-frequency operation
- Academics and business students interested in high-frequency trading
- Individual investors looking for a new way to trade
- Aspiring high-frequency traders, risk managers, and government regulators

The book has five parts. The first part describes the history and business environment of high-frequency trading systems. The second part reviews the statistical and econometric foundations of the common types of

high-frequency strategies. The third part addresses the details of modeling high-frequency trading strategies. The fourth part describes the steps required to build a quality high-frequency trading system. The fifth and last part addresses the issues of running, monitoring, and benchmarking high-frequency trading systems.

The book includes numerous quantitative trading strategies with references to the studies that first documented the ideas. The trading strategies discussed illustrate practical considerations behind high-frequency trading. Chapter 10 considers strategies of the highest frequency, with position-holding periods of one minute or less. Chapter 11 looks into a class of high-frequency strategies known as the market microstructure models, with typical holding periods seldom exceeding 10 minutes. Chapter 12 details strategies capturing abnormal returns around ad hoc events such as announcements of economic figures. Such strategies, known as "event arbitrage" strategies, work best with positions held from 30 minutes to 1 hour. Chapter 13 addresses a gamut of other strategies collectively known as "statistical arbitrage" with positions often held up to one trading day. Chapter 14 discusses the latest scientific thought in creating multistrategy portfolios.

The strategies presented are based on published academic research and can be readily implemented by trading professionals. It is worth keeping in mind, however, that strategies made public soon become obsolete, as many people rush in to trade upon them, erasing the margin potential in the process. As a consequence, the best-performing strategies are the ones that are kept in the strictest of confidence and seldom find their way into the press, this book being no exception. The main purpose of this book is to illustrate how established academic research can be applied to capture market inefficiencies with the goal of stimulating readers' own innovations in the development of new, profitable trading strategies.

CHAPTER 2

Evolution of High-Frequency Trading

Advances in computer technology have supercharged the transmission and execution of orders and have compressed the holding periods required for investments. Once applied to quantitative simulations of market behavior conditioned on large sets of historical data, a new investment discipline, called "high-frequency trading," was born.

This chapter examines the historical evolution of trading to explain how technological breakthroughs impacted financial markets and facilitated the emergence of high-frequency trading.

FINANCIAL MARKETS AND TECHNOLOGICAL INNOVATION

Among the many developments affecting the operations of financial markets, technological innovation leaves the most persistent mark. While the introduction of new market securities, such as EUR/USD in 1999, created large-scale one-time disruptions in market routines, technological changes have a subtle and continuous impact on the markets. Over the years, technology has improved the way news is disseminated, the quality of financial analysis, and the speed of communication among market participants. While these changes have made the markets more transparent and reduced the number of traditional market inefficiencies, technology has also made available an entirely new set of arbitrage opportunities.

Many years ago, securities markets were run in an entirely manual fashion. To request a quote on a financial security, a client would contact

his sales representative in person or via messengers and later via telegraph and telephone when telephony became available. The salesperson would then walk over or shout to the trading representative a request for prices on securities of interest to the client. The trader would report back the market prices obtained from other brokers and exchanges. The process would repeat itself when the client placed an order.

The process was slow, error-prone, and expensive, with the costs being passed on to the client. Most errors arose from two sources:

1. Markets could move significantly between the time the market price was set on an exchange and the time the client received the quote.
2. Errors were introduced in multiple levels of human communication, as people misheard the market data being transmitted.

The communication chain was as costly as it was unreliable, as all the links in the human chain were compensated for their efforts and market participants absorbed the costs of errors.

It was not until the 1980s that the first electronic dealing systems appeared and were immediately heralded as revolutionary. The systems aggregated market data across multiple dealers and exchanges, distributed information simultaneously to a multitude of market participants, allowed parties with preapproved credits to trade with each other at the best available prices displayed on the systems, and created reliable information and transaction logs. According to Leinweber (2007), designated order turnaround (DOT), introduced by the New York Stock Exchange (NYSE), was the first electronic execution system. DOT was accessible only to NYSE floor specialists, making it useful only for facilitation of the NYSE's internal operations. Nasdaq's computer-assisted execution system, available to broker-dealers, was rolled out in 1983, with the small-order execution system following in 1984.

While computer-based execution has been available on selected exchanges and networks since the mid-1980s, systematic trading did not gain traction until the 1990s. According to Goodhart and O'Hara (1997), the main reasons for the delay in adopting systematic trading were the high costs of computing as well as the low throughput of electronic orders on many exchanges. NASDAQ, for example, introduced its electronic execution capability in 1985, but made it available only for smaller orders of up to 1,000 shares at a time. Exchanges such as the American Stock Exchange (AMEX) and the NYSE developed hybrid electronic/floor markets that did not fully utilize electronic trading capabilities.

Once new technologies are accepted by financial institutions, their applications tend to further increase demand for automated trading. To wit, rapid increases in the proportion of systematic funds among all hedge

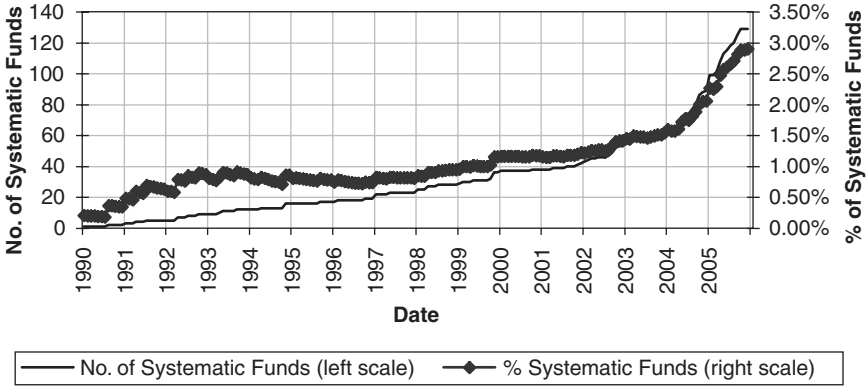

FIGURE 2.1 Absolute number and relative proportion of hedge funds identifying themselves as "systematic."
Source: Aldridge (2009b).

funds coincided with important developments in trading technology. As Figure 2.1 shows, a notable rise in the number of systematic funds occurred in the early 1990s. Coincidentally, in 1992 the Chicago Mercantile Exchange (CME) launched its first electronic platform, Globex. Initially, Globex traded only CME futures on the most liquid currency pairs: Deutsche mark and Japanese yen. Electronic trading was subsequently extended to CME futures on British pounds, Swiss francs, and Australian and Canadian dollars. In 1993, systematic trading was enabled for CME equity futures. By October 2002, electronic trading on the CME reached an average daily volume of 1.2 million contracts, and innovation and expansion of trading technology continued henceforth, causing an explosion in systematic trading in futures along the way.

The first fully electronic U.S. options exchange was launched in 2000 by the New York–based International Securities Exchange (ISE). As of mid-2008, seven exchanges offered either fully electronic or a hybrid mix of floor and electronic trading in options. These seven exchanges are ISE, Chicago Board Options Exchange (CBOE), Boston Options Exchange (BOX), AMEX, NYSE's Arca Options, and Nasdaq Options Market (NOM).

According to estimates conducted by Boston-based Aite Group, shown in Figure 2.2, adoption of electronic trading has grown from 25 percent of trading volume in 2001 to 85 percent in 2008. Close to 100 percent of equity trading is expected to be performed over the electronic networks by 2010.

Technological developments markedly increased the daily trade volume. In 1923, 1 million shares traded per day on the NYSE, while just over 1 billion shares were traded per day on the NYSE in 2003, a 1,000-times increase.

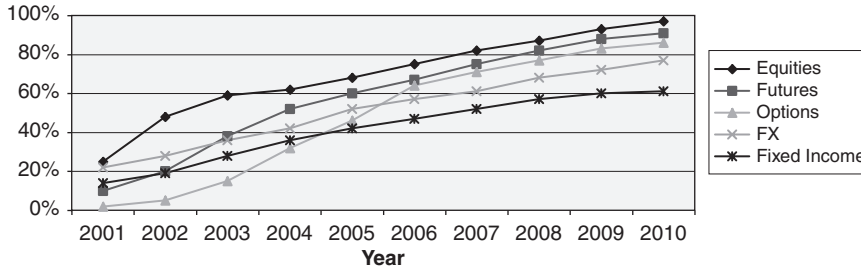

FIGURE 2.2 Adoption of electronic trading capabilities by asset class. *Source:* Aite Group.

Technological advances have also changed the industry structure for financial services from a rigid hierarchical structure popular through most of the 20th century to a flat decentralized network that has become the standard since the late 1990s. The traditional 20th-century network of financial services is illustrated in Figure 2.3. At the core are the exchanges or, in the case of foreign exchange trading, inter-dealer networks. Exchanges are the centralized marketplaces for transacting and clearing securities orders. In decentralized foreign exchange markets, inter-dealer networks consist of inter-dealer brokers, which, like exchanges, are organizations that ensure liquidity in the markets and deal between their peers and broker-dealers.

Broker-dealers perform two functions—trading for their own accounts (known as "proprietary trading" or "prop trading") and transacting and clearing trades for their customers. Broker-dealers use inter-dealer brokers to quickly find the best price for a particular security among the network of other broker-dealers. Occasionally, broker-dealers also deal directly with other broker-dealers, particularly for less liquid instruments such as customized option contracts. Broker-dealers' transacting clients are investment banking clients (institutional clients), large corporations (corporate clients), medium-sized firms (commercial clients), and high-net-worth individuals (HNW clients). Investment institutions can in turn be brokerages providing trading access to other, smaller institutions and individuals with smaller accounts (retail clients).

Until the late 1990s, it was the broker-dealers who played the central and most profitable roles in the financial ecosystem; broker-dealers controlled clients' access to the exchanges and were compensated handsomely for doing so. Multiple layers of brokers served different levels of investors. The institutional investors, the well-capitalized professional investment outfits, were served by the elite class of institutional sales brokers that sought volume; the individual investors were assisted by the retail brokers that charged higher commissions. This hierarchical structure existed from the early 1920s through much of the 1990s when the advent of the

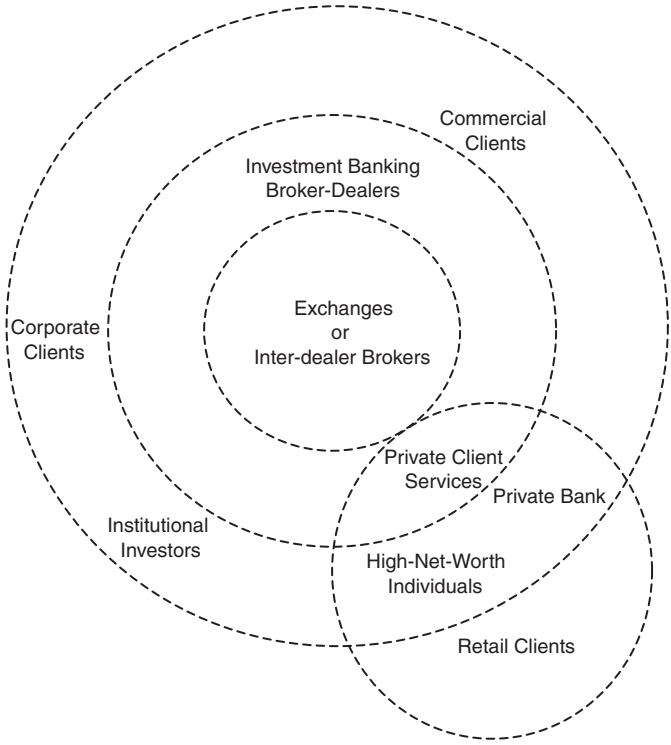

FIGURE 2.3 Twentieth-century structure of capital markets.

Internet uprooted the traditional order. At that time, a garden variety of online broker-dealers sprung up, ready to offer direct connectivity to the exchanges, and the broker structure flattened dramatically.

Dealers trade large lots by aggregating their client orders. To ensure speedy execution for their clients on demand, dealers typically "run books"—inventories of securities that the dealers expand or shrink depending on their expectation of future demand and market conditions. To compensate for the risk of holding the inventory and the convenience of transacting in lots as small as $100,000, the dealers charge their clients a spread on top of the spread provided by the inter-broker dealers. Because of the volume requirement, the clients of a dealer normally cannot deal directly with exchanges or inter-dealer brokers. Similarly, due to volume requirements, retail clients cannot typically gain direct access either to inter-dealer brokers or to dealers.

Today, financial markets are becoming increasingly decentralized. Competing exchanges have sprung up to provide increased trading liquidity in addition to the market stalwarts, such as NYSE and AMEX.

Following the advances in computer technology, the networks are flattening, and exchanges and inter-dealer brokers are gradually giving way to electronic communication networks (ECNs), also known as "liquidity pools." ECNs employ sophisticated algorithms to quickly transmit orders and to optimally match buyers and sellers. In "dark" liquidity pools, trader identities and orders remain anonymous.

Island is one of the largest ECNs, which traded about 10 percent of NASDAQ's volume in 2002. On Island, all market participants can post their limit orders anonymously. Biais, Bisiere and Spatt (2003) find that the higher the liquidity on NASDAQ, the higher the liquidity on Island, but the reverse does not necessarily hold. Automated Trading Desk, LLC (ATD) is an example of a dark pool. The customers of the pool do not see the identities or the market depth of their peers, ensuring anonymous liquidity. ATD algorithms further screen for disruptive behaviors such as spread manipulation. The identified culprits are financially penalized for inappropriate behavior.

Figure 2.4 illustrates the resulting "distributed" nature of a typical modern network incorporating ECNs and dark pool structures. The lines connecting the network participants indicate possible dealing routes. Typically, only exchanges, ECNs, dark pools, broker-dealers, and retail brokerages have the ability to clear and settle the transactions, although

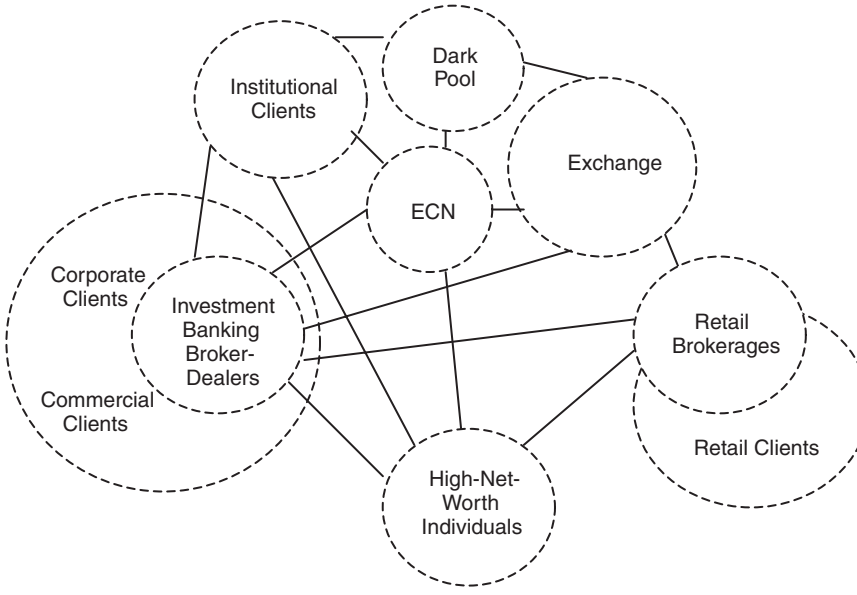

FIGURE 2.4 Contemporary trading networks.

selected institutional clients, such as Chicago-based Citadel, have recently acquired broker-dealer arms of investment banks and are now able to clear all the trades in-house.

EVOLUTION OF TRADING METHODOLOGY

One of the earlier techniques that became popular with many traders was technical analysis. Technical analysts sought to identify recurring patterns in security prices. Many techniques used in technical analysis measure current price levels relative to the rolling moving average of the price, or a combination of the moving average and standard deviation of the price. For example, a technical analysis technique known as moving average convergence divergence (MACD) uses three exponential moving averages to generate trading signals. Advanced technical analysts may look at security prices in conjunction with current market events or general market conditions to obtain a fuller idea of where the prices may be moving next.

Technical analysis prospered through the first half of the 20th century, when trading technology was in its telegraph and pneumatic-tube stages and the trading complexity of major securities was considerably lower than it is today. The inability to transmit information quickly limited the number of shares that changed hands, curtailed the pace at which information was incorporated into prices, and allowed charts to display latent supply and demand of securities. The previous day's trades appeared in the next morning's newspaper and were often sufficient for technical analysts to successfully infer future movement of the prices based on published information. In post-WWII decades, when trading technology began to develop considerably, technical analysis developed into a self-fulfilling prophecy.

If, for example, enough people believed that a "head-and-shoulders" pattern would be followed by a steep sell-off in a particular instrument, all the believers would place sell orders following a head-and-shoulders pattern, thus indeed realizing the prediction. Subsequently, institutional investors began modeling technical patterns using powerful computer technology, and trading them away before they became apparent to the naked eye. By now, technical analysis at low frequencies, such as daily or weekly intervals, is marginalized to work only for the smallest, least liquid securities, which are traded at very low frequencies—once or twice per day or even per week. However, several researchers find that technical analysis still has legs: Brock, Lakonishok, and LeBaron (1992) find that moving averages can predict future abnormal returns, while Aldridge (2009a) shows

that moving averages, "stochastics" and relative strength indicators (RSI) may succeed in generating profitable trading signals on intra-day data sampled at hourly intervals.

In a way, technical analysis was a precursor of modern microstructure theory. Even though market microstructure applies at a much higher frequency and with a much higher degree of sophistication than technical analysis, both market microstructure and technical analysis work to infer market supply and demand from past price movements. Much of the contemporary high-frequency trading is based on detecting latent market information from the minute changes in the most recent price movements. Not many of the predefined technical patterns, however, work consistently in the high-frequency environment. Instead, high-frequency trading models are built on probability-driven econometric inferences, often incorporating fundamental analysis.

Fundamental analysis originated in equities, when traders noticed that future cash flows, such as dividends, affected market price levels. The cash flows were then discounted back to the present to obtain the fair present market value of the security. Graham and Dodd (1934) were one of the earliest purveyors of the methodology and their approach is still popular. Over the years, the term *fundamental analysis* expanded to include pricing of securities with no obvious cash flows based on expected economic variables. For example, fundamental determination of exchange rates today implies equilibrium valuation of the rates based on macroeconomic theories.

Fundamental analysis developed through much of the 20th century. Today, fundamental analysis refers to trading on the expectation that the prices will move to the level predicted by supply and demand relationships, the fundamentals of economic theory. In equities, microeconomic models apply; equity prices are still most often determined as present values of future cash flows. In foreign exchange, macroeconomic models are most prevalent; the models specify expected price levels using information about inflation, trade balances of different countries, and other macroeconomic variables. Derivatives are traded fundamentally through advanced econometric models that incorporate statistical properties of price movements of underlying instruments. Fundamental commodities trading analyzes and matches available supply and demand.

Various facets of the fundamental analysis are active inputs into many high-frequency trading models, alongside market microstructure. For example, event arbitrage consists of trading the momentum response accompanying the price adjustment of the security in response to new fundamental information. The date and time of the occurrence of the news event is typically known in advance, and the content of the news is usually revealed at the time of the news announcement. In high-frequency event

arbitrage, fundamental analysis can be used to forecast the fundamental value of the economic variable to be announced, in order to further refine the high-frequency process.

Technical and fundamental analyses coexisted through much of the 20th century, when an influx of the new breed of traders armed with advanced degrees in physics and statistics arrived on Wall Street. These warriors, dubbed quants, developed advanced mathematical models that often had little to do with the traditional old-school fundamental and technical thinking. The new quant models gave rise to "quant trading," a mathematical model–fueled trading methodology that was a radical departure from established technical and fundamental trading styles. "Statistical arbitrage" strategies (*stat-arb* for short) became the new stars in the money-making arena. As the news of great stat-arb performances spread, their techniques became widely popular, and the constant innovation arms race ensued; the people who kept ahead of the pack were likely to reap the highest gains.

The most obvious aspect of competition was speed. Whoever was able to run a quant model the fastest was the first to identify and trade upon a market inefficiency and was the one to capture the biggest gain. To increase trading speed, traders began to rely on fast computers to make and execute trading decisions. Technological progress enabled exchanges to adapt to the new technology-driven culture and offer docking convenient for trading. Computerized trading became known as "systematic trading" after the computer systems that processed run-time data and made and executed buy-and-sell decisions.

High-frequency trading developed in the 1990s in response to advances in computer technology and the adoption of the new technology by the exchanges. From the original rudimentary order processing to the current state-of-the-art all-inclusive trading systems, high-frequency trading has evolved into a billion-dollar industry.

To ensure optimal execution of systematic trading, algorithms were designed to mimic established execution strategies of traditional traders. To this day, the term "algorithmic trading" usually refers to the systematic execution process—that is, the optimization of buy-and-sell decisions once these buy-and-sell decisions were made by another part of the systematic trading process or by a human portfolio manager. Algorithmic trading may determine how to process an order given current market conditions: whether to execute the order aggressively (on a price close to the market price) or passively (on a limit price far removed from the current market price), in one trade or split into several smaller "packets." As mentioned previously, algorithmic trading does not usually make portfolio allocation decisions; the decisions about when to buy or sell which securities are assumed to be exogenous.

High-frequency trading became a trading methodology defined as quantitative analysis embedded in computer systems processing data and making trading decisions at high speeds and keeping no positions overnight.

The advances in computer technology over the past decades have enabled fully automated high-frequency trading, fueling the profitability of trading desks and generating interest in pushing the technology even further. Trading desks seized upon cost savings realized from replacing expensive trader headcount with less expensive trading algorithms along with other advanced computer technology. Immediacy and accuracy of execution and lack of hesitation offered by machines as compared with human traders have also played a significant role in banks' decisions to switch away from traditional trading to systematic operations. Lack of overnight positions has translated into immediate savings due to reduction in overnight position carry costs, a particular issue in crisis-driven tight lending conditions or high-interest environments.

Banks also developed and adopted high-frequency functionality in response to demand from buy-side investors. Institutional investors, in turn, have been encouraged to practice high-frequency trading by the influx of capital following shorter lock-ups and daily disclosure to investors. Both institutional and retail investors found that investment products based on quantitative intra-day trading have little correlation with traditional buy-and-hold strategies, adding pure return, or alpha, to their portfolios.

As computer technology develops further and drops in price, high-frequency systems are bound to take on an even more active role. Special care should be taken, however, to distinguish high-frequency trading from electronic trading, algorithmic trading, and systematic trading. Figure 2.5 illustrates a schematic difference between high-frequency, systematic, and traditional long-term investing styles.

Electronic trading refers to the ability to transmit the orders electronically as opposed to telephone, mail, or in person. Since most orders in today's financial markets are transmitted via computer networks, the term electronic trading is rapidly becoming obsolete.

Algorithmic trading is more complex than electronic trading and can refer to a variety of algorithms spanning order-execution processes as well as high-frequency portfolio allocation decisions. The execution algorithms are designed to optimize trading execution once the buy-and-sell decisions have been made elsewhere. Algorithmic execution makes decisions about the best way to route the order to the exchange, the best point in time to execute a submitted order if the order is not required to be executed immediately, and the best sequence of sizes in which the order should be optimally processed. Algorithms generating high-frequency trading signals make portfolio allocation decisions and decisions to enter or close a

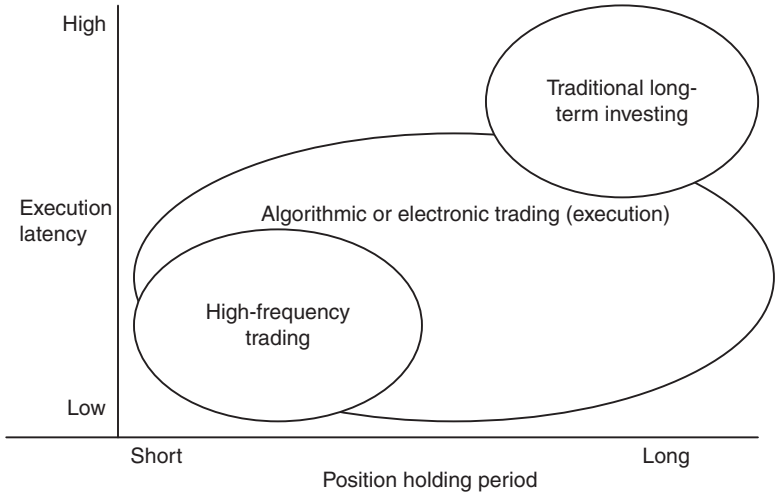

FIGURE 2.5 High-frequency trading versus algorithmic (systematic) trading and traditional long-term investing.

position in a particular security. For example, algorithmic execution may determine that a received order to buy 1,000,000 shares of IBM is best handled using increments of 100 share lots to prevent a sudden run-up in the price. The decision fed to the execution algorithm, however, may or may not be high-frequency. An algorithm deployed to generate high-frequency trading signals, on the other hand, would generate the decision to buy the 1,000,000 shares of IBM. The high-frequency signals would then be passed on to the execution algorithm that would determine the optimal timing and routing of the order.

Successful implementation of high-frequency trading requires both types of algorithms: those generating high-frequency trading signals and those optimizing execution of trading decisions. Algorithms designed for generation of trading signals tend to be much more complex than those focusing on optimization of execution. Much of this book is devoted to algorithms used to generate high-frequency trading signals. Common algorithms used to optimize trade execution in algorithmic trading are discussed in detail in Chapter 18.

The intent of algorithmic execution is illustrated by the results of a TRADE Group survey. Figure 2.6 shows the full spectrum of responses from the TRADE survey. The proportion of buy-side traders using algorithms in their trading increased from 9 percent in 2008 to 26 percent in 2009, with algorithms at least partially managing over 40 percent of the

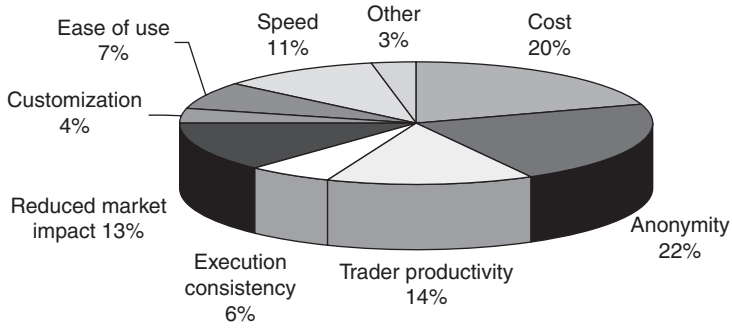

FIGURE 2.6 Reasons for using algorithms in trading.
Source: The TRADE Annual Algorithmic Survey.

total order flow, according to the 2009 Annual Algorithmic Trading Survey conducted by the TRADE Group. In addition to the previously mentioned factors related to adoption of algorithmic trading, such as productivity and accuracy of traders, the buy-side managers also reported their use of the algorithms to be driven by the anonymity of execution that the algorithmic trading permits. Stealth execution allows large investors to hide their trading intentions from other market participants, thus deflecting the possibilities of order poaching and increasing overall profitability.

Systematic trading refers to computer-driven trading positions that may be held a month or a day or a minute and therefore may or may not be high-frequency. An example of systematic trading is a computer program that runs daily, weekly, or even monthly; accepts daily closing prices; outputs portfolio allocation matrices; and places buy-and-sell orders. Such a system is not a high-frequency system.

True high-frequency trading systems make a full range of decisions, from identification of underpriced or overpriced securities, through optimal portfolio allocation, to best execution. The distinguishing characteristic of high-frequency trading is the short position holding times, one day or shorter in duration, usually with no positions held overnight. Because of their rapid execution nature, most high-frequency trading systems are fully systematic and are also examples of systematic and algorithmic trading. All systematic and algorithmic trading platforms, however, are not high-frequency.

Ability to execute a security order algorithmically is a prerequisite for high-frequency trading in a given security. As discussed in Chapter 4, some markets are not yet suitable for high-frequency trading, inasmuch as most trading in these markets is performed over the counter (OTC). According to research conducted by Aite Group, equities are the most algorithmically

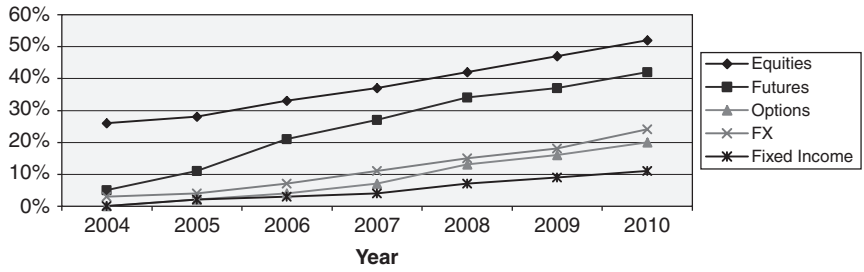

FIGURE 2.7 Adoption of algorithmic execution by asset class. *Source:* Aite Group.

executed asset class, with over 50 percent of the total volume of equities expected to be handled by algorithms by 2010. As Figure 2.7 shows, equities are closely followed by futures. Advances in algorithmic execution of foreign exchange, options, and fixed income, however, have been less visible. As illustrated in Figure 2.7, the lag of fixed income instruments can be explained by the relative tardiness of electronic trading development for them, given that many of them are traded OTC and are difficult to synchronize as a result.

While research dedicated to the performance of high-frequency trading is scarce, due to the unavailability of system performance data relative to data on long-term buy-and-hold strategies, anecdotal evidence suggests that most computer-driven strategies are high-frequency strategies. Systematic and algorithmic trading naturally lends itself to trading applications demanding high speed and precision of execution, as well as high-frequency analysis of volumes of tick data. Systematic trading, in turn, has been shown to outperform human-led trading along several key metrics. Aldridge (2009b), for example, shows that systematic funds consistently outperform traditional trading operations when performance is measured by Jensen's alpha (Jensen, 1968), a metric of returns designed to measure the unique skill of trading by abstracting performance from broad market influences. Aldridge (2009b) also shows that the systematic funds outperform nonsystematic funds in raw returns in times of crisis. That finding can be attributed to the lack of emotion inherent in systematic trading strategies as compared with emotion-driven human traders.

CHAPTER 3

Overview of the Business of High-Frequency Trading

According to the Technology and High-Frequency Trading Survey conducted by FINalternatives.com, a leading hedge fund publication, in June 2009, 90 percent of the 201 asset managers surveyed thought that high-frequency trading had a bright future. In comparison, only 50 percent believed that the investment management industry has favorable prospects, and only 42 percent considered the U.S. economy as having a positive outlook.

The same respondents identified the following key characteristics of high-frequency trading:

- Tick-by-tick data processing
- High capital turnover
- Intra-day entry and exit of positions
- Algorithmic trading

Tick-by-tick data processing and high capital turnover define much of high-frequency trading. Identification of small changes in the quote stream sends rapid-fire signals to open and close positions. The term "high-frequency" itself refers to fast entry and exit of trading positions. An overwhelming 86 percent of respondents in the FINalternatives survey thought that the term "high-frequency trading" referred strictly to holding periods of one day or less. (See Figure 3.1.)

Intra-day position management deployed in high-frequency trading results in considerable savings of overnight position carrying costs. The carry is the cost of holding a margined position through the night; it is usually

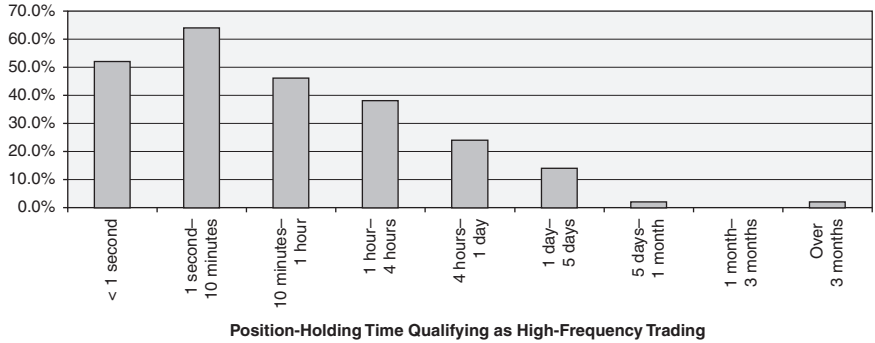

FIGURE 3.1 Details of the FINalternatives July 2009 Technology and Trading Survey responses to the question "What position-holding time qualifies as high-frequency trading?"

computed on the margin portion of account holdings after the close of the North American trading sessions. Overnight carry charges can substantially cut into the trading bottom line in periods of tight lending or high interest rates.

Closing down positions at the end of each trading day also reduces the risk exposure resulting from the passive overnight positions. Smaller risk exposure again results in considerable risk-adjusted savings.

Finally, algorithmic trading is a necessary component of high-frequency trading platforms. Evaluating every tick of data separated by milliseconds, processing market information, and making trading decisions in a consistent continuous manner is not well suited for a human brain. Affordable algorithms, on the other hand, can make fast, efficient, and emotionless decisions, making algorithmic trading a requirement in high-frequency operations.

COMPARISON WITH TRADITIONAL APPROACHES TO TRADING

High-frequency trading is a relatively novel approach to investing. As a result, confusion and questions often arise as to how high-frequency trading relates to other, older investment styles. This section addresses these issues.

Technical, Fundamental, or Quant?

As discussed in Chapter 2, technical trading is based on technical analysis, the objective of which is to identify persistent price change patterns.

Technical analysis may suggest that a price is too high or too low given its past trajectory. Technical trading would then imply buying a security the price of which was deemed too low in technical analysis, and selling a security the price of which was deemed too high. Technical analysis can be applied at any frequency and can be perfectly suitable in high-frequency trading models.

Fundamental trading is based on fundamental analysis. Fundamental analysis derives the equilibrium price levels, given available information and economic equilibrium theories. As with technical trading, fundamental trading entails buying a security the price of which was deemed too low relative to its analytically determined fundamental value and selling a security the price of which is considered too high. Like technical trading, fundamental trading can also be applied at any frequency, although price formation or microstructure effects may result in price anomalies at ultra-high frequencies.

Finally, *quant* (short for quantitative) trading refers to making portfolio allocation decisions based on scientific principles. These principles may be fundamental or technical or can be based on simple statistical relationships. The main difference between quant analyses and technical or fundamental styles is that quants use little or no discretionary judgments, whereas fundamental analysts may exercise discretion in rating the management of the company, for example, and technical analysts may "see" various shapes appearing in the charts. Given the availability of data, quant analysis can be run in high-frequency settings.

Quant frameworks are best suited to high-frequency trading for one simple reason: high-frequency generation of orders leaves little time for traders to make subjective nonquantitative decisions and input them into the system. Aside from their inability to incorporate discretionary inputs, high-frequency trading systems can run on quant analyses based on both technical and fundamental models.

Algorithmic, Systematic, Electronic, or Low-Latency?

Much confusion exists among the terms "high-frequency trading" and "algorithmic," "systematic," "electronic," and "low-latency" trading.

High-frequency trading refers to fast reallocation or turnover of trading capital. To ensure that such reallocation is feasible, most high-frequency trading systems are built as algorithmic trading systems that use complex computer algorithms to analyze quote data, make trading decisions, and optimize trade execution. All algorithms are run electronically and, therefore, automatically fall into the "electronic trading" subset.

While all algorithmic trading qualifies as electronic trading, the reverse does not have to be the case; many electronic trading systems only route

orders that may or may not be placed algorithmically. Similarly, while most high-frequency trading systems are algorithmic, many algorithms are not high-frequency.

"Low-latency trading" is another term that gets confused with "high-frequency trading." In practice, "low-latency" refers to the speed of executing an order that may or may not have been placed by a high-frequency system; "low-latency trading" refers to the ability to quickly route and execute orders irrespective of their position-holding time. High-frequency, on the other hand, refers to the fast turnover of capital that may require low-latency execution capability. Low-latency can be a trading strategy in its own right when the high speed of execution is used to arbitrage instantaneous price differences on the same security at different exchanges.

MARKET PARTICIPANTS

Competitors

High-frequency trading firms compete with other investment management firms for quick access to market inefficiencies, for access to trading and operations capital, and for recruiting of talented trading strategists. Competitive investment management firms may be proprietary trading divisions of investment banks, hedge funds, and independent proprietary trading operations. The largest independent firms deploying high-frequency strategies are DE Shaw, Tower Research Capital, and Renaissance Technologies.

Investors

Investors in high-frequency trading include fund of funds aiming to diversify their portfolios, hedge funds eager to add new strategies to their existing mix, and private equity firms seeing a sustainable opportunity to create wealth. Most investment banks offer leverage through their "prime" services.

Services and Technology Providers

Like any business, a high-frequency trading operation requires specific support services. This section identifies the most common and, in many cases, critical providers to the high-frequency business community.

Electronic Execution High-frequency trading practitioners rely on their executing brokers and electronic communication networks (ECNs)

to quickly route and execute their trades. Goldman Sachs and Credit Suisse are often cited as broker-dealers dominating electronic execution. Today's major ECN players are ICAP and Thomson/Reuters, along with several others.

Custody and Clearing In addition to providing connectivity to exchanges, broker-dealers typically offer special "prime" services that include safekeeping of trading capital (known as custody) and trade reconciliation (known as clearing). Both custody and clearing involve a certain degree of risk. In a custody arrangement, the broker-dealer takes the responsibility for the assets, whereas in clearing, the broker-dealer may act as insurance against the default of trading counterparties. Transaction cost mark-ups compensate broker-dealers for their custody and clearing efforts and risk.

Software High-frequency trading operations deploy the following software that may or may not be built in-house:

- Computerized generation of trading signals refers to the core functionality of a high-frequency trading system; the generator accepts and processes tick data, generates portfolio allocations and trade signals, and records profit and loss (P&L).
- Computer-aided analysis represents financial modeling software deployed by high-frequency trading operations to build new trading models. MatLab and R have emerged as the industry's most popular quantitative modeling choices.
- Internet-wide information-gathering software facilitates high-frequency fundamental pricing of securities. Promptly capturing rumors and news announcements enhances forecasts of short-term price moves. Thomson/Reuters has a range of products that deliver real-time news in a machine-readable format.
- Trading software incorporates optimal execution algorithms for achieving the best execution price within a given time interval through timing of trades, decisions on market aggressiveness, and sizing orders into optimal lots. New York–based MarketFactory provides a suite of software tools to help automated traders get an extra edge in the market, help their models scale, increase their fill ratios, reduce slippage, and thereby improve profitability (P&L). Chapter 18 discusses optimization of execution.
- Run-time risk management applications ensure that the system stays within prespecified behavioral and P&L bounds. Such applications may also be known as system-monitoring and fault-tolerance software.

- Mobile applications suitable for monitoring performance of high-frequency trading systems alert administration of any issues.
- Real-time third-party research can stream advanced information and forecasts.

Legal, Accounting, and Other Professional Services Like any business in the financial sector, high-frequency trading needs to make sure that "all i's are dotted and all t's are crossed" in the legal and accounting departments. Qualified legal and accounting assistance is therefore indispensable for building a capable operation.

Government

In terms of government regulation, high-frequency trading falls under the same umbrella as day trading. As such, the industry has to abide by common trading rules—for example, no insider trading is allowed. An unsuccessful attempt to introduce additional regulation through a surcharge on transaction costs was made in February 2009.

OPERATING MODEL

Overview

Surprisingly little has been published on the best practices to implement high-frequency trading systems. This chapter presents an overview of the business of high-frequency trading, complete with information on planning the rollout of the system and the capital required to develop and deploy a profitable operation.

Three main components, shown in Figure 3.2, make up the business cycle:

- Highly quantitative, econometric models that forecast short-term price moves based on contemporary market conditions
- Advanced computer systems built to quickly execute the complex econometric models
- Capital applied and monitored within risk and cost-management frameworks that are cautious and precise

The main difference between traditional investment management and high-frequency trading is that the increased frequency of opening and closing positions in various securities allows the trading systems to profitably

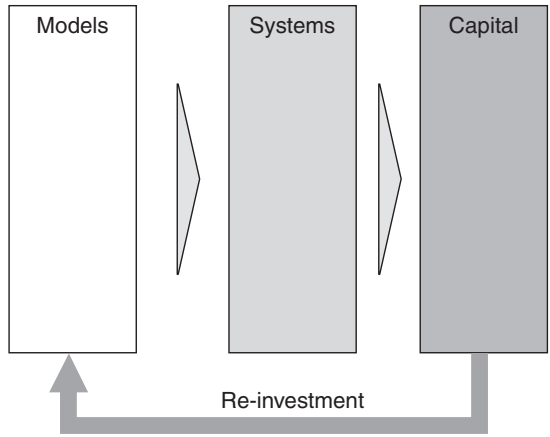

FIGURE 3.2 Overview of the development cycle of a high-frequency trading process.

capture small deviations in securities prices. When small gains are booked repeatedly throughout the day, the end-of-day result is a reasonable gain.

Developing a high-frequency trading business follows a process unusual for most traditional financial institutions. Designing new high-frequency trading strategies is very costly; executing and monitoring finished high-frequency products costs close to nothing. By contrast, traditional proprietary trading businesses incur fixed costs from the moment an experienced senior trader with a proven track record begins running the trading desk and training promising young apprentices, through the time when the trained apprentices replace their masters.

Figure 3.3 illustrates the cost curves for rolling out computerized and traditional trading systems. The cost of traditional trading remains fairly constant through time. With the exception of trader "burn-outs" necessitating hiring and training new trader staff, costs of staffing the traditional trading desk do not change. Developing computerized trading systems, however, requires an up-front investment that is costly in terms of labor and time. One successful trading system takes on average 18 months to develop. The costs of computerized trading decline as the system moves into production, ultimately requiring a small support staff that typically includes a dedicated systems engineer and a performance monitoring agent. Both the systems engineer and a monitoring agent can be responsible for several trading systems simultaneously, driving the costs closer to zero.

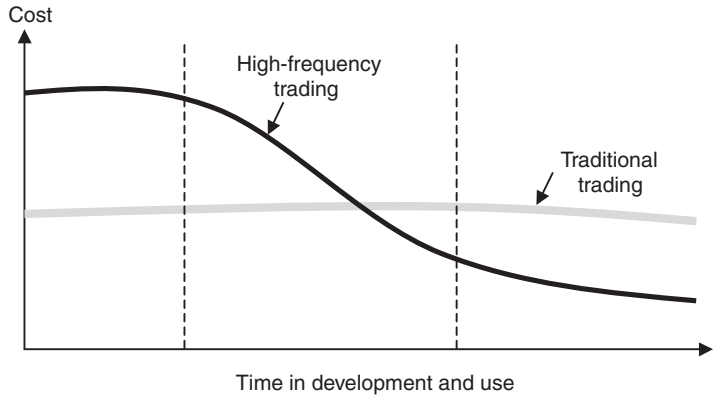

FIGURE 3.3 The economics of high-frequency versus traditional trading businesses.

Model Development

The development of a high-frequency trading business begins with the development of the econometric models that document persistent relationships among securities. These relationships are then tested on lengthy spans of tick-by-tick data to verify the forecasting validity in various market situations. This process of model verification is referred to as "back-testing." Standard back-testing practices require that the tests be run on data of at least two years in duration. The typical modeling process is illustrated in Figure 3.4.

System Implementation

The models are often built in computer languages such as MatLab that provide a wide range of modeling tools but may not be suited perfectly for high-speed applications. Thus, once the econometric relationships are ascertained, the relationships are programmed for execution in a fast computer language such as C++. Subsequently, the systems are tested in "paper-trading" with make-believe capital to ensure that the systems work as intended and any problems (known as "bugs") are identified and fixed. Once the systems are indeed performing as expected, they are switched to live capital, where they are closely monitored to ensure proper execution and profitability.

High-frequency execution systems tend to be complex entities that detect and react to a variety of market conditions. Figure 3.5 documents the standard workflow of a high-frequency trading system operating on live capital.

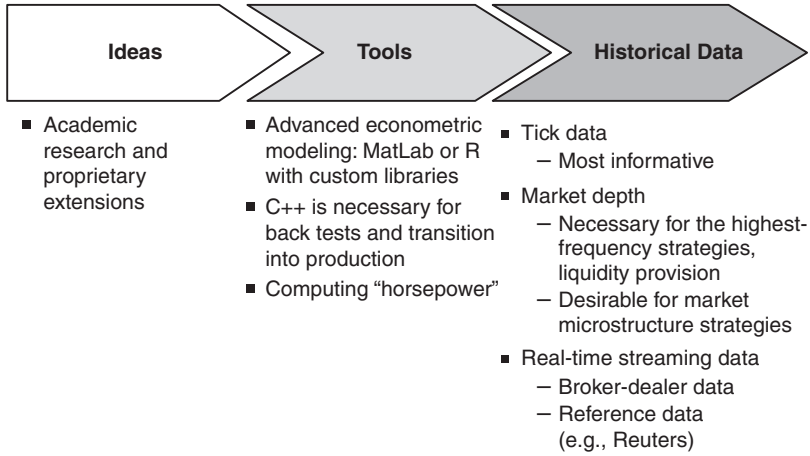

FIGURE 3.4 The process for development of econometric models for high-frequency trading.

As Figure 3.5 shows, a typical high-frequency trading system in production encompasses six major tasks, all of which are interrelated and operate in unison.

- Block A receives and archives real-time tick data on securities of interest.
- Block B applies back-tested econometric models to the tick data obtained in Block A.

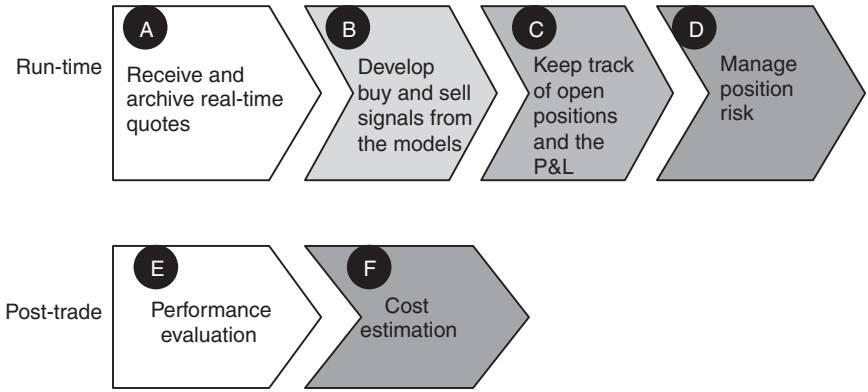

FIGURE 3.5 Run-time and post-trade workflows of a typical high-frequency operation.

- Block C sends orders and keeps track of open positions and P&L values.
- Block D monitors run-time trading behavior, compares it with predefined parameters, and uses the observations to manage the run-time trading risk.
- Block E evaluates trading performance relative to a host of predetermined benchmarks.
- Block F ensures that the trading costs incurred during execution are within acceptable ranges.

Each of the six functional blocks is built with an independent alert system that notifies the monitoring personnel of any problems or other unusual patterns, including unforeseen market behavior, disruptions in the market data process, unexpectedly high trading costs, failure to transmit orders or to receive acknowledgments, and the like.

Given the complexity of the execution process, the development of the six tasks is hardly trivial. It is most successfully approached using a continuous iterative implementation cycle whereby the execution capability is gradually expanded in scope. Figure 3.6 illustrates a

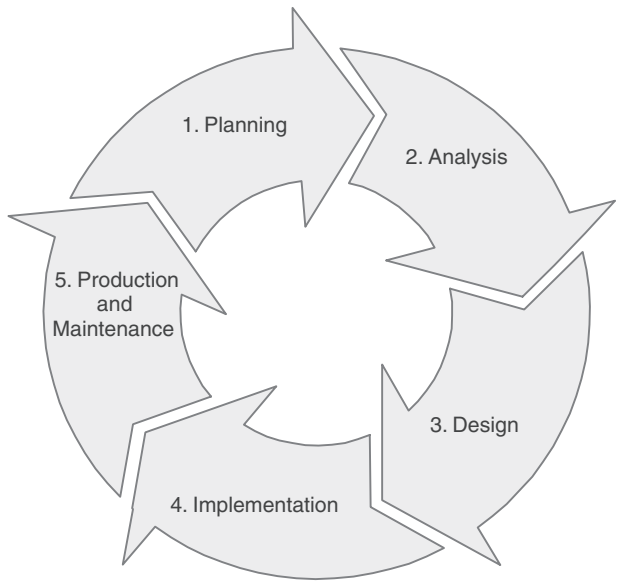

FIGURE 3.6 A typical implementation process of run-time high-frequency trading systems.

standard approach for designing and implementing high-frequency trading systems.

The implementation of the run-time components of the high-frequency trading systems begins with careful planning that establishes core functionalities and budgets for the system. Once the planning phase is complete, the process moves into the analysis phase, where the scope of the initial iteration of the project is determined, feedback from all the relevant stakeholders is aggregated, and senior management signs off on the high-level project specifications. The next stage—the design—breaks the system into manageable modules, outlines the functionality of each module, and specifies the desired behavior. In the following stage (known as the implementation stage) the modules are programmed by teams of dedicated software engineers and are tested against specifications determined in the design stage. Once behavior is found to be satisfactory, the project moves into the production and maintenance phase, where deviations from the desired behavior are addressed. When the project is considered stable, a new iteration of planning begins to incorporate enhancements and other desired features into the project. See Chapter 16 for the details of best practices in design and implementation of the high-frequency trading systems.

Trading Platform

Most high-frequency trading systems today are built to be "platform-independent"—that is, to incorporate flexible interfaces to multiple broker-dealers, ECNs, and even exchanges. The independence is accomplished through the use of FIX language, a special sequence of codes optimized for exchange of financial trading data. With FIX, at a flip of a switch the trading routing can be changed from one executing broker to another or to several brokers simultaneously.

Risk Management

Competent risk management is key to the success of any high-frequency trading system. A seemingly harmless glitch in the code, market data, market conditions, or the like can throw off the trading dynamic and result in large losses. The objective of risk management is to assess the extent of potential damages and to create infrastructure to mitigate damaging conditions during the system run-time. Risk management is discussed in detail in Chapter 17.

ECONOMICS

Revenue Driven by Leverage and the Sharpe Ratio

For the business to remain viable, revenues must be sufficient to cover expenses of running the business. The business of high-frequency trading is no exception. Accounting for trading costs, a portion of revenues (80 percent on average) is paid out to investors in the trading operation, leaving the management with "performance fees." In addition, the management may collect "management fees," which are a fixed percentage of assets designated to cover administrative expenses of the trading business regardless of performance.

Even the most cost-effective high-frequency trading operation has employee salaries, administrative services, and trading costs, as well as legal and compliance expenses. The expenses easily run $100,000 per average employee in base salaries and benefits, not considering a negotiated incentive structure; this is in addition to the fixed cost overhead of office space and related expenses.

To compensate for these expenses, what is the minimum level of return on capital that a high-frequency manager should generate each year to remain a going concern? The answer depends on the leverage of the trading platform. Consider a trading operation with five employees. Fixed expenses of such a business may total $600,000 per year, including salaries and office expenses. Suppose further that the business charges a 0.5 percent management fee on its capital equity and a 20 percent incentive fee on returns it produces above the previous high value, or "watermark." The minimum capital/return conditions for breaking even for such a trading business under different leverage situations are shown in Figure 3.7. As illustrated there, a $20 million unlevered fund with five employees needs to generate at least a 12 percent return per year in order to break even, while the same fund levered 500 percent (borrowing four times its investment equity) needs to generate just 3 percent per year to survive.

Conventional wisdom, however, tells us that leverage increases the risk of losses. To evaluate the risk associated with higher leverage, we next consider the risks of losing at least 20 percent of the capital equity of the business. As shown in Figures 3.7 and 3.8, the probability of severe losses is much more dependent on the aggregate Sharpe ratio of the trading strategies than it is on the leverage used by the hedge fund.

The Sharpe ratio of a high-frequency trading strategy, discussed in detail in Chapter 5, is the ratio of the average annualized return of the strategy to the annualized standard deviation of the strategy's returns. The higher the Sharpe ratio, the lower the probability of severe losses. As Figure 3.8

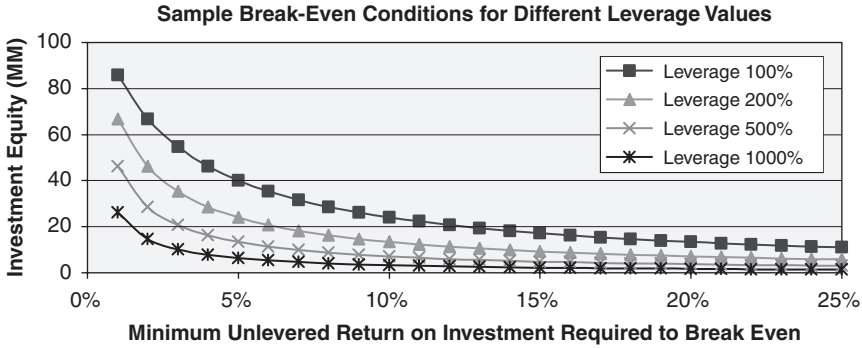

FIGURE 3.7 Sample break-even conditions for a high-frequency trading business employing five workers.

shows, an annualized Sharpe ratio of 0.5 for an unlevered trading operation expecting to make 20 percent per year translates into a 15 percent risk of losing at least one-fifth of the fund's equity capital. Levering the same fund nine-fold only doubles the risk of losing at least one-fifth of equity. In comparison, the annualized Sharpe ratio of 2.0 for an unlevered trading business expecting to make 20 percent per year translates into a miniscule 0.1 percent risk of losing at least one-fifth of the equity capital, and levering the same trading business only increases the risk of losing at least one-fifth of the fund to 1.5 percent, as shown in Figure 3.9.

Furthermore, as Figures 3.8 and 3.9 show, for any given Sharpe ratio, the likelihood of severe losses actually increases with increasing expected returns, reflecting the wider dispersion of returns. From an investor's perspective, a 5 percent expected return with a Sharpe over 2 is much preferable to a 35 percent expected return with a low Sharpe of, say, 0.5.

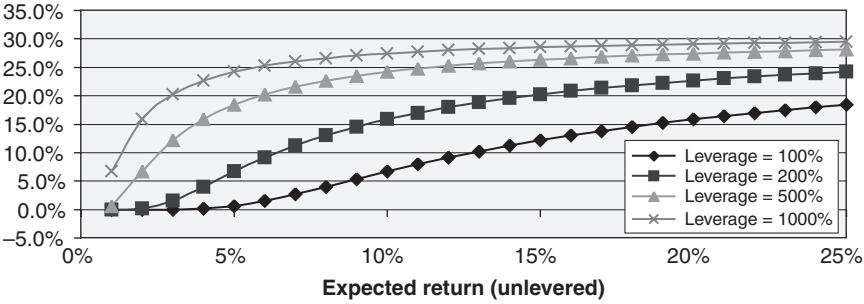

FIGURE 3.8 Probability of losing 20 percent or more of the investment capital equity running strategies with Sharpe ratio of 0.5.

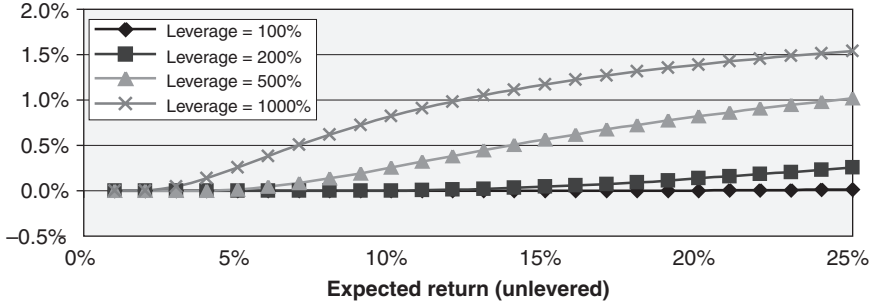

FIGURE 3.9 Probability of losing 20 percent or more of the investment capital equity running strategies with Sharpe ratio of 2.

In summary, a high-frequency trading operation is more likely to survive and prosper if it has leverage and high Sharpe ratios. High leverage increases the likelihood of covering costs, and the high Sharpe ratio reduces the risk of a catastrophic loss.

Transparent and Latent Costs

Understanding the cost structure of trading becomes especially important in high-frequency settings, where the sheer number of transactions can eliminate gains. As Chapter 19 notes, in addition to readily available or transparent costs of trading, high-frequency operations should account for a wide range of unobservable, or latent, costs. For details on various cost types, please see Chapter 19.

Staffing

Initial development of high-frequency systems is both risky and pricey, and the staff required to design trading models should understand PhD-level quantitative research in finance and econometrics. In addition, programming staff should be experienced enough to handle complex issues of system inter-operability, computer security, and algorithmic efficiency.

CAPITALIZING A HIGH-FREQUENCY TRADING BUSINESS

Capital used for trading in high-frequency operations comprises equity and leverage. The equity normally comprises contributions from the founders

of the firm, private equity capital, investor capital, or capital of the parent company. Leverage is debt that can be obtained through a simple bank loan or margin lending or through other loans offered by broker-dealers.

CONCLUSION

Developing a high-frequency business involves challenges that include issues surrounding the "gray box" or "black box" nature of many systems. The low transparency of fast and complex algorithm decisions may frustrate human traders accustomed to having a thorough understanding of decisions prior to placing trades. High trading frequency may make it difficult to spot a malfunction with the algorithm. And we will not even go into the whole issue of computer security!

Despite the complexity of successfully implementing high-frequency operations, the end results make it all worthwhile. The deployment and execution costs decrease considerably with time, leaving the profit-generating engines operating consistently, with no emotion, sickness, or other human factors. High-frequency trading is particularly well suited for markets where traditional long-term investment strategies may not work at all; high geopolitical and economic uncertainty may render many such traditional investing venues unprofitable. Well-designed and -executed high-frequency systems, capitalizing on multiple short-term moves of security prices, are capable of generating solid profitability in highly uncertain markets.

CHAPTER 4

Financial Markets Suitable for High-Frequency Trading

A wide range of securities and numerous market conditions fit the profile for trading at high frequencies. Some securities markets, however, are more appropriate than others. This chapter examines the topic of market suitability for high-frequency trading.

To be appropriate for this type of trading, two requirements must be met: the ability to quickly move in and out of positions and sufficient market volatility to ensure that changes in prices exceed transaction costs. The volatilities of different markets have been shown to be highly interrelated and dependent on the volume of macroeconomic news reaching the markets. The ability to quickly enter into positions as well as to close them is in turn determined by two factors: market liquidity and availability of electronic execution.

Liquid assets are characterized by readily available supply and demand. Liquid securities such as major foreign exchange pairs are traded 24 hours a day, 5 days a week. Less liquid securities, such as penny stocks, may trade only once every few days. Between trades, the prices on illiquid assets may change substantially, making less liquid securities more risky as compared with more liquid assets.

High-frequency strategies focus on the most liquid securities; a security requiring a holding period of 10 minutes may not be able to find a timely counterparty in illiquid markets. While longer-horizon investors can work with either liquid or illiquid securities, Amihud and Mendelson (1986) show that longer-horizon investors optimally hold less liquid assets. According to these authors, the key issue is the risk/return consideration; longer-term

investors (already impervious to the adverse short-term market moves) will obtain higher average gains by taking on more risk in less liquid investments.

According to Bervas (2006), a perfectly liquid market is the one where the quoted bid or ask price can be achieved irrespective of the quantities traded. Market liquidity depends on the presence of trading counterparties in the market, as well as the counterparties' willingness to trade. The market participants' willingness to trade in turn depends on their risk aversions and expectations of impending price movements, along with other market information.

One way to compare the liquidity of different securities is to use the average daily volume of each security as the measure of liquidity. In terms of daily average trading volume, foreign exchange is the most liquid market, followed by recently issued U.S. Treasury securities; then come equities, options, commodities, and futures. Of the most liquid securities, only spot foreign exchange, equities, options, and futures markets have enabled fully automated execution; the remaining markets still tend to negotiate on a contract-by-contract basis over the counter (OTC), slowing down the trading process. Table 4.1 enumerates current market volumes and execution methods for different securities. As the demand for high-frequency trading increases, the development of electronic trading in the OTC markets may prove highly profitable. Figure 4.1 graphically illustrates optimal trading frequencies for various securities where the optimal trading frequency is a function of available market liquidity. The following sections discuss the pros and cons of high-frequency trading in each security market in detail.

FINANCIAL MARKETS AND THEIR SUITABILITY FOR HIGH-FREQUENCY TRADING

This section discusses the availability of various financial markets for high-frequency trading. As discussed in the first section of this chapter, for a market to be suitable, it must be both liquid and electronic to facilitate the quick turnover of capital. In the following subsections, we consider three key elements of each market:

- Available liquidity
- Electronic trading capability
- Regulatory considerations

TABLE 4.1 Average Daily Volume and Dominant Execution Method for Major Security Classes

Market	Average Daily Volume (Billions)	Dominant Execution Method
Foreign exchange swaps*	1,714.4	OTC
Foreign exchange spot*	1,004.9	Electronic
Foreign exchange outright forwards*	361.7	OTC
U.S. Treasury**	570.2	OTC
Agency MBS**	320.1	OTC
Federal agency securities**	83.0	OTC
Municipal**	25.0	OTC
Corporate debt**	24.3	OTC
NYSE***	2.6	Electronic
Options****	1.6	Electronic and OTC

*Information on the global volume of foreign exchange is for April 2007 as reported in the Triennial Central Bank Survey.
**Information on the U.S. debt daily volume is quoted from 2007 data reported by the Securities Industry and Financial Markets Association (SIFMA). By January 2009, in the aftermath of the credit crisis, the average daily volume in U.S. Treasuries decreased to USD 358 billion, Agency MBS volume increased to 358 billion, federal agency securities volume decreased to 75 billion, municipal bonds to 12 billion, and corporate debt to 12 billion.
***The average daily volume is computed for the month of April 2009 from the daily volume reported by the NYSE.
****The trading volume for options is quoted from the average daily volume reported by the Options Clearing Corporation for May 2009.

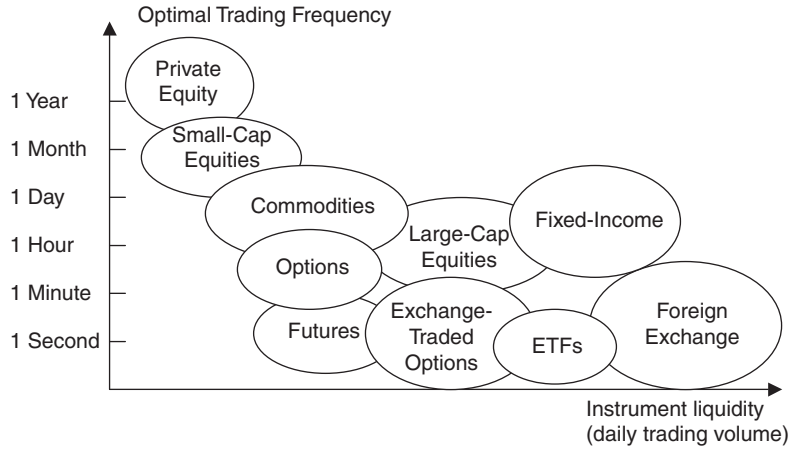

FIGURE 4.1 Optimal trading frequency for various trading instruments, depending on the instrument's liquidity.

Fixed-Income Markets

The fixed-income markets include the interest rate market and the bond market. The interest rate market trades short- and long-term deposits, and the bond market trades publicly issued debt obligations. Interest rate products and bonds are similar in that they both pay fixed or prespecified income to their holders. Aside from their fixed-income quality, bonds and interest rate products exhibit little similarity.

Both interest rate and bond markets use spot, futures, and swap contracts. Spot trading in both interest rate products and bonds implies instantaneous or "on-the-spot" delivery and transfer of possession of the traded security. Futures trading denotes delivery and transfer of possession at a prespecified date. Swap trading is a contractual transfer of cash flows between two parties. Interest rate swaps may specify swapping of a fixed rate for a floating rate; bond swaps refer mostly to a trading strategy whereby the investor sells one bond and buys another at a comparable price, but with different characteristics.

In fixed-income markets, many investors are focused on the product payouts rather than on the prices of the investments themselves. High-frequency traders taking advantage of short-term price deviations win, as do longer-term investors.

Interest Rate Markets The spot interest rate market comprises quotes offered by banks to other banks, and can be known as "spot interest rates," "cash interest rates," or "interbank interest rates." As other financial products, interbank interest rates are quoted as a bid and an ask. A bid interest rate is quoted to banks wanting to make a deposit, whereas the ask quote is offered to banks to take a credit.

The quoted interest rates are not necessarily the rates at which banks lend each other money. The actual lending rate is the quoted interest rate plus a credit spread, where the credit spread is the amount that compensates the lending bank for the risk it takes while lending. The risk of the lending bank in turn depends on the creditworthiness of the borrowing bank. The lower the creditworthiness of the borrowing bank, the higher the risk that the lending bank takes by lending out the money, and the higher the credit spread intended to compensate the lending bank for the risk of lending.

Spot interest rates have fixed maturity periods denominated in days or months. Current maturity periods constitute the following set:

- Overnight: O/N
- The next business day after tomorrow, known as "tomorrow next": T/N
- One week: S/W

- One month: 1M
- Two months: 2M
- Three months: 3M
- Six months: 6M
- Nine months: 9M
- One year: 1Y

Interest rate futures are contracts to buy and sell underlying interest rates in the future. Short-term interest rate futures are more liquid than spot interest rate futures. The liquidity of the interest rate futures market is reflected in the bid-ask spread of the interest rate futures; a bid-ask spread on interest rate futures is on average one-tenth of the bid-ask spread on the underlying spot interest rate.

Interest rate futures are commonly based on the 3-month deposit rate. The actual quotation for a futures bid or ask prices, f_{bid} and f_{ask}, respectively, depends on the annualized bid or ask OTC forward rates, r_{bid} and r_{ask}, as follows:

$$f_{\text{bid}} = 100\left(1 - \frac{r_{\text{bid}}}{100\%}\right)$$

$$f_{\text{ask}} = 100\left(1 - \frac{r_{\text{ask}}}{100\%}\right)$$

A 3-percent forward rate, for example, results in a futures price of 97.00. The forward rates underlying the futures contracts typically mature in three months. The futures contracts usually have four standardized settlements per year—in March, June, September, and December.

Unlike spot interest rates, interest rate futures do not vary according to the creditworthiness of the borrower. Instead of pricing default risk into the rate explicitly, exchanges trading interest rate futures require borrowers to post collateral accounts that reflect the creditworthiness of the borrower.

Swap products are the most populous interest rate category, yet most still trade OTC. Selected swap products have made inroads into electronic trading. CME Group, for example, has created electronic programs for 30-day Fed Funds futures and CBOT 5-year, 10-year, and 30-year interest rate swap futures; 30-day Fed Funds options; 2-year, 5-year and 10-year Treasury note options; and Treasury bond options. As Table 4.2 shows, however, electronic trading volumes of interest rate products remain limited.

Bond Markets Bonds are publicly issued debt obligations. Bonds can be issued by a virtual continuum of organizations ranging from federal governments through local governments to publicly held corporations. Bonds

TABLE 4.2 Daily Dollar Volume in Most Active Interest Rate Products on CME Electronic Trading (Globex) on 6/12/2009 Computed as Average Price Times Total Contract Volume Reported by CME

Instrument	Futures Daily Volume (in USD thousands)
Eurodollar deposits	196,689.9
LIBOR	163.5
30-Year Swap	934.5
5-Year Swap	1,295.2
10-Year Swap	978.6
30-Day Fed Funds	2,917.7

typically pay interest throughout their lifetimes and pay back the principal at the end of the bond contract, known as the maturity of the bond. Bonds can also embed various options, to suit both the needs of the issuer and the needs of the target buyer. For example, a company in the midst of a turmoil may decide to issue bonds that embed an option to convert the bond into the company's stock at some later date. Such a bond type is known as a convertible bond and is designed to give prospective investors the security of preferred redemptions should the company be liquidated; should the company fully recover, the bond gives investors the ability to convert it into equity and thus obtain a higher return in the long run. In addition to risk-averse investors, convertible bonds may attract investors desiring a conservative investment profile at present and a riskier equity profile in the long run.

Despite the advantageous breadth of the bond market, spot bonds are transacted mostly OTC and do not generate a readily observable stream of high-frequency data.

Unlike bonds that can be custom tailored to the buyer's specifications, bond futures contracts are standardized by the exchange and are often electronic. (See Table 4.3.) Bond futures have characteristics similar to those of the interest rate futures. Like interest rate futures, bond futures settle four times a year—in March, June, September, and December. The exact settlement and delivery rules vary from exchange to exchange. Bond futures with the nearer expiry dates are more liquid than their counterparts with longer maturities.

While interest rate futures are based on notional 3-month deposits, bond futures are typically based on government bonds with multiyear maturities. As such, bond futures register less influence from the central banks that issue them. Bonds issued with maturity of less than two years are referred to as "bills," whereas bonds issued with maturity of two to ten years are often called "notes."

TABLE 4.3 Daily Dollar Volume in Most Active Bond Futures Products on CME Electronic Trading (Globex) on 6/12/2009 Computed as Average Price Times Total Contract Volume Reported by CME

Instrument	Futures Daily Volume (in USD thousands)
30-Year U.S. Treasury Bond, Futures	19,486.6
10-Year U.S. Treasury Note, Futures	82,876.5
5-Year U.S. Treasury Note, Futures	31,103.0
2-Year U.S. Treasury Note, Futures	14,187.1

Foreign Exchange Markets

In a nutshell, a foreign exchange rate is a swap of interest rates denominated in different currencies. Foreign exchange trading originated in 1971 when the gold standard collapsed under the heft of U.S. debt. From 1971 until the late 1980s, foreign exchange traded entirely among commercial banks that made deposit arrangements in different currencies. Commercial banks had exclusive access to inter-dealer networks, consisting of loose groups of third-party agents facilitating quick distribution of orders among different commercial banking clients. Investment banks, such as Goldman Sachs, had no direct access to the inter-dealer networks and transacted their foreign exchange trades through commercial banks instead.

In the early 1990s, investment banks were able to gain access to broker-dealer networks. In the late 1990s non-bank companies and non–U.S. investment banks connected directly to the inter-dealer pools. Since 2003, hedge funds and proprietary trading funds have also been granted access to the inter-dealer liquidity. Currently, spot, forward, and swap foreign exchange products trade through this decentralized and unregulated mechanism. Only foreign exchange futures and selected options contracts can be found on exchanges.

The decentralization of foreign exchange trading has had two key consequences: the absence of "one price" and the absence of volume measures.

The absence of a single coherent price at any given time is a direct consequence of decentralization. Different dealers receive different information and price their securities accordingly. The lack of one price can present substantial arbitrage opportunities at high trading frequencies. Another consequence of decentralization is that the market-wide measure of volume at any given time in foreign exchange is not available. To monitor developments in foreign exchange markets, central banks conduct financial institution surveys every three years. These surveys are then aggregated and published by the Bank for International Settlements (BIS).

BIS estimates that the total foreign exchange (FX) market in 2007 had a daily trading volume of $3 trillion. This includes the spot market and

forwards, futures, options, and swaps. The spot market accounts for about 33 percent of the total daily turnover or about $1 trillion. According to BIS Triennial Surveys, the proportion of spot transactions among all FX trades has been decreasing; in 1989, spot represented 59 percent of all FX trades. In 1998, spot accounted for only 40 percent of all FX trades. Of the $2 trillion of daily FX volume that is not spot, $1.7 trillion is contributed by FX swaps.

Some FX futures and options are traded on exchanges. Table 4.4 shows daily electronic trading volumes in most common foreign exchange futures on CME.

Foreign exchange markets profitably accommodate three types of players with distinct goals: high-frequency traders, longer-term investors, and corporations. The main objective of high-frequency traders is to capture small intra-day price changes. The main objective of longer-term investors is to gain from global macro changes. Finally, the main objective of corporate currency managers is usually hedging of cross-border flows against adverse currency movements—for example, a Canadian firm selling in the United States may choose to hedge its revenue stream by purchasing puts on USD/CAD futures. The flows of the three parties can be quite distinct, as Table 4.5 illustrates.

Table 4.5 reports summary statistics for EUR/USD order flows observed by Citibank and sampled at the weekly frequency between January 1993 and July 1999: A) statistics for weekly EUR/USD order flow aggregated across Citibank's corporate, trading, and investing customers; and B) order flows from end-user segments cumulated over a week. The last four columns on the right report autocorrelations i at lag i and p-values for the null that $(i = 0)$. The summary statistics on the order flow data are from Evans and Lyons (2007), who define order flow as the total value of EUR/USD purchases (in USD millions) initiated against Citibank's quotes.

TABLE 4.4 Daily Dollar Volume in Most Active Foreign Exchange Products on CME Electronic Trading (Globex) on 6/12/2009 Computed as Average Price Times Total Contract Volume Reported by CME

Currency	Futures Daily Volume (in USD thousands)	Mini-Futures Daily Volume (in USD thousands)
Australian Dollar	5,389.8	N/A
British Pound	17,575.6	N/A
Canadian Dollar	6,988.1	N/A
Euro	32,037.9	525.3
Japanese Yen	8,371.5	396.2
New Zealand Dollar	426.5	N/A
Swiss Franc	4,180.6	N/A

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TABLE 4.5 Summary Statistics of Weekly EUR/USD Order Flow observed by Citibank between January 1993 and July 1999

	Summary Statistics (Citibank weekly EUR/USD order flow 1993–1999)						
				Autocorrelations Lag			
Order Flow	Mean Standard	Maximum Minimum	Skewness or Kurtosis*	1	2	4	8
	−0.043	3.722	0.105	−0.061	0.027	0.025	−0.015
A: Total for EUR/USD	1.234	−3.715	3.204	(0.287)	(0.603)	(0.643)	(0.789)
B: EUR/USD Order Flows per Customer Type							
	−16.774	549.302	−0.696	−0.037	−0.04	0.028	−0.028
(i) Corporate U.S.	108.685	−529.055	9.246	(0.434)	(0.608)	(0.569)	(0.562)
(ii) Corporate Non-U.S.	−59.784	634.918	−0.005	0.072	0.089	−0.038	0.103
	196.089	−692.419	3.908	(0.223)	(0.124)	(0.513)	(0.091)
	−4.119	1710.163	0.026	−0.021	0.024	0.126	−0.009
(iii) Traders U.S.	346.296	−2024.28	8.337	(0.735)	(0.602)	(0.101)	(0.897)
	11.187	972.106	0.392	−0.098	0.024	0.015	0.083
(iv) Traders Non-U.S.	183.36	−629.139	5.86	(0.072)	(0.660)	(0.747)	(0.140)
(v) Investors U.S.	19.442	535.32	−1.079	0.096	−0.024	−0.03	−0.016
	146.627	−874.15	11.226	(0.085)	(0.568)	(0.536)	(0.690)
	15.85	1881.284	0.931	0.061	0.107	−0.03	−0.014
(vi) Investors Non-U.S.	273.406	−718.895	9.253	(0.182)	(0.041)	(0.550)	(0.825)

*Skewness of order flows measures whether the flows skew toward either the positive or the negative side of their mean, and kurtosis indicates the likelihood of extremely large or small order flows. Statistical properties of skewness and kurtosis are discussed in detail in Chapter 8.

TABLE 4.6 Daily Dollar Volume in Most Active Equity Futures on CME Electronic Trading (Globex) on 6/12/2009 Computed as Average Price Times Total Contract Volume Reported by CME

Instrument	Futures Daily Volume (in USD thousands)	Mini-Futures Daily Volume (in USD thousands)
S&P 500	2,331.6	N/A
Nasdaq100	172.4	N/A
E-mini	2,300,537.2	N/A
E-Mid-cap	40,820.9	N/A
E-Nasdaq	515,511.8	N/A
Nikkei	56,570.3	N/A
GSCI	211.0	N/A
MSCI EAFE	24,727.7	2,126.9

Equity Markets

Equity markets are popular among high-frequency players due to the market inefficiencies presented by the markets' sheer breadth; in 2006, 2,764 stocks were listed on NYSE alone. In addition to stocks, equity markets trade exchange-traded funds (ETFs), warrants, certificates, and even structured products. There are stock futures and options, as well as index futures and options. Most stock exchanges provide full electronic trading functionality for all of their offerings. Table 4.6 documents sample daily electronic trading volumes in most active equity futures trading on Globex.

Equity markets display diversity in investment objectives. Many equity market participants invest in long-term buy-and-hold patterns. Short-term opportunities for high-frequency traders abound.

Commodity Markets

Commodities products also include spot, futures, and options. Spot commodity contracts provide physical delivery of goods (e.g., a bushel of corn) and are therefore ill suited for high-frequency trading. Electronically traded and liquid commodity futures and options, on the other hand, can provide viable and profitable trading strategies.

Like other types of futures, commodity futures are contracts to buy or sell the underlying security—in this case a commodity, at a prespecified point in time in the future. Futures of agricultural commodities may have irregular expiry dates due to the seasonality of harvests. Commodity futures contracts tend to be smaller than FX futures or interest rate futures contracts.

TABLE 4.7 Daily Dollar Volume of Commodity Products in CME Electronic Trading (Globex) on 6/12/2009 Computed as Average Price Times Total Contract Volume Reported by CME

Commodity	Futures Daily Volume (in USD thousands)	Mini-Futures Daily Volume (in USD thousands)
Corn	121,920.9	178.0
Wheat	33,399.5	N/A
Soybeans	147,168.6	423.1
Soybean Meal	13,089.6	N/A
Soybean Oil	4,334.1	N/A
Oats	212.9	N/A
Rough Rice	447.0	N/A
Milk	12.9	N/A
Dry Milk	0.1	N/A
Cash Butter	14.3	N/A
Pork Belly	1.6	N/A
Lean Hog	1,385.0	N/A
Live Cattle	728.0	N/A
Feeder	179.8	N/A
Lumber	167.3	N/A
Ethanol	40.8	N/A

In 2009, CME offered electronic futures and options trading in commodities shown in Table 4.7. Table 4.7 also shows daily volumes on selected electronically traded futures recorded on CME on June 12, 2009.

CONCLUSION

As previous sections have shown, electronic trading is rapidly advancing to bring instantaneous execution to most securities. The advantages of high-frequency trading in the developing electronic markets are two-fold:

- First-to-market high-frequency traders in the newly electronic markets are likely to capture significant premiums on their speculative activity simply because of the lack of competition.
- In the long term, none of the markets is a zero-sum game. The diverse nature of market participants ensures that all players are able to extract value according to their own metrics.

CHAPTER 5

Evaluating Performance of High-Frequency Strategies

The field of strategy performance measurement is quite diverse. Many different metrics have been developed over time to illuminate a strategy's performance. This chapter summarizes the most popular approaches for performance measurement and discusses strategy capacity and the length of time required to evaluate a strategy.

BASIC RETURN CHARACTERISTICS

Trading strategies may come in all shapes and sizes, but they share one characteristic that makes comparison across different strategies feasible—return.

The return itself, however, can be measured across a wide array of frequencies: hourly, daily, monthly, quarterly, and annually, among others. Care should be exercised to ensure that all returns used for inter-strategy comparisons are generated at the same frequency.

Returns of individual strategies can be compared using a variety of performance measures. Average annual return is one such metric. An average return value is a simplistic summary of the location of the mean of the return distribution. Higher average returns may be potentially more desirable than lower returns; however, the average return itself says nothing about dispersion of the distribution of returns around its mean, a measure that can be critical for risk-averse investors.

Volatility of returns measures the dispersion of returns around the average return; it is most often computed as the standard deviation of returns.

Volatility, or standard deviation, is often taken to proxy risk. Standard deviation, however, summarizes the average deviation from the mean and does not account for the risk of extreme negative effects that can wipe out years of performance.

A measure of tail risk popular among practitioners that documents the maximum severity of losses observed in historical data is maximum drawdown. Maximum drawdown records the lowest peak-to-trough return from the last global maximum to the minimum that occurred prior to the next global maximum that supersedes the last global maximum. The global maximum measured on the past data at any point in time is known as "high water mark." A drawdown is then the lowest return in between two successive high water marks. The lowest drawdown is known as the maximum drawdown.

Figure 5.1 illustrates the concepts of high water mark and maximum drawdown graphically. The graph presents an evolution of a sample cumulative return of a particular investment model over time. At time t_A, the return R_A is the highest cumulative return documented on the chart, so it is our high water mark at time t_A. The cumulative return subsequently drops to level R_B at time t_B, but the value of our high water mark remains the same: R_A. Since R_B at time t_B presents the lowest drop in cumulative return on record, the value $(R_B - R_A)$ constitutes our maximum drawdown at time t_B.

Subsequently, the model reaches a new high water mark at time t_A' as soon as the cumulative return reached passes the previous high water mark R_A. The value of the high water mark continues to increase until point C, where it reaches a peak value to date: R_C. At this point, the maximum drawdown remains $(R_B - R_A)$.

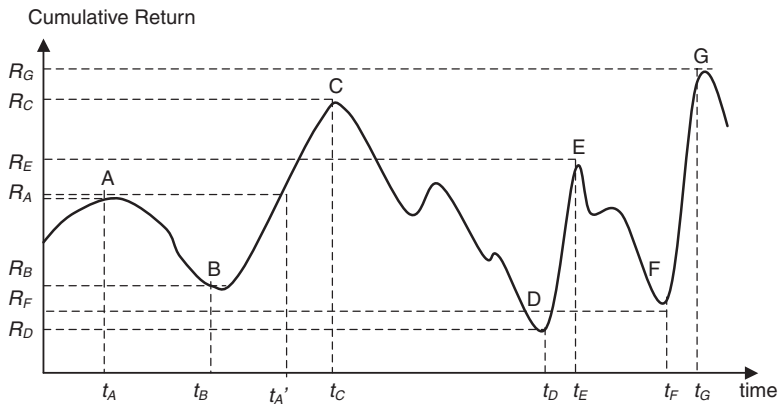

FIGURE 5.1 Calculation of maximum drawdowns.

Following point C, the cumulative return drops considerably, reaching progressively lower troughs. The new maximum drawdown is computed at a point X as soon as the following condition holds: $R_X - R_C < R_B - R_A$. Point C and the corresponding cumulative return R_C remain the high water mark until it is exceeded at point G. Point D sets the new maximum drawdown $(R_D - R_C)$ that remains in effect for the duration of the time shown in the graph.

The average return, volatility, and maximum drawdown over a prespecified window of time measured at a predefined frequency are the mainstays of performance comparison and reporting for different trading strategies. In addition to the average return, volatility, and maximum drawdown, practitioners sometimes quote skewness and kurtosis of returns when describing the shape of their return distributions. As usual, skewness illustrates the position of the distribution relative to the return average; positive skewness indicates prevalence of positive returns, while negative skewness indicates that a large proportion of returns is negative. Kurtosis indicates whether the tails of the distribution are normal; high kurtosis signifies "fat tails," a higher than normal probability of extreme positive or negative events.

COMPARATIVE RATIOS

While average return, standard deviation, and maximum drawdown present a picture of the performance of a particular trading strategy, the measures do not lend to an easy point comparison among two or more strategies. Several comparative performance metrics have been developed in an attempt to summarize mean, variance, and tail risk in a single number that can be used to compare different trading strategies. Table 5.1 summarizes the most popular point measures.

The first generation of point performance measures were developed in the 1960s and include the Sharpe ratio, Jensen's alpha, and the Treynor ratio. The Sharpe ratio is probably the most widely used measure in comparative performance evaluation; it incorporates three desirable metrics—average return, standard deviation, and the cost of capital.

The Sharpe ratio was designed in 1966 by William Sharpe, later a winner of the Nobel Memorial Prize in Economics; it is a remarkably enduring concept used in the study and practice of finance. A textbook definition of the Sharpe ratio is $SR = \frac{\bar{R} - R_F}{\sigma_R}$, where $\bar{R}$ is the annualized average return from trading, σ_R is the annualized standard deviation of trading returns, and R_F is the risk-free rate (e.g., Fed Funds) that is included to capture the opportunity cost as well as the position carrying costs associated with

TABLE 5.1 Performance Measure Summary

Sharpe Ratio (Sharpe [1966])	$SR = \frac{E[r]-r_f}{\sigma[r]}$, where $E[r] = \frac{r_1+\cdots+r_T}{T}$ $\sigma[r] = \sqrt{\frac{(r_1-E[r])^2+\cdots+(r_T-E[r])^2}{T-1}}$ The Sharpe ratio of high-frequency trading strategies: $SR = \frac{E[r]}{\sigma[r]}$	Adequate if returns are normally distributed.
Treynor Ratio (Treynor [1965])	$Treynor_i = \frac{E[r_i]-r_f}{\beta_i}$ β_i is the regression coefficient of trading returns on returns of the investor's reference portfolio, such as the market portfolio.	Adequate if returns are normally distributed and the investor wishes to split his holdings between one trading strategy and the market portfolio.
Jensen's Alpha (Jensen [1968])	$\alpha_i = E[r_i] - r_f - \beta_i(r_M - r_f)$ β_i is the regression coefficient of trading returns on returns of the investor's reference portfolio, such as the market portfolio.	Measures trading return in excess of the return predicted by CAPM. Adequate if returns are normally distributed and the investor wishes to split his holdings between one trading strategy and the market portfolio, but can be manipulated by leveraging the trading strategy.

Measures based on lower partial moments (LPMs):

LPM of order n for security i:

$$LPM_{ni}(\tau) = \frac{1}{T}\sum_{t=1}^{T} \max[\tau - r_{it}, 0]^n$$

where τ is the minimal acceptable return;

n is the moment: $n = 0$ is the shortfall probability, $n = 1$ is the expected shortfall, $n = 2$ for $\tau = E[r]$ is the semi-variance.

According to Eling and Schuhmacher (2007), more risk-averse investors should use higher order n.

LPMs consider only negative deviations of returns from a minimal acceptable return. As such, LPMs are deemed to be a better measure of risk than standard deviation, which considers both positive and negative deviations (Sortino and van der Meer [1991]). Minimal acceptable return can be 0, risk-free rate, or average return.

TABLE 5.1 (*Continued*)

Omega (Shadwick and Keating [2002]), (Kaplan and Knowles [2004])	$\Omega_i = \frac{E[r_i]-\tau}{LPM_{1i}(\tau)} + 1$	$E[r_i] - \tau$ is the average return in excess of the benchmark rate.
Sortino Ratio (Sortino and van der Meer [1991])	$Sortino_i = \frac{E[r_i]-\tau}{(LPM_{2i}(\tau))^{1/2}}$	
Kappa 3 (Kaplan and Knowles [2004])	$\mathrm{K3}_i = \frac{E[r_i]-\tau}{(LPM_{3i}(\tau))^{1/3}}$	
Upside Potential Ratio (Sortino, van der Meer, and Plantinga [1999])	$UPR_i = \frac{HPM_{1i}(\tau)}{\left(LPM_{2i}(\tau)\right)^{1/2}}$ where HPM = higher partial moment $HPM_{ni}(\tau) = \frac{1}{T}\sum_{t=1}^{T} \max[r_{it} - \tau, 0]^n$	According to Eling and Schuhmacher (2007), this ratio gains from the consistent application of the minimal acceptable return τ in the numerator as well as in the denominator.
Measures based on drawdown: frequently used by CTAs, according to Eling and Schuhmacher (2007, p. 5), "because these measures illustrate what the advisors are supposed to do best—continually accumulating gains while consistently limiting losses (see Lhabitant, 2004)." MD_{i1} denotes the lowest maximum drawdown, MD_{i2} the second lowest maximum drawdown, and so on.		
Calmar Ratio (Young [1991])	$Calmar_i = \frac{E[r_i]-r_f}{-MD_{i1}}$	MD_{i1} is the maximum drawdown.
Sterling Ratio (Kestner [1996])	$Sterling_i = \frac{E[r_i]-r_f}{-\frac{1}{N}\sum_{k=1}^{N} MD_{ij}}$	$-\frac{1}{N}\sum_{k=1}^{N} MD_{ij}$ is the average maximum drawdown.
Burke Ratio (Burke [1994])	$Burke_i = \frac{E[r_i]-r_f}{\left[\sum_{k=1}^{N} \left(MD_{ij}\right)^2\right]^{1/2}}$	$\left[\sum_{k=1}^{N} \left(MD_{ij}\right)^2\right]^{1/2}$ is a type of variance below the N^{th} largest drawdown; accounts for very large losses.
Value-at-risk–based measures. Value at risk (VaR_i) describes the possible loss of an investment, which is not exceeded with a given probability of $1-\alpha$ in a certain period. For normally distributed returns, $VaR_i = -(E[r_i] + z_\alpha \sigma_i)$, where z_α is the α-quantile of the standard normal distribution.		

(*Continued*)

TABLE 5.1 (*Continued*)

Excess return on value at risk (Dowd, [2000])	*Excess R on VaR* $= \frac{E[r]-r_f}{VaR_i}$	Not suitable for non-normal returns.
Conditional Sharpe ratio (Agarwal and Naik [2004])	*Conditional Sharpe* $= \frac{E[r]-r_f}{CVaR_i}$ $CVaR_i = E[-r_{it} \mid r_{it} \leq -VaR_i]$	The advantage of CVaR is that it satisfies certain plausible axioms (Artzner et al. [1999]).
Modified Sharpe ratio (Gregoriou and Gueyie [2003])	*Modified Sharpe* $= \frac{E[r]-r_f}{MVaR_i}$ Cornish-Fisher expansion is calculated as follows: $MVaR_i = -(E[r_i] + \sigma_i(z_\alpha + (z_\alpha^2 - 1)S_i/6 + (z_\alpha^3 - 3z_\alpha)EK_i/24 - (2z_\alpha^3 - 5z_\alpha)S_i^2/36))$ where Si denotes skewness and EK_i the excess kurtosis for security i	Suitable for non-normal returns.

the trading activity. It should be noted that in high-frequency trading with no positions carried overnight, the position carrying costs are 0. Therefore, the high-frequency Sharpe ratio is computed as follows:

$$SR = \frac{\overline{R}}{\sigma_R}.$$

What makes the Sharpe ratio an appealing measure of performance, in comparison with, say, raw absolute return? Surprisingly, the Sharpe ratio is an effective metric for selecting mean-variance efficient securities.

Consider Figure 5.2, for example, which illustrates the classic mean-variance frontier. In the figure, the Sharpe ratio is the slope of the line emanating from the risk-free rate and passing through a point corresponding to a given portfolio (M for market portfolio), or trading strategy, or individual security. The bold line tangent to the mean-variance set of all portfolio combinations is the efficient frontier itself. It has the highest slope and, correspondingly, the highest Sharpe ratio of all the portfolios in the set. For any other portfolio, trading strategy, or individual security A, the higher the Sharpe ratio, the closer the security is to the efficient frontier.

Sharpe himself came up with the metric when developing a portfolio optimization mechanism for a mutual fund for which he was consulting. Sharpe's mandate was to develop a portfolio selection framework for the

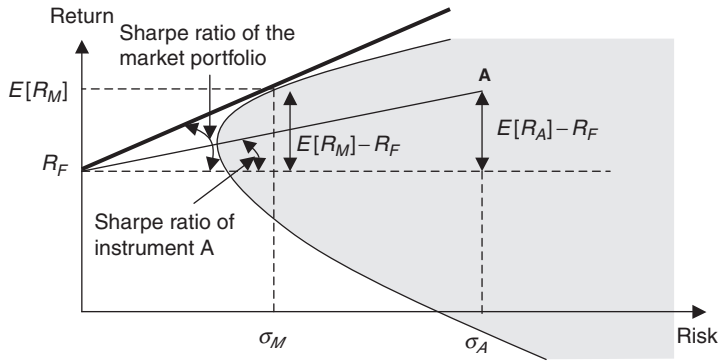

FIGURE 5.2 Sharpe ratio as a mean-variance slope. The market portfolio has the highest slope and, correspondingly, the highest Sharpe ratio.

fund with the following constraint: no more than 5 percent of the fund's portfolio could be allocated to a particular financial security. Sharpe then created the following portfolio solution: he first ranked the security universe on what now is known as Sharpe ratio, then picked the 20 securities with the best performance according to the Sharpe ratio measure, and invested 5 percent of the fund into each of the 20 securities. Equally weighted portfolio allocation in securities with the highest Sharpe ratios is just one example of a successful Sharpe ratio application.

Jensen's alpha is a measure of performance that abstracts from broad market influences, CAPM-style. Jensen's alpha implicitly takes into consideration the variability of returns in co-movement with chosen market indices.

The third ratio, the Treynor ratio, measures the average return in excess of the chosen benchmark per unit of risk proxied by beta from the CAPM estimation.

While these three metrics remain popular, they do not take into account the tail risk of extreme adverse returns. Brooks and Kat (2002), Mahdavi (2004), and Sharma (2004), for example, present cases against using Sharpe ratios on non-normally distributed returns. The researchers' primary concerns surrounding the use of the Sharpe ratio are linked to the use of derivative instruments that result in an asymmetric return distribution and fat tails. Ignoring deviations from normality may underestimate risk and overestimate performance. New performance measures have been subsequently developed to capture the tail risk inherent in the returns of most trading strategies.

A natural extension of the Sharpe ratio is to change the measure of risk from standard deviation to a drawdown-based methodology in an effort to

capture the tail risk of the strategies. The Calmar ratio, Sterling ratio, and Burke ratio do precisely that. The Calmar ratio, developed by Young (1991), uses the maximum drawdown as the measure of volatility. The Sterling ratio, first described by Kestner (1996), uses the average drawdown as a proxy for volatility. Finally, the Burke ratio, developed by Burke (1994), uses the standard deviation of maximum drawdowns as a volatility metric.

In addition to ignoring the tail risk, the Sharpe ratio is also frequently criticized for including positive returns in the volatility measure. The argument goes that only the negative returns are meaningful when estimating and comparing performance of trading strategies. In response, a "Greek" class of ratios extended the Sharpe ratio by replacing volatility with the average metrics of adverse returns only. These adverse return metrics are known as lower partial moments (LPMs) and are computed as regular moments of a distribution (i.e., mean, standard deviation, and skewness), except that the data used in computation comprises returns below a specified benchmark only. Thus, a metric known as Omega, developed by Shadwick and Keating (2002) and Kaplan and Knowles (2004), replaces the standard deviation of returns in the Sharpe ratio calculation with the first lower partial moment, the average of the returns that fell below the selected benchmark. The Sortino ratio, developed by Sortino and van der Meer (1991), uses the standard deviation of the returns that fell short of the benchmark, the second LPM, as a measure of return volatility in the Sharpe ratio calculation. The Kappa 3 measure, developed by Kaplan and Knowles (2004), replaces the standard deviation in the Sharpe ratio with the third LPM of the returns, the skewness of the returns below the benchmark. Finally, the Upside Potential ratio, produced by Sortino, van der Meer, and Plantinga (1999), measures the average return above the benchmark (the first higher partial moment) per unit of standard deviation of returns below the benchmark.

Value-at-risk (VaR) measures also gained considerable popularity as metrics able to summarize the tail risk in a convenient point format within a statistical framework. The VaR measure essentially identifies the 90 percent, 95 percent, or 99 percent Z-score cutoff in distribution of returns (the metric is also often used on real dollar distributions of daily profit and loss). VaR companion measure, the conditional VaR (CVaR), also known as expected loss (EL), measures the average value of return within the cut-off tail. Of course, the original VaR assumes normal distributions of returns, whereas the returns are known to be fat-tailed. To address this issue, a modified VaR (MVaR) measure was proposed by Gregoriou and Gueyie (2003) and takes into account deviations from normality. Gregoriou and Gueyie (2003) also suggest using MVaR in place of standard deviation in Sharpe ratio calculations.

How do these performance metrics stack up against each other? It turns out that all metrics deliver comparable rankings of trading strategies.

Eling and Schuhmacher (2007) compare hedge fund ranking performance of the 13 measures listed and conclude that the Sharpe ratio is an adequate measure for hedge fund performance.

PERFORMANCE ATTRIBUTION

Performance attribution analysis, often referred to as "benchmarking," goes back to the arbitrage pricing theory of Ross (1977) and has been applied to trading strategy performance by Sharpe (1992) and Fung and Hsieh (1997), among others. In a nutshell, performance attribution notes that t-period return on strategy i that invests into individual securities with returns r_{jt} in period t, with $j = 1, \ldots, J$, has an underlying factor structure:

$$R_{it} = \sum_j x_{jt} r_{jt} \tag{5.1}$$

where x_{jt} is the relative weight of the jth financial security in the portfolio at time t, $\sum_j x_{jt} = 1$. The jth financial security, in turn, has a period-t return that can be explained by K systematic factors:

$$r_{jt} = \sum_k \lambda_{jk} F_{kt} + \varepsilon_{jt} \tag{5.2}$$

where F_{kt} is one of K underlying systematic factors in period t, $k = 1, \ldots, K$, λ is the factor loading, and ε_{jt} is the security j idiosyncratic return in period t. Following Sharpe (1992), factors can be assumed to be broad asset classes, as well as individual stocks or other securities. Combining equations (1) and (2) we can express returns as follows:

$$R_{it} = \sum_{j,k} x_{jt} \lambda_{jk} F_{kt} + \sum_j x_{jt} \varepsilon_{jt} \tag{5.3}$$

reducing the large number of financial securities potentially underlying strategy i's returns to a small group of global factors. Performance attribution to various factors then involves regressing the strategy's returns on a basket of factors:

$$R_{it} = \alpha_i + \sum_k b_{ik} F_{kt} + u_{it} \tag{5.4}$$

where b_k measures the performance of the strategy that can be attributed to factor k, α_i measures the strategy's persistent ability to generate abnormal returns, and u_{it} measures the strategy's idiosyncratic return in period t.

Performance attribution is a useful measure of strategy returns for the following reasons:

- The technique may accurately capture investment styles of black-box strategies in addition to the details reported by the designer of the strategy.
- Performance attribution is a measure of true added value of the strategy and lends itself to easy comparison with other strategies.
- Near-term persistence of trending factors allows forecasting of strategy performance based on performance attribution (see, for example, Jegadeesh and Titman [1993]).

In the performance attribution model, the idiosyncratic value-added of the strategy is the strategy's return in excess of the performance of the basket of weighted strategy factors.

Fung and Hsieh (1997) find that the following eight global groups of asset classes serve well as performance attribution benchmarks:

- Three equity classes: MSCI U.S. equities, MSCI non–U.S. equities, and IFC emerging market equities
- Two bond classes: JP Morgan U.S. government bonds and JP Morgan non–U.S. government bonds
- One-month Eurodollar deposit representing cash
- The price of gold proxying commodities and the Federal Reserve's trade-weighted dollar index measuring currencies in aggregate.

OTHER CONSIDERATIONS IN STRATEGY EVALUATION

Strategy Capacity

Strategy performance may vary with the amount of capital deployed. Size-induced changes in observed performance are normally due to limits in market liquidity for each trading instrument. Large position sizes consume available pools of liquidity, driving market prices into adverse directions and reducing the profitability of trading strategies. The capacity of individual strategies can be estimated through estimation of market impact, discussed in detail in Chapter 19. Extensive research on the impact of investment size on performance has been documented for hedge funds utilizing portfolios of strategies. This section notes the key findings in the studies of the impact of investment size on fund performance.

Fransolet (2004) shows that fast increase in capital in the entire industry may erode capacities of many profitable strategies. In addition, per Brown, Goetzmann, and Park (2004), strategy capacity may depend on a manager's skills. Furthermore, strategy capacity is a function of trading costs and asset liquidity, as shown by Getmansky, Lo, and Makarov (2004). As a result, Ding et al. (2008) conjecture that when the amount of capital deployed is lower than the strategy capacity, the strategy performance may be positively related to its capitalization. However, once capitalization exceeds strategy capacity, performance becomes negatively related to the amount of capital involved.

Length of the Evaluation Period for High-Frequency Strategies

Most portfolio managers face the following question in evaluating candidate trading strategies for inclusion in their portfolios: how long does one need to monitor a strategy in order to gain confidence that the strategy produces the Sharpe ratio advertised?

Some portfolio managers have adopted an arbitrarily long evaluation period: six months to two years. Some investors require a track record of at least six years. Yet others are content with just one month of daily performance data. It turns out that, statistically, any of the previously mentioned time frames is correct if it is properly matched with the Sharpe ratio it is intended to verify. The higher the Sharpe ratio, the shorter the strategy evaluation period needed to ascertain the validity of the Sharpe ratio.

If returns of the trading strategy can be assumed to be normal, Jobson and Korkie (1981) showed that the error in Sharpe ratio estimation is normally distributed with mean 0 and standard deviation

$$s = [(1/T)(1 + 0.5SR^2)]^{1/2}$$

For a 90 percent confidence level, the claimed Sharpe ratio should be at least 1.645 times greater than the standard deviation of the Sharpe ratio errors, s. As a result, the minimum number of evaluation periods used for Sharpe ratio verification is

$$T_{\min} = (1.645^2/SR^2)(1 + 0.5SR^2)$$

The Sharpe ratio SR used in the calculation of $T_{\min}$, however, should correspond to the frequency of estimation periods. If the annual Sharpe ratio claimed for a trading strategy is 2, and it is computed based on the basis of monthly data, then the corresponding monthly Sharpe ratio SR is $2/(12)^{0.5} = 0.5774$. On the other hand, if the claimed Sharpe ratio is computed based on daily data, then the corresponding Sharpe ratio SR

TABLE 5.2 Minimum Trading Strategy Performance Evaluation Times Required for Verification of Reported Sharpe Ratios

Claimed Annualized Sharpe Ratio	No. of Months Required (Monthly Performance Data)	No. of Months Required (Daily Performance Data)
0.5	130.95	129.65
1.0	33.75	32.45
1.5	15.75	14.45
2.0	9.45	8.15
2.5	6.53	5.23
3.0	4.95	3.65
4.0	3.38	2.07

is $2/(250)^{0.5} = 0.1054$. The minimum number of monthly observations required to verify the claimed Sharpe ratio with 90 percent statistical confidence is then just over nine months for monthly performance data and just over eight months for daily performance data. For a claimed Sharpe ratio of 6, less than one month of daily performance data is required to verify the claim. Table 5.2 summarizes the minimum performance evaluation times required for verification of performance data for key values of Sharpe ratios.

CONCLUSION

Statistical tools for strategy evaluation allow managers to assess the feasibility and appropriateness of high-frequency strategies to their portfolios. Although several statistical strategy evaluation methods have been developed, the Sharpe ratio remains the most popular measure.

CHAPTER 6

Orders, Traders, and Their Applicability to High-Frequency Trading

High-frequency trading aims to identify and arbitrage temporary market inefficiencies that are created by the competing interests of market participants. Understanding the types of orders that traders can place to achieve their goals allows insights into the strategies of various traders. Ultimately, this understanding can inform the forecasting of impending actions of market participants, which itself is key to success in high-frequency trading. This chapter examines various types of orders present in today's markets.

ORDER TYPES

Contemporary exchanges and electronic communication networks (ECNs) offer a vast diversity of ordering capabilities. The order types differ as to execution price, timing, size, and even disclosure specifications. This section considers each order characteristic in detail.

Order Price Specifications

Market Orders versus Limit Orders Orders can be executed at the best available price or at a specified price. Orders to buy or sell a security at the best available price when the order is placed are known as market orders. Orders to buy or sell a security at a particular price are known as limit orders.

When a market order arrives at an exchange or an ECN, the order is immediately matched with the best available opposite order or several best orders, depending on the size of the arriving order. For example, if a market order to sell 100,000 shares of SPY arrives at an exchange, and the exchange has the following buy orders outstanding from best to worst: 10,000 shares at $935, 40,000 shares at $930, and 50,000 at $925, then the arriving market sell order is "walked through the order book" until it is filled: the first 10,000 shares are sold at $935, the next 40,000 shares at $930 and the final 50,000 shares at $925, capturing the weighted-average price of $928 per share:

$$P = \frac{10{,}000 \times \$935 + 40{,}000 \times \$930 + 50{,}000 \times \$925}{100{,}000} = \$928$$

Transaction costs, such as the broker-dealer and exchange fees, are accounted for separately, further reducing the profitability of the trade. For details on cost types and values, please see Chapter 19.

Limit orders are executed at a specified limit price or at a better price, if one is available. When a limit order arrives at an exchange or an ECN, the order is first compared with the best available opposite orders to determine whether the newly arrived order can be filled immediately. For example, when a limit order to sell SPY at 930 arrives at an exchange, the exchange first checks whether the exchange already has matching orders to buy SPY at or above 930. If orders to buy SPY at 930 or at a higher price are present, the arriving order is treated as a regular market order; it is filled immediately and charged the market order fees. If no matching orders exist, the arriving limit order is placed in the limit order book, where it remains until it becomes the best available order and is matched with an incoming market order.

The aggregate size of limit orders available in the limit order book is often thought to be the liquidity of the market. The total size of limit orders available at a particular price is referred to as the market depth. The number of different price points at which limit orders exist in the limit order book is known as the breadth of the market. Figure 6.1 illustrates components of the liquidity in the limit order book, according to Bervas (2006).

Limit orders can be seen as pre-commitments to buy or sell a specified number of shares of a particular security at a prespecified price, whereas market orders are requests to trade the specified quantity of a given security as soon as possible at the best price available in the market. As a result, market orders execute fast, with certainty, at uncertain prices and relatively high transaction costs. Limit orders, on the other hand, have a positive probability of no execution, lower transaction costs, and

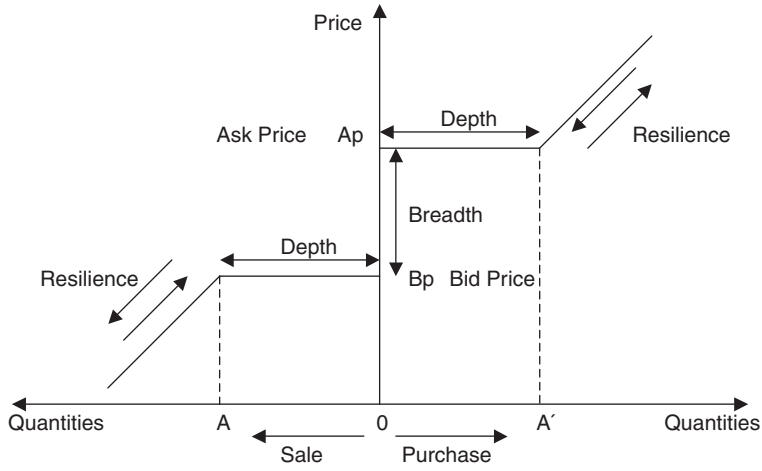

FIGURE 6.1 Aspects of market liquidity (Bervas, 2006). The bid price and the ask price are defined for liquidity quantities OA and OA′ that represent market depths at bid and ask prices, respectively.

encompass an option to resubmit the order at a different price. Table 6.1 key outlines key differences between market and limit orders.

Market orders specify the desired exchange for the order, the exchange code of the security to be traded, the quantity of the security to be bought or sold, and whether the order is to buy or to sell. Limit orders specify the same parameters as do market orders along with the desired execution price.

Profitability of Limit Orders A trader placing a buy limit order writes a put option available freely, with no premium, to the market. Similarly, a trader placing a sell limit order writes a free call option on the security. In addition to foregoing the premium on the option, the limit trader opens himself up to being "picked off" by better-informed traders. For

TABLE 6.1 Limit Orders versus Market Orders

	Market Orders	**Limit Orders**
Order Execution	Guaranteed	Uncertain
Time to Execution	Short	Uncertain
Execution Price	Uncertain	Certain
Order Resubmission	None	Infinite prior to execution
Transaction Costs	High	Low

example, suppose that the limit trader places a limit buy order on a security at $30.00, and that another, better-informed, trader knows that $30.00 is the maximum price the security can reach within the time interval under consideration. The better-informed trader then jumps at the opportunity to sell the security to the limit trader, leaving the limit trader in a losing position.

Despite the seeming lack of profitability of limit order traders, limit trading has prospered. Most exchanges now offer limit order capabilities, and limit order–based exchange alternatives known as the electronic communication networks (ECNs) experience a boom. The ability of exchanges and ECNs to attract numerous limit order traders suggests that limit order trading is profitable for many market participants.

Handa and Schwartz (1996) examine the profitability of limit order traders, also known as liquidity traders, and find that limit order strategies can capture economic rents in excess of market order strategies. Specifically, Handa and Schwartz (1996) find that the buy limit order strategies allow limit traders to obtain a premium of 0.1 percent to 1.6 percent per trade on average, depending on the type of the limit order strategy. Handa and Schwartz (1996) consider four types of buy limit order strategies: those with buy limit orders placed at prices 0.5 percent, 1 percent, 2 percent, and 3 percent below the corresponding market price.

To measure the profitability of limit order trades, Handa and Schwartz (1996) conducted the following experiment:

- The authors break down the trading data into equally spaced profitability evaluation periods.
- In every evaluation period, the simulated market trader executes all trades at the opening price of the evaluation period.
- The limit order trader sets limit orders at prices x percent below the market opening price.
- The limit order is considered executed when it is crossed by the market price.
- If the limit orders are not executed within the evaluation period, the limit trader is forced to execute his orders at the opening price of the next evaluation period.

Handa and Schwartz (1996) measure the profitability of the limit order strategy as the average difference between the prices obtained using the limit order strategy and the prices obtained using the market order strategy during each evaluation period.

The limit order strategy is profitable if the average cost of realized limit orders is lower than that of realized market orders. The profitability of the

buy limit order strategy within each evaluation period is then measured according to equation (6.1):

$$\Pi_t = P_{t,M} - P_{t,L} \tag{6.1}$$

where Π_t is the profitability of the buy limit order strategy in evaluation period t, $P_{t,M}$ is the opening price of the evaluation period at which the market buy order is executed, and $P_{t,L}$ is the obtained limit price—either a price obtained when the market price crosses the best limit sell order or the opening price of the next evaluation period, as defined previously. The average buy limit order profitability is then computed as an average of profitability for individual evaluation periods:

$$\overline{\Pi}_t = \frac{1}{T}\sum_{t=1}^{T}(P_{t,M} - P_{t,L}) \tag{6.2}$$

Handa and Schwartz (1996) assess the performance of the following buy limit orders: orders with prices set 0.5 percent, 1 percent, 2 percent, and 3 percent below the opening market price in each evaluation period. The authors run their experiments on stocks of 30 Dow Jones Industrial firms that traded on the NYSE and find that, on average, limit order strategies outperform market orders. Table 6.2 replicates the average results for limit order strategies appearing in Handa and Schwartz (1996). The results show the average percentage value by which the limit order strategy outperforms the market order strategy.

In summary, limit order strategies can bring clear profitable outcomes to traders. The limit order strategy works particularly well in the volatile range-bound markets, such as those we are currently experiencing.

Delays in Limit Order Execution Limit orders, when executed, are usually executed at prices more favorable than otherwise identical

TABLE 6.2 Average Profitability of Limit Order Strategies in Excess of the Market Order Strategies

	The Distance of Limit Order Prices Away (in the Favorable Direction) from the Market Order Prices			
	0.5 percent	**1 percent**	**2 percent**	**3 percent**
Average profitability	0.100 percent	0.361 percent*	0.516 percent*	1.605 percent*

*Indicates statistical significance at the 95 percent confidence level.

market orders that are executed with certainty in most cases. The time duration of limit orders, however, is unpredictable; limit orders can be "hit" by market orders right away or can fail to be executed if the market price moves away from the price of the limit order. Failure to execute a limit order can be quite costly when the limit order is placed to close a position, particularly when the position is a loss that needs to be liquidated. For example, consider a trading strategy that is currently long USD/CAD. Suppose that the position was opened with a market buy order at 1.2005, the current market price is 1.1955, and a stop-loss order arrives to close the position. If the stop-loss order is placed as a market order, the order is executed with probability 1 at or below the current market price. If the order is placed as a limit order, and if the market price for USD/CAD suddenly drops, the order never gets filled and the losses exacerbate dramatically. As a result, stop-losses are most often executed at market to ensure that the negative exposure is reliably limited.

Failure to execute a limit order can also be costly when the order is placed to open a position, because the trading strategy incurs the opportunity cost corresponding to the average expected gain per trade. For example, consider a trading strategy that is "flat," or fully in cash, at time 0. Suppose further that at time 1, a buy order arrives to buy USD/CAD while USD/CAD is at 1.2005. If the buy order is placed as a market order, the order will be executed with a probability of 100 percent but at 1.2005 at best (latency in execution and other slippage issues may push the price even further in the adverse direction). On the other hand, if the buy order is placed as a limit order at 1.2000, the order may be executed at 1.2000 if the market price drops to that level, or it may not be executed at all if the market price stays above the limit level. If the limit order never gets hit, the system loses the trade opportunity value equal to the gain from the trade initiated at the market price.

Foucault (1999) and Parlour (1998) model dynamic conditions that ensure that limit orders get hit by market orders, while resulting in the profitable outcome for the trader placing the limit orders. The main questions answered by the two research articles are

1. At what prices will traders post limit orders?
2. How often do traders modify their limit orders?

Such issues in order flow dynamics impact the traders' bargaining power and affect their profitability through transaction costs. The main finding of the studies is that limit orders are preferred to market orders in high-volatility conditions. Thus, in high-volatility situations, the proportion of limit orders in the marketplace increases, simultaneously reducing cumulative profitability of agents resulting from a larger number of trades

that are left not executed, as well as from the increased market bid-ask spreads. Parlour (1998) further explains the diagonal effect observed in Biais, Hillion and Spatt (1995): a market buy reduces the liquidity available at the ask, inducing sellers to post additional limit sell orders instead of market sell orders and subsequently triggering more market buy orders. Thus, serial autocorrelation in order flow arises from liquidity dynamics in addition to dynamics in informed trading. Ahn, Bae, and Chan (2001) find that the volume of limit orders increases with increases in volatility on the Stock Exchange of Hong Kong.

Both Foucault, Kadan, and Kandel (2005) and Rosu (2005) assume that investors care about execution time and submit orders based on their expectations of execution time.

Kandel and Tkatch (2006) find that investors indeed take execution delay into account when submitting limit orders on the Tel-Aviv Stock Exchange. The duration of limit orders appears to decrease with increases in aggressiveness of the limit orders (the distance of the limit orders from the current market price), as shown by survival analysis of Lo, MacKinlay, and Zhang (2002).

Goettler, Parlour, and Rajan (2005) extend the analysis to a world where investors can submit multiple limit orders, at different prices and with different order quantities. Lo and Sapp (2005) test the influence of order size along with order aggressiveness in the foreign exchange markets and find that aggressiveness and size are negatively correlated.

Hasbrouck and Saar (2002) document that limit order traders may post fleeting limit orders; according to Rosu (2005), the fleeting orders are offers to split the difference made by patient investors on one side of the book to the patient investors on the other side of the book.

Traders who are confident in their information may choose to place limit orders during the time they expect their information to impact prices. Keim and Madhavan (1995), for example, show that informed traders whose information decays slowly tend to use limit orders. The proportion of limit orders used by a particular trader or trading strategy, therefore, can be used to measure the traders' confidence in their information. The confidence variable may then be used in extracting additional information from the observed trading decisions and order flow of the traders.

Limit Orders and Bid-Ask Spreads A trader may also gear towards limit orders whenever the bid-ask spreads are high. The bid-ask spread may be a greater cost than the opportunity cost associated with non-execution of position entry orders placed as limit orders. Biais, Hillion, and Spatt (1995) show that on the Paris Bourse the traders indeed place limit orders whenever the bid-ask spread is large and market orders whenever the bid-ask spread is small. Chung, Van Ness, and Van Ness (1999) further show

that the proportion of traders entering limit orders increases whenever bid-ask spreads are wide.

Limit Orders and Market Volatility Bae, Jang, and Park (2003) examine traders' propensity to place market and limit orders in varying volatility conditions. They find that the number of limit orders increases following a rise in intra-day market volatility, independently of the relative bid-ask spread size. Handa and Schwartz (1996) show that transitory volatility, the volatility resulting from uninformed or noise trading, induces a higher propensity of traders to place limit orders than do permanent volatility changes, given that traders can get compensated for providing liquidity while limiting the probability of being picked off. Foucault (1999), however, finds that limit orders are always more optimal than market orders, even when the probability of being picked off increases.

Order Timing Specifications

Both market and limit orders can be specified as valid for different lengths of time and even at different times of the trading day. The "good till canceled" orders (GTC) remain active in the order book until completely filled. The "good till date" orders (GTD) remain in the order book until completely filled or until the specified expiry date. The GTC and GTD orders can be "killed," or canceled, by the exchange or the ECN after a predefined time period (e.g., 90 days), whenever certain corporate actions are taken (e.g., bankruptcy or delisting), or as a result of structural changes on the exchange (e.g., a change in minimum order sizes).

The "day" orders, also known as the "good for the day" (GFD) orders, remain in the order book until completely filled or until the end of the trading day, defined as the end of normal trading hours. The "good for the extended day" (GFE) orders allow the day orders to be executed until the end of the extended trading session. Orders even shorter in duration that are particularly well suited for high-frequency trading are the "good till time" (GTT) orders. The GTT orders remain in the order book until completely filled or until the specified expiry date and time and can be used to specify short-term orders. The GTT orders are especially useful in markets where order cancellation or order change fees are common, such as in the options markets. When market conditions change, instead of canceling or changing the order and thus incurring order cancellation or change fees, traders can let their previous orders expire at a predetermined time and place new orders instead.

Order Size Specifications

The straightforward or plain "vanilla," order size specification in both limit and market orders is a simple number of security contracts geared for

execution. Vanilla order sizes are typically placed in "round lots"—that is, the standard contract sizes traded on the exchange. For example, a round lot for common stocks on the New York Stock Exchange (NYSE) is 100 shares. Smaller orders, known as "odd lots," are filled by a designated odd lot dealer (on a per-security basis), and are normally charged higher transaction costs than the round-lot orders. Both market and limit orders can be odd lots.

Orders bigger than round lots, yet not in round-lot multiples, are known as "mixed lots." Mixed-lot orders are typically broken down into the round lots and the odd lots on the exchange and are executed accordingly by a regular dealer and the odd-lot dealer.

When large market orders are placed, there may not be enough liquidity to fill the order, and subsequent liquidity may be attainable only at prices unacceptable to the trader placing the market order. To address the problem, most exchanges and ECNs accept "fill or kill" (FOK) orders that specify that the order be filled immediately in full, or in part, with the unfilled quantity killed in its entirety. If the partial fill of the market order is unacceptable to the trader, the order can be specified as a "fill and kill" (FAK) order, to be either filled immediately in full or killed in its entirety. Alternatively, if the immediacy of the order execution is not the principal concern but the size is, the order can be specified as an "all or none" (AON) order. The AON orders remain in the order book with their original time priorities until they can be filled in full.

Order Disclosure Specifications

The amount of order information that is disclosed varies from exchange to exchange and from ECN to ECN. On some exchanges and ECNs, all market and limit orders are executed with full transparency to all market participants. Other exchanges, such as the NYSE, allow market makers to decide how much of an incoming market order should be executed at a given price. Other exchanges show limit orders only for a restricted set of prices that are near the current market price. Still others permit "iceberg" orders—that is, orders with only a portion of the order size observable to other market participants.

A "standard iceberg" (SI) order is a limit order that specifies a total size and a disclosed size. The disclosed size is revealed as a limit order. Once the disclosed size is completely executed, the new quantity becomes disclosed, and it is instantaneously made available for matching with time priority corresponding to the release time.

An order is often placed anonymously, without disclosing the identity of the trader or the trading institution to other market participants on the given exchange or ECN. Anonymous orders are particularly attractive to

traders processing large orders, as any identifying information may trigger adverse price offers from other market participants.

Stop-Loss and Take-Profit Orders

In addition to previously discussed specifications of order execution, some exchanges offer stop-loss and take-profit order capability. Both the stop-loss and take-profit orders become market or limit orders to buy or sell the security if a specified price, known as the stop price, is reached, or passed.

Administrative Orders

A change order is an order to change a pending limit order, whether a limit open or limits for take-profit or stop-loss. The change order can specify the change in the limit price, the order type (buy or sell), and the number of units to process. A change order can also be placed to cancel an existing limit order. Some execution counterparties may charge a fee for a change order.

A margin call close order is one order traders probably want to avoid. It is initiated by the executing counterparty whenever a trader trades on margin and the amount of cash in the trader's account is not sufficient to cover two times the losses of all open positions. The margin call close is executed at market at the end of day, which varies depending on the financial instrument traded.

Most broker-dealers and ECNs provide phone support for clients. If customer computer system or network connectivity breaks down for whatever reason, a customer can phone in an order. Such phone-in orders are sometimes referred to as "orders by hand," and are often charged a transaction cost premium relative to the electronic ordering.

Finally, several cancel orders can be initiated either by the customer or by the executing counterparty. An insufficient funds cancel can be enacted by the executing broker in the event that the customer does not have enough funds to open a new position. Limit orders can be canceled if the price of the underlying instrument moves outside the preselected bounds; such orders are known as bound violation cancel orders.

ORDER DISTRIBUTIONS

Order statistics, such as Oanda's FX Trade presented in Table 6.3, are seldom, if ever, distributed to the public. It should be noted, however, that the mean and median size of Oanda FXTrade transactions indicate that the

majority of Oanda's customers are retail and that the numbers are thus not necessarily representative of order flows at broker-dealers and ECNs. Nevertheless, the data offers an interesting point for comparison.

As Table 6.3 shows, on an average day between October 1, 2003 and May 14, 2004, the most common orders—both by number of orders and by volume—were stop-loss or take-profit (22 percent and 23 percent,

TABLE 6.3 Daily Distributions of Trades per Trade Category in FX Spot of Oanda FXTrade, a Toronto-Based Electronic FX Brokerage, as Documented by Lechner and Nolte (2007)

Transaction Record	Percentage of Orders per Order Count	Mean Daily Trading Volume in EUR	Percentage of Orders by Trade Volume
Buy Market (open)	13.10 percent	167,096	14.13 percent
Sell Market (open)	10.61 percent	135,754	11.48 percent
Buy Market (close)	8.27 percent	110,461	9.34 percent
Sell Market (close)	10.27 percent	140,263	11.86 percent
Limit Order: Buy	5.41 percent	61,856	5.23 percent
Limit Order: Sell	4.76 percent	48,814	4.13 percent
Buy Limit Order Executed (open)	3.22 percent	23,860	2.02 percent
Sell Limit Order Executed (open)	2.92 percent	14,235	1.20 percent
Buy Limit Order Executed (close)	0.46 percent	6,091	0.52 percent
Sell Limit Order Executed (close)	0.46 percent	6,479	0.55 percent
Buy Take-Profit (close)	3.14 percent	12,858	1.09 percent
Sell Take-Profit (close)	3.49 percent	19,401	1.64 percent
Buy Stop-Loss (close)	2.18 percent	19,773	1.67 percent
Sell Stop-Loss (close)	2.55 percent	23,391	1.98 percent
Buy Margin Call (close)	0.12 percent	733	0.06 percent
Sell Margin Call (close)	0.17 percent	1,213	0.10 percent
Change Order	3.01 percent	61,229	5.18 percent
Change Stop-Loss or Take-Profit	22.36 percent	268,568	22.71 percent
Cancel Order by Hand	2.41 percent	44,246	3.74 percent
Cancel Order: Insufficient Funds	0.28 percent	10,747	0.91 percent
Cancel Order: Bound Violation	0.20 percent	860	0.07 percent
Order Expired	0.65 percent	4,683	0.40 percent
Total:	100.04 percent	1,182,611	100.00 percent

TABLE 6.4 Popularity of Orders as a Percentage of Order Number and Total Volume among Orders Recorded by Oanda between October 1, 2003 and May 14, 2004

Order Type	Number of Orders, Daily Average	Total Volume in EUR
Percent of All Open Orders That Are Buy Orders	55 percent	55 percent
Percent of Market Orders That Are Buy Orders	55 percent	55 percent
Percent of Open Buy Limit Orders Executed	60 percent	39 percent
Percent of Open Sell Limit Orders Executed	61 percent	29 percent
Total Long Positions Opened per Day	**1,647**	**190,956**
Closing the Long Position		
Sell Market (close)	63 percent	73 percent
Sell Take-Profit (close)	21 percent	10 percent
Sell Stop-Loss (close)	16 percent	12 percent
Sell Limit Order Executed (close)	3 percent	3 percent
Sell Margin Call (close)	1 percent	1 percent
Total Short Positions Opened per Day	**1,367**	**149,989**
Closing the Short Position		
Buy Market (close)	61 percent	74 percent
Buy Take-Profit (close)	23 percent	9 percent
Buy Stop-Loss (close)	16 percent	13 percent
Buy Limit Order Executed (close)	3 percent	4 percent
Buy Margin Call (close)	1 percent	0 percent

respectively), buy market open (13 percent and 14 percent), sell market open (11 percent), and sell market close (10 percent and 12 percent by order number and volume, respectively).

Aggregating the data by buy and sell order types provides insightful statistics on the distribution of orders. As Table 6.3. further shows, 55 percent of both market or limit open orders were buy orders. The numbers reflect a slight preference of smaller customers to enter into long positions.

Out of the total number of open limit orders placed, 60 percent and 61 percent were "hit" or executed across both buy limit opens and sell limit opens (three hits for every five orders). By volume, however, the hit percentage on limit orders was significantly lower. Out of all the buy limit open orders, only 39 percent were hit (EUR 390 hit for every EUR 1,000 in buy limit open orders). Out of all the sell limit open orders, the hit rate

was even lower: just 29 percent (EUR 290 hit out of every EUR 1,000 sell limit open orders placed). The observed discrepancy probably reflects the relative propensity of higher-volume traders to seek bargains—that is to place limit open orders farther away from the market in the hope of entering a position at a lower buy or a higher sell.

Among the opened positions, long and short, discrepancies persisted relating to the position closing method. A comparison of columns 2 and 3 in Table 6.4 shows that larger customers were less likely to close their positions using take-profit and stop-loss orders than were smaller customers and that larger customers preferred to close their positions using market orders instead. This finding may reflect the relative sophistication of larger customers: since all the take-profit and stop-loss orders may be artificially triggered by Oanda's proprietary trading team, larger customers may be posting well-timed market orders instead. However, among those customers using take-profit and stop-loss provisions, smaller customers had a higher success ratio: by the number of orders, customers took profit on 21 percent of orders and experienced stop-losses on 16 percent of orders. By volume, customers took profit on only 9 percent of orders and experienced stop-losses on 13 percent of orders.

CONCLUSION

Diversity of order types allows traders to build complex trading strategies by changing price, timing, transparency, and other parameters of orders. Still, simple market and limit orders retain their popularity in the trading community because of their versatility and ease of use.

CHAPTER 7

Market Inefficiency and Profit Opportunities at Different Frequencies

The key feature that unites all types of high-frequency trading strategies is persistence of the underlying tradable phenomena. This part of the book addresses the ways to identify these persistent trading opportunities.

High-frequency trading opportunities range from microsecond price moves allowing a trader to benefit from market-making trades, to several-minute-long strategies that trade on momentum forecasted by microstructure theories, to several-hour-long market moves surrounding recurring events and deviations from statistical relationships. Dacorogna et al. (2001) emphasize a standard academic approach to model development:

1. Document observed phenomena.
2. Develop a model that explains the phenomena.
3. Test the model's predictive properties.

The development of high-frequency trading strategies begins with identification of recurrent profitable trading opportunities present in high-frequency data. The discourse on what is the most profitable trading frequency often ends once the question of data availability emerges and researchers cannot quantify the returns of strategies run at different frequencies. Traders that possess the data shun the public limelight because they are using the data to successfully run high-frequency strategies. Other sources tend to produce data from questionable strategies.

The profitability of a trading strategy is bound by the chosen trading frequency. At the daily trading frequency, the maximum profit and loss are

limited by the daily range of price movements. At the hourly frequency, the possible range of the price movement shrinks, but the number of hourly ranges in the day increases to 7 in most equities and 24 in foreign exchange. The total potential gain is then the sum of all intra-hour ranges recorded during the day. At even higher frequencies, the ranges of price movements tighten further, and the number of ranges increases to provide even higher profitability.

Table 7.1 shows the maximum gain potential and other high-frequency range statistics for SPY (S&P 500 Depository Receipts ETF) and EUR/USD at different frequencies recorded for April 21, 2009. The maximum gain is calculated as the sum of price ranges at each frequency. The maximum gains of SPY and EUR/USD are then normalized by the daily open prices of SPY and EUR/USD, respectively, to show the relative gain in percentages. The maximum gain potential at every frequency is determined by the sum of all per-period ranges at that frequency.

The gain potential in the high-frequency space is nothing short of remarkable, as is the maximum potential loss, which is equal to the negative maximum gain. Careful strategy design, extensive back testing, risk management, and implementation are needed to realize the high-frequency gain potential.

The profitability of a trading strategy is often measured by Sharpe ratios, a risk-adjusted return metric first proposed by Sharpe (1966). As Table 7.2 shows, maximum Sharpe ratios increase with increases in trading frequencies. From March 11, 2009, through March 22, 2009, the maximum possible annualized Sharpe ratio for EUR/USD trading strategies with daily position rebalancing was 37.3, while EUR/USD trading strategies that held positions for 10 seconds could potentially score Sharpe ratios well over the 5,000 mark.

The maximum possible intra-day Sharpe ratio is computed as a sample period's average range divided by the sample period's standard deviation of the range, adjusted by square root of the number of observations in a year:

$$SR = \frac{E[\text{Range}]}{\sigma[\text{Range}]} \times \sqrt{(\#\text{ Intra-day Periods}) \times (\#\text{ Trading Days in a Year})} \tag{7.1}$$

Note that high-frequency strategies normally do not carry overnight positions and, therefore, do not incur the overnight carry cost often proxied by the risk-free rate in Sharpe ratios of longer-term investments.

In practice, well-designed and -implemented strategies trading at the highest frequencies tend to produce the highest profitability with the double-digit Sharpe ratios. Real-life Sharpe ratios for well-executed strategies with daily rebalancing typically fall in the 1–2 range.

TABLE 7.1 Maximum Gain Potential and Other Range Statistics for SPY and EUR/USD at Different Frequencies on April 21, 2009

SPY

	Period Duration				
Statistic	**10 sec**	**1 min**	**10 min**	**1 hour**	**1 day**
Maximum Gain Potential per Day (Sum of All per-Period Ranges)	96.33 percent	44.59 percent	13.96 percent	5.66 percent	1.09 percent
Average Range per Period	0.04 percent	0.11 percent	0.35 percent	0.81 percent	1.09 percent
Number of Intra-Day Periods	2340	390	39	7	1

EUR/USD

	Period Duration				
Statistic	**10 sec**	**1 min**	**10 min**	**1 hour**	**1 day**
Maximum Gain Potential per Day (Sum of All per-Period Ranges)	319.23 percent	90.07 percent	18.48 percent	6.44 percent	0.57 percent
Average Range per Period	0.04 percent	0.06 percent	0.13 percent	0.27 percent	0.57 percent
Number of Intra-Day Periods	8640	1440	144	24	1

TABLE 7.2 Comparison of Maximum Sharpe Ratios Achievable by an Ideal Strategy with Perfect Predictability of Intra-Period Price Movement in EUR/USD. (The results are computed ex-post with 20/20 hindsight on the data for 30 trading days from February 9, 2009 through March 22, 2009.)

	Average Maximum Gain (Range) per Period	Range Standard Deviation per Period	Number of Observations in the Sample Period	Maximum Annualized Sharpe Ratio
10 seconds	0.04 percent	0.01 percent	2,592,000	5879.8
1 minute	0.06 percent	0.02 percent	43,200	1860.1
10 minutes	0.12 percent	0.09 percent	4,320	246.4
1 hour	0.30 percent	0.19 percent	720	122.13
1 day	1.79 percent	0.76 percent	30	37.3

PREDICTABILITY OF PRICE MOVES AT HIGH FREQUENCIES

Predictability and Market Efficiency

Every trader and trading system aims to generate trading signals that result in consistently positive outcomes over a large number of trades. In seeking such signals, both human traders and econometricians designing systematic trading platforms are looking to uncover sources of predictability of future price movements in selected securities. Predictability, both in trading and statistics, is the opposite of randomness. It is, therefore, the objective of every trading system to find ways to distinguish between predictable and random price moves and then to act on the predictable ones.

While the future is never certain, history can offer us some clues about how the future may look given certain recurring events. All successful trading systems, therefore, are first tested on large amounts of past historical data. Technical analysts, for example, pore over historical price charts to obtain insights into past price behaviors. Fundamental analysts often run multiple regressions to determine how one fundamental factor influences another. High-frequency trading system developers run their models through years of tick data to ascertain the validity of their trading signals.

A more scientific method for analyzing a particular financial security may lie in determining whether price changes of the security are random or not. If the price changes are indeed random, the probability of detecting a consistently profitable trading opportunity for that particular security is small. On the other hand, if the price changes are nonrandom, the financial security has persistent predictability, and should be analyzed further.

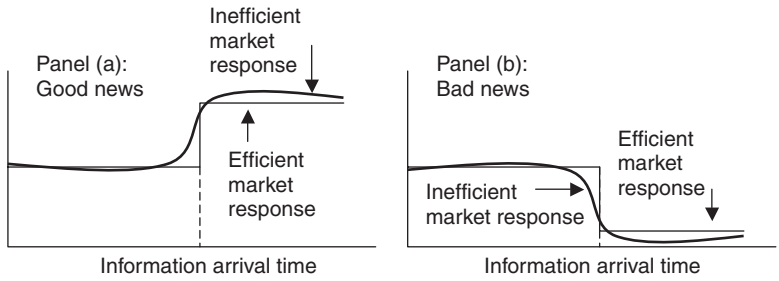

FIGURE 7.1 Incorporation of information in efficient and inefficient markets.

The relative availability of trading opportunities can be measured as a degree of market inefficiency. An efficient market (Fama, 1970) should instantaneously reflect all the available information in prices of the traded securities. If the information is impounded into the securities slowly, then arbitrage opportunities exist, and the market is considered to be inefficient. Figure 7.1 illustrates the idea of efficient versus inefficient markets. To identify markets with arbitrage opportunities is to find inefficient markets. The arbitrage opportunities themselves are market inefficiencies.

In Figure 7.1, panel (a) shows efficient and inefficient market responses to "good" information that pushes the price of the security higher, while panel (b) shows efficient and inefficient market responses to "bad" information that lowers the price of the security. The price of the security in the efficient market adjusts to the new level instantaneously at the time the news comes out. The price of the security in the inefficient market begins adjusting before the news becomes public ("information leakage"), and usually temporarily overreacts ("overshoots") once the news becomes public. Many solid trading strategies exploit both the information leakage and the overshooting to generate consistent profits.

Testing for Market Efficiency and Predictability

The more inefficient the market, the more predictable trading opportunities become available. Tests for market efficiency help discover the extent of predictable trading opportunities. This chapter considers several tests for market efficiency designed to help the researchers to select the most profitable markets. The chapter by no means considers all the market efficiency tests that have been proposed in the academic literature; rather, it summarizes key tests with varying degrees of complexity, from the simplest to the most advanced.

The market efficiency hypothesis has several "levels": weak, semi-strong, and strong forms of market efficiency. Tests for the weak form of market efficiency measure whether returns can be predicted by their past

prices and returns alone. Other forms of market efficiency restrict the kinds of information that can be considered in forecasting prices. The strong form deals with all kinds of public and nonpublic information; the semi-strong form excludes nonpublic information from the information set. As in most contemporary academic literature on market efficiency, we restrict the tests to the weak form analysis only.

Non-Parametric Runs Test Several tests of market efficiency have been developed over the years. The very first test, constructed by Louis Bachelier in 1900, measured the probability of a number of consecutively positive or consecutively negative price changes, or "runs." As with tossing a fair coin, the probability of two successive price changes of the same sign (a positive change followed by a positive change, for example) is $1/(2^2) = 0.25$. The probability of three successive price changes of the same sign is $1/(2^3) = 0.125$. Four successive price changes of the same sign are even less likely, having the probability of $1/(2^4) = 0.0625$ or 6.25 percent. Several price changes of the same sign in a row present a trading opportunity at whichever frequency one chooses to consider, the only requirement being the ability to overcome the transaction costs accompanying the trade.

The test works as follows:

1. Within a sample containing price moves of the desired frequency, note the number of sequences consisting strictly of price moves of the same sign. If the desired frequency is every tick, then a run can be a sequence of strictly positive or strictly negative price increments from one tick to the next. If the desired frequency is one minute, a run can be a sequence of strictly positive or strictly negative price increments measured at 1-minute intervals. Table 7.3 shows 1-minute changes in the AUD/USD exchange rate that occurred from 16:00 GMT to 16:20 GMT on June 8, 2009.

   The 1-minute changes are calculated as follows: if the closing price for AUD/USD recorded for a given minute is higher than that for the previous minute, the change for the given minute is recorded to be positive. If the closing price for AUD/USD recorded for a given minute is lower than that for the previous minute, the change for the given minute is recorded to be negative. As shown in Table 7.3, from 16:00 GMT to 16:20 GMT, there were in total nine minutes with positive change in AUD/USD, eight minutes with negative change in AUD/USD, and three minutes with 0 change in AUD/USD. Altogether, there were six positive runs whereby the price of AUD/USD increased, and four negative runs where the price of AUD/USD decreased. Positive runs are marked "P" in the "Runs" column, and negative runs are marked "N." Thus, minutes marked "P2" correspond to the second run of

sequential positive changes, and minutes marked "N1" correspond to the first run of negative changes.

2. Denote the total number of runs, both positive and negative, observed in the sample as u. Furthermore, denote as n_1 the number of positive 1-minute changes in the sample, and as n_2 the number of negative 1-minute changes in the sample. In the sample shown in Table 7.3, $u = 10$, $n_1 = 9$, and $n_2 = 8$.
3. If price changes were completely random, it can be shown that the expected number of runs in a random sample is $\bar{x} = \frac{2n_1n_2}{n_1+n_2} + 1$, with the standard deviation of runs being

$$s = \sqrt{\frac{2n_1n_2(2n_1n_2 - n_1 - n_2)}{(n_1 + n_2)^2(n_1 + n_2 - 1)}}.$$

In the example of Table 7.3, $\bar{x} = 9.47$, and $s = 1.99$.

TABLE 7.3 One-Minute Closing Price Data on AUD/USD Recorded on June 8, 2009, from 16:00 to 16:20 GMT

DATE	TIME	Close	1-Minute Change	Sign of the Change	Runs
6/8/2009	16:00	0.7851			
6/8/2009	16:01	0.7853	0.0002	+	P1
6/8/2009	16:02	0.7848	−0.0005	−	N1
6/8/2009	16:03	0.7841	−0.0007	−	N1
6/8/2009	16:04	0.784	−1E-04	−	N1
6/8/2009	16:05	0.7842	0.0002	+	P2
6/8/2009	16:06	0.7844	0.0002	+	P2
6/8/2009	16:07	0.7845	1E-04	+	P2
6/8/2009	16:08	0.7845	0		
6/8/2009	16:09	0.7847	0.0002	+	P3
6/8/2009	16:10	0.7849	0.0002	+	P3
6/8/2009	16:11	0.7846	−0.0003	−	N2
6/8/2009	16:12	0.7845	−1E-04	−	N2
6/8/2009	16:13	0.7839	−0.0006	−	N2
6/8/2009	16:14	0.7841	0.0002	+	P4
6/8/2009	16:15	0.7841	0		
6/8/2009	16:16	0.7837	−0.0004	−	N3
6/8/2009	16:17	0.7842	0.0005	+	P5
6/8/2009	16:18	0.784	−0.0002	−	N4
6/8/2009	16:19	0.784	0		
6/8/2009	16:20	0.7842	0.0002	+	P6

4. Next, we test whether the realized number of runs indicates statistical nonrandomness. The runs at the selected frequency are deemed predictable, or nonrandom, with 95 percent statistical confidence if the number of runs is at least 1.654 standard deviations s away from the mean $\bar{x}$. The number of runs is not random if the two-tailed test based on Z-score is rejected. The Z-score is computed as $Z = \frac{|u - \bar{x}| - 0.5}{s}$. In other words, the randomness of runs is rejected with 95 percent statistical confidence whenever Z is greater than 1.645. The randomness of runs cannot be rejected if $Z < 1.645$.

In the example of Table 7.3, $Z = 0.0147$; therefore, randomness of 1-minute changes in the sample cannot be rejected.

Table 7.4 summarizes runs test Z-scores obtained for several securities on data of different frequency for all data of June 8, 2009. As Table 7.4 shows, the runs test rejects randomness of price changes at 1-minute frequencies, except for prices on S&P 500 Depository Receipts (SPY). The results imply strong market inefficiency in 1-minute data for the securities shown. Market inefficiency measured by runs test decreases or disappears entirely at a frequency lower than 10 minutes.

Tests of Random Walks Other, more advanced tests for market efficiency have been developed over the years. These tests help traders evaluate the state of the markets and reallocate trading capital to the markets with the most inefficiencies—that is, the most opportunities for reaping profits.

When price changes are random, they are said to follow a "random walk." Formally, a random walk process is specified as follows:

$$\ln P_t = \ln P_{t-1} + \varepsilon_t \tag{7.2}$$

where $\ln P_t$ is the logarithm of the price of the financial security of interest at time t, $\ln P_{t-1}$ is the logarithm of the price of the security one time

TABLE 7.4 Non-Parametric Runs Test Applied to Data on Various Securities and Frequencies Recorded on June 8, 2009.

Ticker	Data Frequency	N_1	N_2	u	Z	Result
SPY	1 minute	179	183	193	1.11	Random
SPY	10 minutes	21	17	20	−0.10	Random
T	1 minute	142	138	187	5.45	Predictable
T	10 minutes	18	17	20	0.35	Random
USD/JPY	1 minute	558	581	775	12.11	Predictable
USD/JPY	10 minutes	68	64	82	2.55	Predictable
XAU/USD	1 minute	685	631	778	6.61	Predictable
XAU/USD	10 minutes	75	66	76	0.73	Random

interval removed at a predefined frequency (minute, hour, etc.), and ε_t is the error term with mean 0. From equation (7.2), log price changes $\Delta \ln P_t$ are obtained as follows:

$$\Delta \ln P_t = \ln P_t - \ln P_{t-1} = \varepsilon_t$$

At any given time, the change in log price is equally likely to be positive and negative. The logarithmic price specification ensures that the model does not allow prices to become negative (logarithm of a negative number does not exist).

The random walk process can drift, and be specified as shown in equation (7.3):

$$\ln P_t = \mu + \ln P_{t-1} + \varepsilon_t \tag{7.3}$$

In this case, the average change in prices equals the drift rather than 0, since $\Delta \ln P_t = \ln P_t - \ln P_{t-1} = \mu + \varepsilon_t$. The drift can be due to a variety of factors; persistent inflation, for example, would uniformly lower the value of the U.S. dollar, inflicting a small positive drift on prices of all U.S. equities. At very high frequencies, however, drifts are seldom noticeable.

Lo and MacKinlay (1988) developed a popular test for whether or not a given price follows a random walk. The test can be applied to processes with or without drift. The test procedure is built around the following principle: if price changes measured at a given frequency (e.g., one hour) are random, then price changes measured at a lower frequency (e.g., two hours) should also be random. Furthermore, the variances of the 1-hour and 2-hour changes should be deterministically related. Note that the reverse does not apply; randomness in 1-hour price changes does not imply randomness in 10-minute price changes, nor does it imply a relationship in variances between the 1-hour and 10-minute samples.

The test itself is based on the following estimators:

$$\hat{\mu} = \frac{1}{2n}\sum_{k=1}^{2n}(\ln P_k - \ln P_{k-1}) = \frac{1}{2n}(\ln P_{2n} - \ln P_0) \tag{7.4}$$

$$\hat{\sigma}_a^2 = \frac{1}{2n}\sum_{k=1}^{2n}(\ln P_k - \ln P_{k-1} - \hat{\mu})^2 \tag{7.5}$$

$$\hat{\sigma}_b^2 = \frac{1}{2n}\sum_{k=1}^{n}(\ln P_{2k} - \ln P_{2k-2} - 2\hat{\mu})^2 \tag{7.6}$$

If the error term ε_t in equations (7.2) or (7.3) is a sequence of independent, identically normally distributed numbers with mean 0 and variance

$\sigma_0^2,\ \varepsilon_t \sim i.i.d.N(0, \sigma_0^2)$, then Lo and MacKinlay (1988) show that the differences in parameters σ_0^2, $\hat{\sigma}_a^2$ (7.5), and $\hat{\sigma}_b^2$ (7.6) are asymptotically distributed as follows:

$$\sqrt{2n}(\hat{\sigma}_a^2 - \sigma_0^2) \overset{a}{\sim} N(0, 2\sigma_0^4) \tag{7.7}$$

$$\sqrt{2n}(\hat{\sigma}_b^2 - \sigma_0^2) \overset{a}{\sim} N(0, 4\sigma_0^4) \tag{7.8}$$

The test for market efficiency is then performed as specified by equation (7.9):

$$J_r \equiv \frac{\hat{\sigma}_b^2}{\hat{\sigma}_a^2} - 1,\ \sqrt{2n}J_r \overset{a}{\sim} N(0, 2) \tag{7.9}$$

Lo and MacKinlay (1988) subsequently use the test on daily, weekly, and monthly equity data and conclude that while market efficiency cannot be rejected for weekly and monthly frequency, daily equity prices are not efficient.

Table 7.5 summarizes the results of the variance ratio test applied to several foreign exchange instruments across different frequencies. The rows represent the estimated values for the variance ratio test J_r, as defined by equation (7.9). The parentheses show the value of the test statistic measuring the deviation of J_r from 0. If the time series follows a random walk, then the test statistic J_r will have a normal distribution.

When applied to daily data of the S&P 500 index, the variance ratio test produced mean J_r of 0.7360 with the corresponding test statistic of 13.84, significant at 0.001 percent. As Table 7.5 shows, each of the six major USD crosses are more efficient than the S&P 500: deviations of the variance ratio test statistic, J_r, from 0 are less statistically significant for major USD crosses and their derivatives than they are for S&P 500, even at high frequencies. The relative inefficiency of S&P 500 signifies that S&P 500 has more arbitrage opportunities than do the six USD currencies.

According to Table 7.5, daily spot in USD/CAD is the most efficient currency pair with the fewest number of arbitrage opportunities among the six major USD-crosses. USD/CAD together with USD/JPY are the most efficient USD-based pairs in put options written on JPY/USD and CAD/USD futures with the nearest expiration date.

As Table 7.5 shows, the efficiency of spot instruments decreases—that is, the number of arbitrage opportunities increases—with increases in data sampling frequency. For example, the inefficiency of EUR/USD daily spot rate is higher when EUR/USD is measured at 1-hour intervals than when it is measured at daily intervals, as evidenced by a higher t-statistic accompanying the daily and hourly estimates.

TABLE 7.5 The variance ratios J_r defined by equation (7.9) are reported in the rows, with the heteroscedasticity-robust test statistics $z^*(q)$ given in parentheses immediately below each row. Under the random walk null hypothesis, the value of J_r is 0 and the test statistics have a standard normal distribution (asymptotically). Test statistics marked with asterisks indicate that the corresponding variance ratios are statistically different from 0 at the 5 percent level of significance. The estimation was conducted on data spanning the two-month period of November and December 2008.

Currency Pair	Daily Spot	5-min Spot	15-min Spot	1-Hr Spot	1-Hr Futures	1-Hr Call Options	1-Hr Put Options
AUD/USD	0.6354	0.7320	0.7310	0.7128	0.7176	0.6851	0.6937
	(2.27)*	(8.74)*	(9.21)*	(4.90)*	(5.48)*	(3.72)*	(3.75)*
USD/CAD	0.5672	0.7301	0.7278	0.7124	0.7181	0.6919	0.6614
	(1.89)	(7.97)*	(8.61)*	(4.90)*	(5.48)*	(3.87)*	(2.98)*
USD/CHF	0.6356	0.7325	0.7315	0.7123	0.6952	0.6796	0.6850
	(2.27)*	(8.85)*	(9.77)*	(5.00)*	(4.14)*	(3.57)*	(3.80)*
EUR/USD	0.6355	0.7359	0.7337	0.7135	0.7189	0.6912	0.6836
	(2.27)*	(15.07)*	(10.08)*	(5.00)*	(5.57)*	(3.87)*	(3.77)*
GBP/USD	0.6658	0.7346	0.7263	0.7068	0.6927	0.6962	0.6871
	(2.27)*	(11.00)*	(7.01)*	(4.84)*	(3.72)*	(3.88)*	(3.81)*
USD/JPY	0.6353	0.7386	0.7322	0.7129	0.7123	0.6580	0.6384
	(2.27)*	(17.37)*	(9.83)*	(5.00)*	(5.36)*	(2.88)*	(2.61)*

As Table 7.5 further shows, spot foreign exchange rates at daily frequencies have the fewest number of persistent trading opportunities. Surprisingly, 1-hr put and call options have fewer trading opportunities than do spot and futures at the same frequency. Higher frequency markets, however, show more persistent profit pockets, further strengthening the case for already popular high-frequency trading styles.

As documented in Table 7.5, even in foreign exchange, value opportunities exist at high frequencies. Extracting that value requires much ingenuity, speed, and precision and can be a time-consuming and expensive process. The tests for market efficiency save traders time and money by enabling the selection of the most profitable financial instruments and frequencies prior to committing to development of trading models.

Autoregression-Based Tests Trading strategies perform best in the least efficient markets, where abundant arbitrage opportunities exist. Perfectly efficient markets instantaneously incorporate all available market information, allowing no dependencies from past price movements. One way to measure the relative degree of market efficiency, therefore, is to estimate the explanatory power of past prices. Mech (1993) and Hou and Moskowitz (2005), for example, propose to measure market efficiency as the difference between *Adjusted* R^2 coefficients of an unrestricted model attempting to explain returns with lagged variables and of a restricted model involving no past data.

The unrestricted model is specified as follows:

$$r_{i,t} = \alpha_i + \beta_{i,1} r_{i,t-1} + \beta_{i,2} r_{i,t-2} + \beta_{i,3} r_{i,t-3} + \beta_{i,4} r_{i,t-4} + \varepsilon_{i,t} \tag{7.10}$$

where $r_{i,t}$ is the return on security i at time t (see Chapter 8 for a detailed discussion on the computation of returns).

The restricted model restricts all coefficients $\beta_{i,j}$ to be 0:

$$r_{i,t} = \alpha_i + \varepsilon_{i,t} \tag{7.11}$$

Market inefficiency is next calculated as the relative difference between Ordinary Least Squares (OLS) R^2 coefficients of the two models:

$$\textit{Market Inefficiency} = 1 - \frac{R^2_{\text{Restricted}}}{R^2_{\text{Unrestricted}}} \tag{7.12}$$

The closer the difference is to 0, the smaller the influence of past price movements and the higher the market efficiency.

Market Efficiency Tests Based on the Martingale Hypothesis A classic definition of market efficiency in terms of security returns is due to

Samuelson (1965), who showed that properly anticipated prices fluctuate randomly in an efficient market. In other words, if all of the news is incorporated instantaneously into the price of a given financial security, the expected price of the security given current information is always the current price of the security itself. This relationship is known as a martingale. Formally, a stochastic price process $\{P_t\}$ is a martingale within information set I_t if the best forecast of P_{t+1} based on current information I_t is equal to P_t:

$$E[P_{t+1}|I_t] = P_t \tag{7.13}$$

Applying the martingale hypothesis to changes in price levels, we can express "abnormal," or returns in excess of expected returns given current information, as follows:

$$Z_{t+1} = \Delta P_{t+1} - E[\Delta P_{t+1}|I_t] \tag{7.14}$$

A market in a particular financial security or a portfolio of financial securities is then said to be efficient when abnormal return Z_{t+1} is a "fair game"—that is,

$$E[Z_{t+1}|I_t] = 0 \tag{7.15}$$

LeRoy (1989) provides an extensive summary of the literature on the subject.

A financial securities market characterized by fair game returns is efficient as it lacks consistent profit opportunities. As Fama (1991) pointed out, in a market with trading costs, equation (7.15) will hold within trading cost deviations.

Fama (1991) also suggested that the efficient markets hypothesis is difficult to test for the following reason: the idea of a market fully reflecting all available information contains a joint hypothesis. On the one hand, expected values of returns are a function of information. On the other hand, differences of realized returns from their expected values are random. Incorporating both issues in the same test is difficult. Nevertheless, martingale-based tests for market efficiencies exist.

Froot and Thaler (1990), for example, derive a specification for a test of market efficiency of a foreign exchange rate. In equilibrium, foreign exchange markets follow the uncovered interest rate parity hypothesis that formulates the price of a foreign exchange rate as a function of interest rates in countries on either side of the interest rate. Under the uncovered interest rate parity, an expected change in the equilibrium spot foreign exchange rate S, given that the information set I_t is a function of the interest

rate differential between domestic and foreign interest rates, $r_t - r_t^d$ and risk premium ξ_t of the exchange rate:

$$E[\Delta S_{t+1}|I_t] = r_t - r_t^d + \xi_t \tag{7.16}$$

where the risk premium ξ_t is zero for risk-neutral investors and is diversifiable to zero for others.

In addition, following the martingale hypothesis, realized spot exchange rate at time $t+1$, S_{t+1} is related to its ex-ante expectation $E[S_{t+1}|I_t]$ as follows:

$$S_{t+1} = E[S_{t+1}|I_t] + u_{t+1} \tag{7.17}$$

where $E[u_{t+1}|I_t] = 0$. Combining equations (7.16) and (7.17) yields the following, information-independent test for market efficiency of a foreign exchange rate:

$$\Delta S_{t+1} = r_t - r_t^d + \varepsilon_{t+1} \tag{7.18}$$

where $\{\varepsilon_t\}$ series is independent, identically distributed with mean 0.

Taking exponents of both sides of equation (7.18), the test can be specified in a forward-rate form as follows:

$$\log S_{t+1} = \log F_t + \upsilon_{t+1} \tag{7.19}$$

where mean of υ_t is $E[\upsilon_t] = 0$ and variance of υ_t is σ_υ^2.

The specification of equation (7.19) produces a testable hypothesis at low frequencies as forward contracts and their open market counterparts (i.e., futures) typically mature once a quarter. Most low-frequency tests reject predictability of spot rates with forward rates. For example, Hodrick (1987) notes that none of the pre-1987 tests involving forward rates to forecast spot rates fit the data. Engel (1996) supports Hodrick's (1987) conclusions and further notes that even when the risk premium in the specification of equation (7.18) is assumed to differ from 0, the risk premium fails to explain the lack of predictability of the forward model. Alexakis and Apergis (1996), however, find that the forward rates indeed accurately predict spot rates when predictability is measured in an ARCH specification (ARCH is discussed in Chapter 8). A high-frequency specification, nonetheless, is easy to derive as a differential between two subsequent realizations of equation (7.19):

$$\log\left[\frac{S_{t+1}}{S_t}\right] = \log\left[\frac{F_t}{F_{t-1}}\right] + \Delta\upsilon_{t+1} \tag{7.20}$$

with mean of $\Delta\upsilon_t$ is $E[\Delta\upsilon_t] = 0$ and variance of $\Delta\upsilon_t$ is $2\sigma_\upsilon^2$.

Cointegration-Based Tests of Market Efficiency Another test of market efficiency is based on the Engle and Granger (1987) representation theorem that suggests that cointegration between two variables implies systematic predictability. For example, if some market factor X, say log forward rate, predicts spot exchange rate S according to specification $S_t = b_0 + b_1 X_t + \varepsilon_t$, where ε_t is stationary (has a consistent distribution over time) and $E[\varepsilon_t] = 0$, then a cointegration-based test for ascertaining dependency of S on X has the following specification:

$$\Delta S_t = \alpha(b_0 + b_1 X_{t-1} - S_{t-1}) + \beta \Delta X_{t-1} + \gamma \Delta S_{t-1} + \eta_t \tag{7.21}$$

where η_t is an independent, identically distributed error term with mean 0, α measures the speed of the model's adjustment to its long-term equilibrium, and β and γ measure short-term impact of lagged changes in X and S.

Evidence of cointegration on daily closing rates of three or more currency pairs has been documented by Goodhart (1988), Hakkio and Rush (1989), Coleman (1990), and Alexander and Johnson (1992), among others.

Literature on the efficient markets hypothesis in foreign exchange further distinguishes between speculative and arbitraging efficiencies. The speculative efficiency hypothesis due to Hansen and Hodrick (1980) proposes that the expected rate of return from speculation in the forward market conditioned on available information is zero. The arbitraging efficiency hypothesis puts forward that the expected return on a portfolio composed of long one unit of currency and short one future contract on that unit of currency is zero. The arbitrage strategy of buying one unit of currency and selling one futures contract is known as uncovered interest arbitrage. The strategy attempts to arbitrage the uncovered interest parity.

CONCLUSION

The tests of market efficiency illuminate different aspects of a security's price and return dependency on other variables. Taking advantage of market inefficiency requires an understanding of the different tests that identified the inefficiency in the first place.

The same security may be predictable at one frequency and fully random at another frequency. Various combinations of securities may have different levels of efficiency. While price changes of two or more securities may be random when securities are considered individually, the price changes of a combination of those securities may be predictable, and vice versa.

CHAPTER 8

Searching for High-Frequency Trading Opportunities

This chapter reviews the most important econometric concepts used in the subsequent parts of the book. The treatment of topics is by no means exhaustive; it is instead intended as a high-level refresher on the core econometric concepts applied to trading at high frequencies. Yet, readers relying on software packages with preconfigured statistical procedures may find the level of detail presented here to be sufficient for quality analysis of trading opportunities. The depth of the statistical content should be also sufficient for readers to understand the models presented throughout the remainder of this book. Readers interested in a more thorough treatment of statistical models may refer to Tsay (2002); Campbell, Lo, and MacKinlay (1997); and Gouriéroux and Jasiak (2001).

This chapter begins with a review of the fundamental statistical estimators, moves on to linear dependency identification methods and volatility modeling techniques, and concludes with standard nonlinear approaches for identifying and modeling trading opportunities.

STATISTICAL PROPERTIES OF RETURNS

According to Dacorogna et al. (2001, p. 121), "high-frequency data opened up a whole new field of exploration and brought to light some behaviors that could not be observed at lower frequencies." Summary statistics about aggregate behavior of data, known as "stylized facts," help distill particularities of high-frequency data. Dacorogna et al. (2001) review stylized facts

for foreign exchange rates, interbank money market rates, and Eurofutures (futures on Eurodollar deposits).

Financial data is typically analyzed using returns. A return is a difference between two subsequent price quotes normalized by the earlier price level. Independent of the price level, returns are convenient for direct performance comparisons across various financial instruments. A simple return measure can be computed as shown in equation (8.1):

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} - 1 \tag{8.1}$$

where R_t is the return for period t, P_t is the price of the financial instrument of interest in period t, and P_{t-1} is the price of the financial instrument in period $t - 1$. As discussed previously, determination of prices in high-frequency data may not always be straightforward; quotes arrive at random intervals, but the analysis demands that the data be equally spaced.

Despite the intuitiveness of simple returns, much of the financial literature relies on log returns. Log returns are defined as follows:

$$r_t = \ln(R_t) = \ln(P_t) - \ln(P_{t-1}) \tag{8.2}$$

Log returns are often preferred to simple returns for the following reasons:

1. If log returns are assumed to follow a normal distribution, then the underlying simple returns and the asset prices used to compute simple returns follow a lognormal distribution. Lognormal distributions better reflect the actual distributions of asset prices than do normal distributions. For example, asset prices are generally positive. Lognormal distribution models this property perfectly, whereas normal distributions allow values to be negative.
2. Like distributions of asset prices, lognormal distributions have fatter tails than do normal distributions. Although lognormal distributions typically fail to model the fatness of the tails of asset prices exactly, lognormal distributions better approximate observed fat tails than do normal distributions.
3. Once log prices have been computed, log returns are easy and fast to manipulate.

Returns can be computed on bid prices, ask prices, last trade prices, or mid prices. Mid prices can be taken to be just an arithmetic average, or a mid-point between a bid and an ask price at any point in time. In the

absence of synchronous quotes, mid prices can be computed using the last bid and ask quotes.

Both simple and log returns can be averaged over time to obtain lower-frequency return estimates. An average of simple and log returns can be computed as normal arithmetic averages:

$$E[R] = \frac{1}{T}\sum_{t=1}^{T} R_t \tag{8.3}$$

$$\mu = \frac{1}{T}\sum_{t=1}^{T} r_t \tag{8.4}$$

Variation in sequential returns is known as volatility. Volatility can be measured in a variety of ways. The simplest measure of volatility is variance of simple or log returns, computed according to equations (8.5) and (8.6).

$$\text{var}\,[R] = \frac{1}{T-1}\sum_{t=1}^{T} (R_t - E[R])^2 \tag{8.5}$$

$$\sigma^2 = \frac{1}{T-1}\sum_{t=1}^{T} (r_t - \mu)^2 \tag{8.6}$$

Note that the division factor in volatility computation is $(T-1)$, not T. The reduced number of normalizing observations accounts for reduced number of degrees of freedom—the variance equation includes the average return, which in most cases is itself estimated from the sample data. Standard deviation is a square root of the variance.

Other common statistics used to describe distributions of prices or simple or log returns are skewness and kurtosis. Skewness measures whether a distribution skews towards either the positive or the negative side of the mean, as compared with the standardized normal distribution. Skewness of the standardized normal distribution is 0. Skewness can be measured as follows:

$$S[R] = \frac{1}{T-1}\frac{\sum_{t=1}^{T} (R_t - E[R])^3}{(\text{var}[R])^{3/2}} \tag{8.7}$$

Kurtosis is a measure of fatness of the tails of a distribution. The fatter the tails of a return distribution, the higher the chance of an extreme positive or negative return. Extreme negative returns can be particularly damaging to a trading strategy, potentially wiping out all previous profits

and even equity capital. The standardized normal distribution has a kurtosis of 3. Kurtosis can be computed as follows:

$$K[R] = \frac{1}{T-1}\frac{\sum_{t=1}^{T}(R_t - E[R])^4}{(\text{var}[R])^2} \tag{8.8}$$

If returns indeed follow a lognormal distribution (i.e., log returns r_t are normally distributed with mean μ and standard deviation σ^2), then the mean and the standard deviation of simple returns has the following properties:

$$\begin{aligned} E[R_t] &= \exp\left(\mu - \frac{\sigma^2}{2}\right) - 1 \\ \text{var}[R_t] &= \exp\left(2\mu + \sigma^2\right)\left[\exp(\sigma^2) - 1\right] \end{aligned} \tag{8.9}$$

Table 8.1 shows statistics for log returns of EUR/USD. The log returns are calculated from closing trade prices observed during each period at different frequencies. If no trades are observed during a particular time period, the closing trade price from the previous period is used in the estimation. As Table 8.1 illustrates, the higher the data frequency, the higher the kurtosis of the data—that is, the fatter the tails of the data distribution.

Another metric useful to describe distributions of returns is autocorrelation, which is a measure of serial dependence between subsequent

TABLE 8.1 Summary Statistics for Log Returns for Different Instruments of EUR/USD at Various Frequencies and for Different Securities (The log returns are computed from closing trade prices sampled at different frequencies on data for August–November 2008.)

Frequency	Mean	Max	Median	Min	Standard Deviation	Skewness	Kurtosis
Daily Spot	−0.0003	0.0119	0.0001	−0.0179	0.0052	−0.4389	3.3051
5-min Spot	−0.0001	0.0184	0.0000	−0.0225	0.0009	−1.4723	83.7906
15-min Spot	−0.0001	0.0186	0.0000	−0.0226	0.0015	−1.0278	32.6960
1-Hr Spot	−0.0002	0.0213	−0.0001	−0.0228	0.0029	−0.8184	14.5028
1-Hr Futures	−0.0002	0.0213	−0.0001	−0.0300	0.0032	−0.7207	13.5256
1-Hr Call Options	−0.0001	0.6551	−0.0031	−0.6555	0.1411	0.3333	8.3108
1-Hr Put Options	0.0000	2.1115	0.0022	−1.7717	0.1736	0.9859	51.8545

returns sampled at a specific frequency. For example, autocorrelation of order 1 measures of 1-minute returns is a correlation of 1-minute returns with 1-minute returns that occurred 1 minute earlier. Autocorrelation of order 2 measures of 1-minute returns is a correlation of 1-minute returns with 1-minute returns that occurred 2 minutes earlier. The autocorrelation value of order p can be determined as follows:

$$\rho(p) = \frac{\sum_{t=p+1}^{T} \left[(R_t - E[R])(R_{t-p} - E[R])\right]}{\left(\sum_{t=p+1}^{T} (R_t - E[R])\right)^{1/2} \left(\sum_{t=p+1}^{T} (R_{t-p} - E[R])\right)^{1/2}} \tag{8.10}$$

As any other correlation function, $\rho(p)$ ranges from -1 to 1. Equation (8.10) uses simple returns to compute autocorrelation, but returns of any type can be used instead.

Autocorrelation is of interest because its results indicate a persistent behavior in returns. For example, Dacorogna et al. (2001) report negative first-order autocorrelation in 10-minute spot foreign exchange data as evidence of persistent trends in price formation.

Autocorrelation allows us to check whether there are any persistent momentum/reversal relationships in the data that we could trade upon. For example, it is a well-known stylized fact that a large swing, or momentum, in the price of a financial security is typically followed by a reversal. Using autocorrelation at different frequencies we can actually establish whether the patterns persist and whether we can trade upon them.

Autocorrelation, like any correlation function, can range from -1 to 1. High autocorrelation, say 0.5 and higher, implies a significant positive relationship between current and lagged observations. Low autocorrelation, say -0.5 and lower, in turn implies a significant negative relationship between current and lagged observations. Thus, if a return today is positive and the lag-1 autocorrelation is greater than 0.5, we can expect that the return tomorrow will be positive as well, at least 50 percent of the time. Thus, if a return today is positive and the lag-1 autocorrelation is less than -0.5, we can expect that the return tomorrow will be negative at least 50 percent of the time. Little, if anything, can be said about lagged relationships characterized by correlations closer to 0.

Of course, we cannot make sweeping inferences without first formally testing the statistical significance of the observed autocorrelation. There are two popular tests: (1) a t-ratio test allows us to check whether autocorrelation is significant at a specific lag, and (2) the Portmanteau test and its variation, the Ljung-Box test, allow us to determine the last significant autocorrelation in the sequence, beginning with $\rho(1)$. The Portmanteau and

Ljung-Box tests are useful in determining the number of lags necessary in the autoregressive process, which is discussed in the following section.

To determine whether an individual $\rho(l)$ is significant, we can apply the following test:

$$t - \text{ratio} = \frac{\hat{\rho}_l}{\sqrt{(1 + 2\sum_{i=1}^{l-1} \hat{\rho}_i)/T}} \tag{8.11}$$

For a significance level of (100-α) percent, we reject the observed autocorrelation $\rho(l)$ if $|t - \text{ratio}| > Z_{\alpha/2}$, where $Z_{\alpha/2}$ is the $100(1 - \alpha/2)$th percentile of the standard normal distribution.

As the t-ratio test indicates, autocorrelation is only 90 percent+ statistically significant at lags 1 and 2, meaning that we can reliably use this analysis to make statistically accurate predictions for at most two days ahead.

To find the optimal number of autocorrelations or the number of lags to use in forecasting future values, many researchers use the Portmanteau test and its finite-sample enhancement, the Ljung-Box test. Unlike the t-ratio test, which tests the statistical significance of an individual autocorrelation at a particular lag, both the Portmanteau and Ljung-Box tests help us determine whether correlations at lags $1, 2, \ldots, l$ are jointly significant.

The end-goal of the test is to find the lag m where autocorrelations $1, 2, \ldots, m$ are jointly significant, but autocorrelations $1, 2, \ldots, m, m + 1$ are no longer jointly significant. Such a lag m, once identified, signals the optimal number of lags to be used in further modeling for the security under consideration.

Both Portmanteau and Ljung-Box tests produce similar results when the number of observations is large. The Ljung-Box test is optimized for samples with at least 200 observations or 30 at the very minimum.

Formally, the tests are specified as follows: the null hypothesis is that the m autocorrelations are not jointly statistically significant—that is, $H_0 : \rho_1 = \ldots \rho_m = 0$, and the alternative hypothesis is $H_a : \rho_i \neq 0$ for some $i \in \{1, \ldots, m\}$. To establish that the mth autocorrelation is statistically significant, we need to be able to reject the null hypothesis.

Portmanteau test (Box and Pierce [1970]): $$Q^*(m) = T\sum_{l=1}^{m} \hat{\rho}_l^2 \tag{8.12}$$

Ljung-Box test (Ljung and Box [1978]):

$$Q(m) = T(T + 2)\sum_{l=1}^{m} \frac{\hat{\rho}_l^2}{T - l} \tag{8.13}$$

Assuming that the underlying data sequence is independently and identically distributed, both tests are asymptotically chi-squared random

TABLE 8.2 Critical Values for the Chi-Squared Distributions with Different Degrees of Freedom

Degrees of Freedom	Statistical Significance			
	90 percent	95 percent	99 percent	99.99 percent
1	2.705541	3.841455	6.634891	15.13429
2	4.605176	5.991476	9.210351	18.42474
3	6.251394	7.814725	11.34488	21.10402
4	7.779434	9.487728	13.2767	23.5064
5	9.236349	11.07048	15.08632	25.75065
6	10.64464	12.59158	16.81187	27.85266
7	12.01703	14.06713	18.47532	29.88138
8	13.36156	15.50731	20.09016	31.82683
9	14.68366	16.91896	21.66605	33.72467
10	15.98717	18.30703	23.20929	35.55716
11	17.27501	19.67515	24.72502	37.36475
12	18.54934	21.02606	26.21696	39.13058
13	19.81193	22.36203	27.68818	40.87346
14	21.06414	23.68478	29.14116	42.57523
15	22.30712	24.9958	30.57795	44.25964
16	23.54182	26.29622	31.99986	45.92551
17	24.76903	27.5871	33.40872	47.55914
18	25.98942	28.86932	34.80524	49.18533
19	27.20356	30.14351	36.19077	50.78726
20	28.41197	31.41042	37.56627	52.38323

variables with m degrees of freedom. The decision rule is to reject the null hypothesis at $100(1-\alpha)$ percent statistical significance if $Q(m) > \chi^2_\alpha$, where χ^2_α is the $100(1-\alpha)$th percentile of a chi-squared distribution with m degrees of freedom. Table 8.2 lists cut-off values for the chi-squared distributions with different levels of α.

High-frequency trading relies on fast, almost instantaneous, execution of orders. In this respect, high-frequency trading works best when all orders are initiated, sent through, and executed via computer networks, bypassing any human interference. Depending on the design of a particular systematic trading mechanism, even a second's worth of delay induced by hesitation or distraction on the part of a human trader can substantially reduce the system's profitability.

LINEAR ECONOMETRIC MODELS

Linear econometric models forecast random variables as linear combinations of other contemporaneous or lagged random variables with

well-defined distributions. In equation terms, linear models can be expressed as follows:

$$y_t = \alpha + \sum_{i=0}^{\infty} \beta_i x_{t-i} + \sum_{j=0}^{\infty} \gamma_j z_{t-i} + \cdots + \varepsilon_t \tag{8.14}$$

where $\{y_t\}$ is the time series of random variables that are to be forecasted, $\{x_t\}$ and $\{z_t\}$ are factors significant in forecasting $\{y_t\}$, and α, β, and γ are coefficients to be estimated.

Technical analysts like to refer to periods of momentums and reversals. While both can be readily observed on the charts, accurately predicting when the next momentum or reversal begins or ends is not simple. Autoregressive moving average (ARMA) is an estimation framework designed to detect consistent momentum and reversal patterns in data of selected frequencies.

Most linear models require that the distributional properties of data remain approximately constant through time, or stationary.

Stationarity

Stationarity is measured on the residuals (error terms) of econometric models. Stationarity requires that the distribution of the residuals remains stable through time; it is a necessary condition of most linear models.

Stationarity describes the stability of a distribution of a random variable. Distribution of a stationary time series does not change if shifted in time or space. A number of stationarity tests have been developed, the few examples of which are Choi (1992), Cochrane (1991), Dickey and Fuller (1979), and Phillips and Perron (1988). The Augmented Dickey Fuller (ADF) test frequently appears in the literature and tests several lags of autocorrelation of the dependent variable for unit root ($\rho = 1$). The absence of unit root indicates stability in the inferences obtained in the estimation. On the other hand, presence of the unit root suggests that the obtained results may well be spurious and that the results are invalid.

Autoregressive (AR) Estimation

Autoregressive (AR) estimation models are regressions on the lagged values of the dependent variable:

$$y_t = \alpha + \sum_{i=0}^{\infty} \beta_i y_{t-i} + \varepsilon_t \tag{8.15}$$

Coefficients obtained in autoregressions indicate momentum and reversal patterns in the data. Positive and statistically significant β coefficients, for example, indicate positive serial dependence or momentum. Similarly, negative statistically significant β coefficients indicate reversal.

Moving Average (MA) Estimation

Moving average (MA) models constitute another set of tools for forecasting future movements of a financial instrument. While autoregressive AR models estimate what proportion of past period data is likely to persist in future periods, MA models focus on how future data reacts to innovation in the past data. In other words, AR models estimate future responses to the expected component realized in the past persistence, whereas MA models measure future responses to the unexpected component realized in the past data. Figure 8.1 illustrates the difference between AR and MA estimation.

Unlike the AR models that can be estimated using ordinary least-squares or OLS regressions, estimation of moving average models is more complex. Many off-the-shelf packages provide built-in routines to assist users in the process.

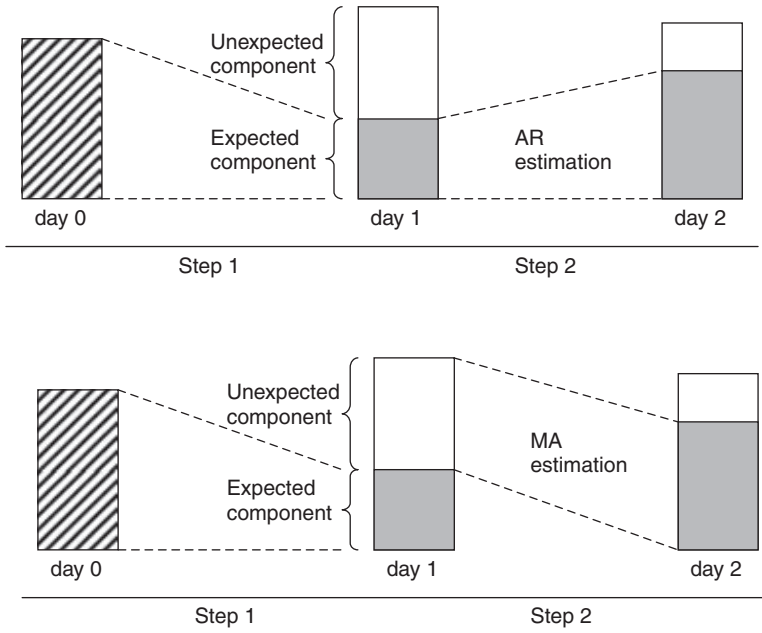

FIGURE 8.1 Illustration of differences between AR and MA estimation.

MA(q) model, with q lags, can be specified as follows:

$$r_t = c_0 + a_t - \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q} \tag{8.16}$$

where c_0 is the intercept, θ_l is the coefficient pertaining to lag l, and a_l is the unexpected component of the return at lag l. The negative signs in front of the θ's are nothing more than a conventional notation of an MA representation.

For the intrepid, there are two main approaches to estimating MA:

1. Assume that the initial unexpected component is 0 and then recursively estimate other unexpected components using OLS.
2. Assume that the unexpected component is an additional parameter to be estimated, and then estimate the model using a technique known as maximum likelihood estimation (MLE).

Using the first approach, estimation proceeds as follows:

1. Run autocorrelation analysis to determine the last statistically significant lag, q.
2. Estimate c_0 by running the following OLS regression of r_t on a vector of 1's: $r_t = c_0 + a_t$, assuming $a_t \sim N(0, \sigma^2)$. Determine a_1's as $a_t = r_t - c_0$.
3. Estimate c_0 and θ_1 using the following OLS regression: $r_t = c_0 - \theta_1 a_1 + a_2$, where a_1 is as determined in Step 2. Find a_2's as $a_2 = r_t - c_0 + \theta_1 a_1$.
4. Repeat Step 3 to find $a_3, \ldots, a_q$, where q is as determined in Step 1.
5. Estimate MA(q) coefficients c_0, θ_1, $\theta_2, \ldots,$ θ_q in the equation $r_t = c_0 - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \cdots - \theta_q a_{t-q} + a_t$, with $a_{t-i} = a_i$ estimated previously, and a_t an error term distributed with mean 0 and variance σ_a^2.

Forecasting with MA models is a pretty straightforward exercise. For a one-period forecast, we are seeking to find $E[r_{t+1}]$, which for MA(q) model is

$$E[r_{t+1}|I_t] = E[c_0 - \theta_1 a_t - \theta_2 a_{t-1} - \cdots - \theta_q a_{t-q} + a_{t+1}|I_t] \tag{8.17}$$

where I_t is the set of all information available at t. The key issue is to remember that forecasts for the unexpected components a_l have the following properties: $E[a_{t+1}|I_t] = 0$, and $Var[a_{t+1}|I_t] = \sigma_a^2$. Keeping these properties in mind, $E[r_{t+1}|I_t]$ now becomes

$$E[r_{t+1}|I_t] = c_0 - \theta_1 a_t - \theta_2 a_{t-1} - \cdots - \theta_q a_{t-q} \tag{8.18}$$

and the forecast error becomes $e(1) = r_{t+1} - E[r_{t+1}|I_t] = a_{t+1}$ with variance $Var[e(1)] = \sigma_a^2$. A two-step-ahead forecast can be computed as follows: $E[r_{t+2}|I_t] = E[c_0 - \theta_1 a_{t+1} - \theta_2 a_t - \cdots - \theta_q a_{t-q+1} + a_{t+2}|I_t] = c_0 - \theta_2 a_t - \cdots - \theta_q a_{t-q+1}$. Finally, an l-step ahead forecast for $l > q$ is $E[r_{t+l}|I_t] = c_0$.

Autoregressive Moving Average (ARMA)

Autoregressive moving average (ARMA) models combine the AR and MA models in a single framework. ARMA(p,q), for example, is specified as follows:

$$r_t = \alpha + \sum_{i=0}^{p} \beta_i r_{t-i} - \sum_{i=0}^{q} \theta_i a_{t-i} + \varepsilon_t \tag{8.19}$$

Like MA models, ARMA models are estimated using maximum likelihood (MLE).

Cointegration

Cointegration is a popular technique used for optimal portfolio construction, hedging, and risk management. Cointegration measures the contemporaneous or lagged effect of one variable on another variable. For example, if both time series {x} and {y} represent price time series of two financial securities, cointegration identifies a lead-lag relationship between the two time series.

The simplest test for lead-lag relationships can be specified using the following equation, first suggested by Engle and Granger (1987):

$$x_t = \alpha + \beta y_t + \varepsilon_t \tag{8.20}$$

Equation (8.20) can be estimated using OLS, with the residuals tested for stationarity.

The cointegration specification of equation (8.20), however, does not reveal whether one variable drives or causes another. To detect causality, a technique known as error correction model (ECM) is often used. In the simplest case with just two variables (e.g., log-price series), the ECM can be specified as the following pair of simultaneous equations:

$$\begin{aligned} \Delta x_t &= \alpha_1 + \beta_1 \Delta x_{t-1} + \beta_2 \Delta y_{t-1} + \gamma_1 z_{t-1} + \varepsilon_1 \\ \Delta y_t &= \alpha_2 + \beta_3 \Delta x_{t-1} + \beta_4 \Delta y_{t-1} + \gamma_2 z_{t-1} + \varepsilon_2 \end{aligned} \tag{8.21}$$

where Δ denotes the first difference operator and z_t is a stationary cointegrating vector, $z_t = x_t - \alpha - \beta y_t$ from equation (8.20). Equations (8.21) are then simultaneously estimated using OLS. Coefficients γ_1 and γ_2 constrain deviations from long-run equilibrium specified by equation (8.20) and explain the "error correction" piece of the ECM.

If β_2, the coefficient on the lagged y returns in the Δx equation is found to be significant, then changes in y lead changes in x. In Granger causality terminology, y "causes" x. Both the direction and strength of causalities may change over time.

Cointegration is widely used in testing for lead-lag relationships in generating cross-asset trading signals. Cointegration can also be an important component of portfolio management and hedging applications.

VOLATILITY MODELING

Most of today's uses of volatility modeling involve forecasting components of future returns. The forecasts range from the point forecasts to quantiles of returns to the probabilistic density of future returns. These forecasts are used by portfolio managers to optimize the performance of their investment vehicles, by risk managers to limit their trading downside, and by quantitative traders to develop superior trading models. According to Engle and Patton (2001, p. 238):

> *A risk manager must know today the likelihood that his portfolio will decline in the future. An option trader will want to know the volatility that can be expected over the future life of the contract. To hedge this contract he will also want to know the volatility of his forecast. A portfolio manager may want to sell a stock or a portfolio before it becomes too volatile. A market maker may want to set the bid ask spread wider when the future is believed to be more volatile.*

A good volatility model is the one that competently forecasts volatility:

1. Volatility is persistent.
2. Volatility is mean-reverting.
3. Market returns may have asymmetric impact on volatility.
4. External (exogenous) variables may affect volatility.

Persistence of volatility is sometimes referred to as "volatility clustering." The phenomenon describes the observed persistence in the levels of

volatility; if volatility is high today, it is likely to be high tomorrow. The converse is also true; volatility that is low during the current observation period is likely to remain low in the next observation period. The observed volatility persistence implies that "shocks" (unusually large price moves) will impact expected volatility measures many observation periods ahead. According to Engle and Patton (2001), the impact of shocks on future volatility expectations declines geometrically but can be seen in options data as long as one year after the occurrence of the shock.

The mean-reversion properties of volatility describe the phenomenon whereby volatility regresses to its optimal intrinsic levels. Thus, if volatility is unusually high one period and, due to persistence, will remain high for several observation periods, it will nevertheless eventually fall to its normal level. A useful tool for comparing volatility models is to compare the models' forecasts many periods ahead. According to the mean-reversion property of volatility, long-term forecasts of all volatility models should converge on the same intrinsic volatility value, a finite number.

Positive and negative shocks to market returns have been found to impact subsequent volatility differently. Market crashes and other negative shocks have been shown to result in higher subsequent volatility levels than rallies and other news favorable to the market. This asymmetric property of volatility generates skews in volatility surfaces constructed of option-implied volatilities for different option strike prices. Engle and Patton (2001) cite the following example of volatility skews: the implied volatilities of in-the-money put options are lower than those of at-the-money put options. Furthermore, the implied volatilities of at-the-money put options are lower than the implied volatilities of out-of-the-money options.

Finally, volatility forecasts may be influenced by external events, such as news announcements. In foreign exchange, for example, price and return volatility of a particular currency pair increase markedly during macroeconomic announcements pertaining to one or both sides of the currency pair.

Volatility can be forecasted in a number of ways. The simplest volatility forecast assumes that the volatility remains constant through time and that, as a result, future volatility will be equal to the volatility estimate obtained from historical data. In this case, the forecast for future squared volatility, σ_{t+1}^2, is just the variance of the past returns of the security or portfolio under consideration:

$$E_t\left[\sigma_{t+1}^2\right] = \sigma_t^2 = \sigma^2 = \frac{1}{t-1}\sum_{\tau=1}^{t}\left(R_\tau - \overline{R}\right)^2 \tag{8.22}$$

Of course, volatility may change with time. Securities that exhibit time-varying volatility are said to possess "heteroscedastic" properties, with the

term "heteroscedasticity" referring to the varying volatility; the term "homoscedasticity" describes constant volatility.

One way to model volatility that changes with time is to assume that it stays constant over short periods of time, known as volatility estimation windows. To do so, the volatility is forecasted as the volatility over the time window of returns on the security of interest:

$$E_t\left[\sigma_{t+1}^2\right] = \frac{1}{T-1}\sum_{\tau=t-T+1}^{t}\left(R_\tau - \overline{R}_t\right)^2 \tag{8.23}$$

where

$$\overline{R}_t = \frac{1}{T}\sum_{\tau=t-T+1}^{t} R_\tau \tag{8.24}$$

Similar to moving average estimation, the window used in volatility estimation is then moved through time to obtain the latest estimates. According to the central limit theorem, the return window used for estimation of each individual volatility forecast should contain at least 30 observations. The time spaces between subsequent returns used within the window can be made as short as required—thirty 1-second returns can be used to estimate intraminute volatility.

The moving window approach to volatility estimation places equal weight on all the observations in the sample. The earliest changes in returns are given the same weights as the latest changes, but the latest changes may possess more relevance to the present time and forecasting of future returns. To address this issue, several weighting schemes for returns within a volatility estimation window have been proposed.

The simplest observation weighting scheme is linear or triangular weighting: each of the T observations within the window is multiplied by a coefficient that reflects the order of the observation within the window. The earliest observation is given the lowest weight, and the latest observations are given the highest significance. The resulting forecast of variance at time $t+1$ is then computed as follows:

$$E_t\left[\sigma_{t+1}^2\right] = \sum_{\tau=t-T+1}^{t}\left(\frac{\tau - t + T}{T}R_\tau - \overline{R}_t\right)^2 \tag{8.25}$$

where

$$\overline{R}_t = \sum_{\tau=t-T+1}^{t}\frac{\tau - t + T}{T}R_\tau \tag{8.26}$$

An exponential weighting scheme also gives special significance to later observations. The scheme uses a geometric coefficient, λ, for weighting observations within the volatility estimation window. The geometric

coefficient is known as the "smoothing parameter" and is used in estimation as follows:

$$E_t\left[\sigma_{t+1}^2\right] = \sum_{\tau=t-T+1}^{t} \left(\lambda^{t-\tau}(1-\lambda)R_\tau - \overline{R}_t\right)^2 \tag{8.27}$$

where

$$\overline{R}_t = \sum_{\tau=t-T+1}^{t} \left[\lambda^{t-\tau}(1-\lambda)R_\tau\right] \tag{8.28}$$

The smoothing parameter is normally estimated on historical data using maximum likelihood. RiskMetrics™ estimates λ to be 0.94, but λ may vary from security to security and with changes in the number of observations, T, used in the volatility estimation window.

Figure 8.2 graphically compares the shapes of the weights used in volatility estimation under the simple, equally weighted moving window, the triangular moving window, and the exponential moving window.

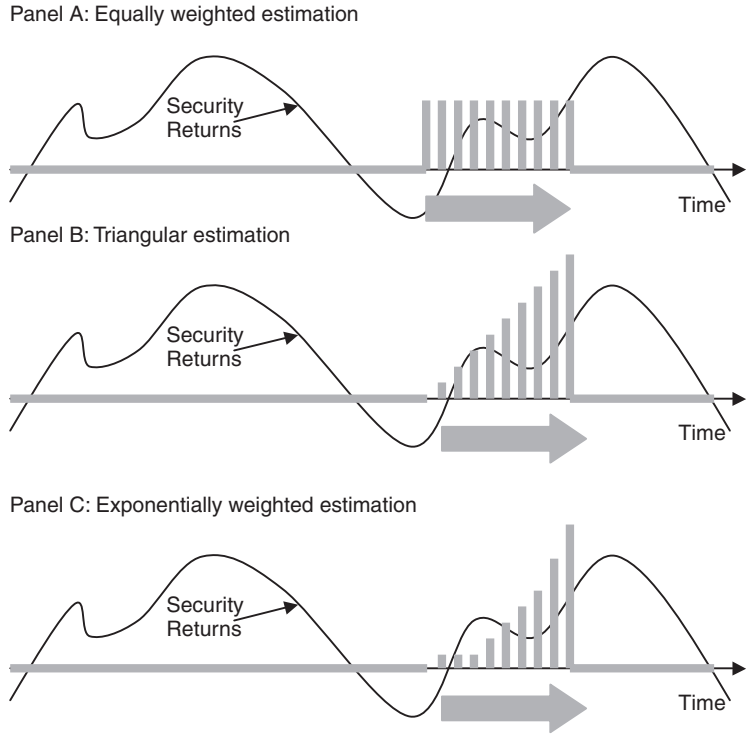

FIGURE 8.2 Panel A: Equally weighted estimation.

The moving window estimators of volatility fail to model an important characteristic of volatility—"volatility clustering." Volatility clustering describes the phenomenon of volatility persistence. Current high volatility does not typically revert to lower volatility levels instantaneously; instead, high volatility persists for several time periods. The same observation holds for low volatility; low volatility at present is likely to lead to low volatility in the immediate future.

To model the observed volatility clustering, researchers use ARMA technique on volatilities:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i a_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2 \tag{8.29}$$

where $a_t = \sigma_t z_t$, where $\{z_t\}$ is a sequence of independent, identically distributed random variables with mean 0 and variance 1. Additional stationarity conditions include $\alpha_0 > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, and $\sum_{k=1}^{\max(m,s)} (\alpha_k + \beta_k) < 1$. Such a volatility model is known as a generalized autoregressive conditional heteroscedasticity (GARCH) process, proposed by Bollerslev (1986), extending the ARCH specification of Engle (1982).

GARCH parameters are typically estimated recursively using maximum likelihood with the model's observation σ_0 "seeded" with a window-estimated volatility value. Various extensions to the GARCH specification include additional explanatory right-hand side variables controlling for external events, an exponential "EGARCH" specification that addresses the asymmetric response of returns to positive and negative shocks (bad news is typically accompanied by a higher volatility than good news), and a "GARCH-M" model in which the return of a security depends on the security's volatility, among numerous other GARCH extensions.

In addition to the moving window and GARCH volatility estimators, popular volatility measurements include the intraperiod volatility estimator, known as the "realized volatility;" several measures based on the intraperiod range of prices; and a stochastic volatility model where volatility is thought to be a random variable drawn from a prespecified distribution. The realized volatility due to Andersen, Bollerslev, Diebold, and Labys (2001) is computed as the sum of squared intraperiod returns obtained by breaking a time period into n smaller time increments of equal duration:

$$RV_t = \sum_{i=1}^{n} r_{t,i}^2 \tag{8.30}$$

The range-based volatility measures are based on combinations of open, high, low, and close prices for every period under consideration.

Garman and Klass (1980), for example, find that all of the following volatility estimators are less noisy than the conventional estimator based on the variance of returns (O_t, H_t, L_t, and C_t denote the open, high, low, and close prices for period t, respectively):

$$\hat{\sigma}_{1,t}^2 = \frac{(O_t - C_{t-1})^2}{2f} + \frac{(C_t - O_t)^2}{2(1-f)},\ 0 < f < 1 \tag{8.31}$$

$$\hat{\sigma}_{2,t}^2 = \frac{(H_t - L_t)^2}{4\ln(2)} \tag{8.32}$$

$$\hat{\sigma}_{3,t}^2 = 0.17\frac{(O_t - C_{t-1})^2}{f} + 0.83\frac{(H_t - L_t)^2}{(1-f)4\ln(2)},\ 0 < f < 1 \tag{8.33}$$

$$\hat{\sigma}_{5,t}^2 = 0.5(H_t - L_t)^2 - [2\ln(2) - 1](C_t - O_t)^2 \tag{8.34}$$

$$\hat{\sigma}_{6,t}^2 = 0.12\frac{(O_t - C_{t-1})^2}{f} + 0.88\frac{\hat{\sigma}_{5,t}^2}{1-f},\ 0 < f < 1 \tag{8.35}$$

GARCH estimators assume that volatility is a deterministic function of lagged observations and their variances. The deterministic condition can be restrictive and fail to reflect the dynamic nature of volatility. A different class of volatility estimators, known as stochastic volatility estimators, have been developed to allow modeling of heteroscedasticity and volatility clustering without the functional form restrictions on volatility specification.

The simplest stochastic volatility estimator can be specified as follows:

$$v_t = \sigma_t \xi_t = \varsigma \exp(\alpha_t/2)\ \xi_t \tag{8.36}$$

where $\alpha_t = \phi\alpha_{t-1} + \eta_t$ is the parameter modeling volatility persistence, $|\phi| < 1$, ξ_t is an identically and independently distributed random variable with mean 0 and variance 1, and ζ is a positive constant.

While stochastic volatility models reflect well the random nature underlying volatility processes, stochastic volatility is difficult to estimate. The parameters of equation (8.36) are often estimated using an econometric technique known as maximum likelihood or its close cousins. Given the randomness of the stochastic volatility estimator, the estimation process is quite complex. Estimation of GARCH can seem trivial in comparison with the estimation of stochastic volatility.

NONLINEAR MODELS

Overview

As their name implies, nonlinear models allow modeling of complex nontrivial relationships in the data.

Unlike linear models discussed in the first section of this chapter, nonlinear models forecast random variables that cannot be expressed as linear combinations of other, contemporaneous or lagged, random variables with well-defined distributions. Instead, nonlinear models can be expressed as some functions $f(.)$ of other random variables. In mathematical terms, if a linear model can be expressed as shown in equation (8.37), reprinted here for convenience, then nonlinear models are best expressed as shown in equation (8.38) which follows:

$$y_t = \alpha + \sum_{i=0}^{\infty} \beta_i x_{t-i} + \varepsilon_t \tag{8.37}$$

$$y_t = f(x_t, x_{t-1}, x_{t-2}, \cdots) \tag{8.38}$$

where $\{y_t\}$ is the time series of random variables that are to be forecasted, $\{x_t\}$ is a factor significant in forecasting $\{y_t\}$, and α and β are coefficients to be estimated.

The one-step-ahead nonlinear forecast conditional on the information available in the previous period is usually specified using a Brownian motion formulation, as shown in equation (8.39):

$$y_{t+1} = \mu_{t+1} + \sigma_{t+1}\xi_{t+1} \tag{8.39}$$

where $\mu_{t+1} = E_t[y_{t+1}]$ is the one-period-ahead forecast of the mean of the variable being forecasted, $\sigma_{t+1} = \sqrt{\text{var}_t[x_{t+1}]}$ is the one-period-ahead forecast of the volatility of the variable being forecasted, and ξ_{t+1} is an identically and independently distributed random variable with mean 0 and variance 1. The term ξ_{t+1} is often referred to as a standardized shock or innovation.

The nonlinear estimation is often used in pricing derivatives and other complex financial instruments. In fact, many readers will recognize equation (8.39) as the cornerstone equation of derivatives pricing models.

Here, we will briefly review the following nonlinear estimation methods:

- Taylor series expansion (bilinear models)
- Threshold autoregressive model

- Markov switching model
- Nonparametric estimation
- Neural networks

For a detailed examination of nonlinear estimation, please see Priestley (1988) and Tong (1990).

Taylor Series Expansion (Bilinear Models)

One of the simplest ways to deal with nonlinear functions is to linearize them using the Taylor series expansion. The Taylor series expansion of a univariate function $f(x)$, for any x in the vicinity of some specific value a, is a derivative-based approximation of the function $f(x)$ and is defined as follows:

$$f(x) = f(a) + (x-a)\frac{df(x)}{dx}\bigg|_{x=a} + \frac{1}{2}(x-a)^2\frac{d^2f(x)}{dx^2}\bigg|_{x=a} + o(f(x)) \tag{8.40}$$

where $f(a)$ is the value of the function $f(x)$ at point a, $\frac{df(x)}{dx}|_{x=a}$ is the slope of the function $f(x)$ at $x=a$, $\frac{d^2f(x)}{dx^2}|_{x=a}$ is the curvature of the function $f(x)$ at point a, and $o(f(x))$ are higher-order derivative terms of the function $f(x)$. The higher-order derivative terms $o(f(x))$ are generally small and are routinely ignored in estimation.[1]

Granger and Andersen (1978) showed that equation (8.40) translates into the following linear econometric equation:

$$y_t = \alpha + \sum_{i=1}^{p}\phi_i y_{t-i} - \sum_{j=1}^{q}\theta_j x_{t-j} + \sum_{i=1}^{m}\sum_{j=1}^{s}\beta_{ij}y_{t-i}x_{t-j} + \varepsilon_t \tag{8.42}$$

where p, q, m, and s are nonnegative integers.

---

[1]The Taylor series expansion of a bivariate function $f(x, z)$, for any x in the vicinity of some points $x = a$ and $z = b$, includes a cross-derivative and is specified as follows:

$$\begin{aligned} f(x,z) = f(a,b) &+ (x-a)\frac{df(x,z)}{dx}\bigg|_{x=a} + (z-b)\frac{df(x,z)}{dz}\bigg|_{z=b} \\ &+ \frac{1}{2}(x-a)^2\frac{d^2f(x,z)}{dx^2}\bigg|_{x=a} + \frac{1}{2}(z-b)^2\frac{d^2f(x,z)}{dz^2}\bigg|_{z=b} \\ &+ (x-a)(z-b)\frac{d^2f(x,z)}{dxdz}\bigg|_{x=a,z=b} + o(f(x,z)) \end{aligned} \tag{8.41}$$

Taylor series expansions can be used in estimation of cross-market derivative/underlying security arbitrage.

Threshold Autoregressive (TAR) Models

Threshold autoregressive (TAR) models approximate nonlinear functions with piecewise linear estimation with thresholds defined on the dependent variable. For example, the model may have different specifications for positive and negative values of the dependent variable, in addition to separate linear models for large positive and large negative values. Such specification can be used in estimation of statistical arbitrage models. Figure 8.3 illustrates the idea.

An example of the TAR of Figure 8.3 may be the following AR(1) specification:

$$y_t = \begin{cases} -5y_{t-1} + \varepsilon_t, \ if \ y_{t-1} < \text{Threshold 1} \\ 5y_{t-1} + \varepsilon_t, \ if \ y_{t-1} \in (\text{Threshold 1}, \ \text{Threshold 2}) \\ -5y_{t-1} + \varepsilon_t, \ if \ y_{t-1} \in (\text{Threshold 2}, \ \text{Threshold 3}) \\ 5y_{t-1} + \varepsilon_t, \ if \ y_{t-1} > \text{Threshold 3} \end{cases} \tag{8.43}$$

The major problem of the TAR models is the models' discontinuity at thresholds. Smooth transition AR (STAR) models have been proposed to address the discontinuities of the TAR models. For detailed treatment of STAR models, please see Chan and Tong (1986) and Teräsvirta (1994).

Markov Switching Models

Markov models are models with a finite number of mutually exclusive "states"; the states can be defined as value intervals between successive thresholds as in TAR models discussed previously, or as some discrete values potentially reflecting exogenous variables such as states of overall economy and the like. Contrary to TAR models, in Markov models each state has a discrete probability of transitioning into another state. The

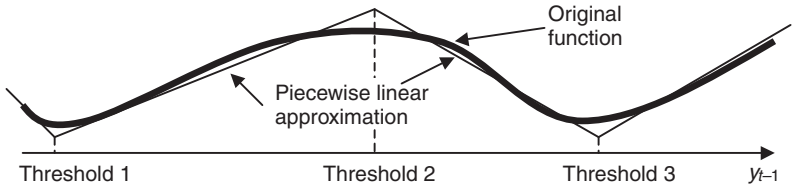

FIGURE 8.3 Piecewise linear approximation of a nonlinear function.

transition probabilities are often estimated from historical data or determined analytically from theory.

An example of a two-state Markov model with a linear AR(1) specification in each state can be of the following nature:

$$y_t = \begin{cases} -5y_{t-1} + \varepsilon_t, if s_t = 1 \\ 5y_{t-1} + \varepsilon_t, if s_t = 2 \end{cases} \tag{8.44}$$

where s_t denotes the "state" of y_t, however the state is defined, and state transition probabilities are specified as follows:

$$\begin{aligned} &P(s_t = 2|s_{t-1} = 1) = p_1 \\ &P(s_t = 1|s_{t-1} = 1) = 1 - p_1 \\ &P(s_t = 1|s_{t-1} = 2) = p_2 \\ &P(s_t = 2|s_{t-1} = 2) = 1 - p_2 \end{aligned} \tag{8.45}$$

Using the advanced statistical properties of the Markov processes, it can be shown that the expected proportion of time that the 2-state Markov process spends in state 1 is $1/p_1$, while the expected proportion of time that the 2-state Markov process spends in state 2 is $1/p_2$.

Markov switching models can be used in estimation of inter-trade durations and execution probabilities in limit order-based trading and other optimizations of execution. Markov switching models can also be applied in estimation of cross-market arbitrage opportunities.

Nonparametric Estimation of Nonlinear Models

Nonparametric estimation denotes a broad class of econometric models that generally refers to econometric estimation without any assumptions as to the distribution of estimation errors or the shape of the function relating dependent and independent variables. One subclass of nonparametric models discussed here allows us to determine the functional relationship between dependent and independent variables directly from the historical data. Such nonparametric techniques boil down to smoothing the data into a functional form.

The nonparametric estimation of nonlinear models is designed to estimate the following function:

$$y_t = f(x_t) + \varepsilon_t \tag{8.46}$$

where $\{\varepsilon_t\}$ is the time series sequence of normally distributed errors and $f(.)$ is an arbitrary, smooth function to be estimated. The simple average smoothing determines the value of $f(x)$ at every point $x = X$ by taking the

across-time averages of both sides of equation (8.46) and utilizing the fact that $E[\varepsilon] = 0$ by assumption:

$$E[y] = f(x) + E[\varepsilon] = f(x) \tag{8.47}$$

or, equivalently,

$$f(x) = \frac{1}{T}\sum_{t=1}^{T} y_t \tag{8.48}$$

where T is the size of the sample.

To make sure that the estimation of $f(x)$ considers only the values around x and not the values of the entire time series, the values of y_t can be weighted by a weight function, $w_t(x)$. The weight function is determined by another function, known as a "kernel function," $K_h(x)$:

$$w_t(x) = \frac{K_h(x - x_t)}{\sum_{t=1}^{T} K_h(x - x_t)} \tag{8.49}$$

The weight function, $w_t(x)$, defines the location of the filter window that includes the y elements closest to the x being estimated at the moment and excludes all the y's outside the filter window. The kernel function $K_h(x)$ defines the shape of the filter window. As the window is moved through the continuum of x's, the y elements fall in and out of the estimation window to reflect their relevance to the estimation. Since the weights w have to add up to 1 for all values of y considered, $K(x)$ can be defined as the probability density function with $K_h(x) \geq 0$ and $\int K_h(z)dz = 1$. Figure 8.4 shows the process of kernel estimation using the Gaussian kernel specified as the density of the normal distribution:

$$K_h(x) = \frac{1}{h\sqrt{2\pi}} \exp\left(-\frac{x^2}{2h^2}\right)$$

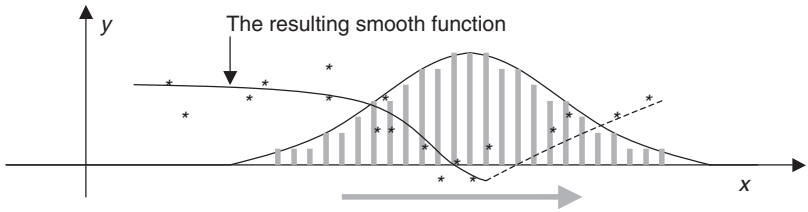

FIGURE 8.4 Kernel smoothing using a normal-density kernel.

The width of the estimation window can be controlled through a parameter known as bandwidth that enters the kernel function as shown in equation (8.50):

$$K_h(x) = \frac{1}{h}K(x/h) \tag{8.50}$$

Fan and Yao (2003) determine the optimal bandwidth parameter h to be $1.06sT^{-0.2}$, where s is the sample standard error of x and T is the total size of the sample.

Kernel smoothing with different kernel functions is commonly used to filter the data—that is, to eliminate outliers and other noise.

Neural Networks

Neural networks are an example of semiparametric estimation. The term "neural network" is sometimes perceived to signal advanced complexity of a high-frequency system. In reality, neural networks are built instead to simplify algorithms dealing with econometric estimation.

A neural network is, in essence, a collection of interconnected rules that are selectively and sequentially triggered, depending on which conditions are satisfied in the real-time data. Caudill (1988, p. 53) defines a neural network as "a computing system made up of a number of simple, highly interconnected processing elements, which process information by their dynamic state response to external inputs." The simplest neural network can be built as shown in Figure 8.5.

Advanced neural networks can comprise multiple decisions based on numerous simultaneous inputs. Neural networks can also incorporate feedback mechanisms whereby outputs of the previous periods are taken as inputs, for example.

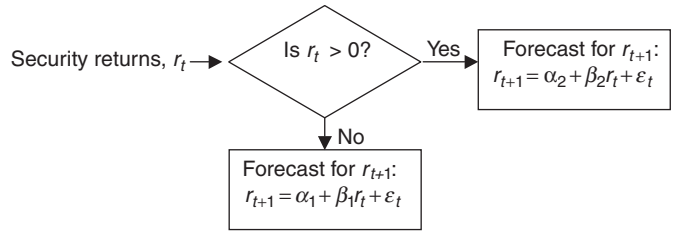

FIGURE 8.5 A simple neural network that forecasts return values on the basis of the value of the previous return value. The forecast parameters $\hat{\alpha}_1$, $\hat{\alpha}_2$, $\hat{\beta}_1$, $\hat{\beta}_2$ are estimated from historical data.

The main advantage of neural networks is their simplified step-by-step structure that can significantly speed up execution of the forecasting algorithm. Neural networks are classified as semiparametric estimation tools, given that the networks may incorporate both distributional assumptions and rule-based systems in estimating and forecasting desired variables.

CONCLUSION

The field of econometrics provides a wide range of tools to model statistical dependencies in the data. Linear models assume that data dependencies are direct, or linear, while nonlinear models comprise a set of functions for more involved relationships. Chapter 9 discusses additional high-frequency estimation models.

CHAPTER 9

Working with Tick Data

Trading opportunities are largely a function of the data that identifies them. As discussed in Chapter 7, the higher the data frequency, the more arbitrage opportunities appear. When researching profitable opportunities, therefore, it is important to use data that is as granular as possible. Recent microstructure research and advances in econometric modeling have facilitated a common understanding of the unique characteristics of tick data. In contrast to traditional low-frequency regularly spaced data, tick data is irregularly spaced with quotes arriving randomly at very short time intervals. The observed irregularities present researchers and traders with a wealth of information not available in low-frequency data sets. Intertrade durations may signal changes in market volatility, liquidity, and other variables, as discussed further along in this chapter.

In addition, the sheer volume of data allows researchers to produce statistically precise inferences. As noted by Dacorogna et al. (2001), large sets of data can support considerably wider ranges of input variables (parameters) because of the expanded number of allowable degrees of freedom.

Finally, the copious quantities of tick data allow researchers to use short-term data samples to make statistically significant inferences pertaining to the latest changes in the markets. Whereas a monthly set of daily data is normally deemed too short a sample to make statistically viable predictions, volumes of tick data in the same monthly sample can make such short-term estimation practical. Other frequency-specific considerations, such as intra-day seasonality, must be taken into account in assessing the sufficient number of observations.

This chapter discusses the following topics:

- Various properties of tick data
- Econometric techniques specific to tick data estimation
- How trading systems can make better trading decisions using tick data
- How trading systems can apply traditional econometric principles

PROPERTIES OF TICK DATA

The highest-frequency data is a collection of sequential "ticks," arrivals of the latest quote, trade, price, and volume information. Tick data usually has the following properties:

- A timestamp
- A financial security identification code
- An indicator of what information it carries:
  - Bid price
  - Ask price
  - Available bid volume
  - Available ask volume
  - Last trade price
  - Last trade size
  - Option-specific data, such as implied volatility
- The market value information, such as the actual numerical value of the price, available volume, or size

A timestamp records the date and time at which the quote originated. It may be the time at which the exchange or the broker-dealer released the quote, or the time when the trading system has received the quote. The quote travel time from the exchange or the broker-dealer to the trading system can be as small as 20 milliseconds. All sophisticated systems, therefore, include milliseconds as part of their timestamps.

Part of the quote is an identifier of the financial security. In equities, the identification code can be a ticker, or, for tickers simultaneously traded on multiple exchanges, a ticker followed by the exchange symbol. For futures, the identification code can consist of the underlying security, futures expiration date, and exchange code.

The last trade price shows the price at which the last trade in the security cleared. Last trade price can differ from the bid and ask. The differences can arise when a customer posts a favorable limit order that is immediately matched by the broker without broadcasting the customer's quote. Last trade size shows the actual size of the last executed trade.

The bid quote is the highest price available for sale of the security in the market. The ask quote is the lowest price entered for buying the security at any particular time. Both bid and ask quotes are provided by other market participants through limit orders. A yet-to-be-executed limit order to buy with the highest price becomes the market bid, and a limit order to sell with the lowest price among other limit orders in the same book becomes the market ask. Available bid and ask volumes indicate the total demand and supply, respectively, at the bid and ask prices.

QUANTITY AND QUALITY OF TICK DATA

High-frequency data is voluminous. According to Dacorogna et al. (2001), the number of observations in a single day of tick-by-tick data is equivalent to 30 years of daily observations. The quality of data does not always match its quantity. Centralized exchanges generally provide accurate data on bids, asks, and volume of any trade with a reasonably timely timestamp. The information on the limit order book is less commonly available. In decentralized markets, such as foreign exchange and the interbank money market, no market-wide quotes are available at any given time. In such markets, participants are aware of the current price levels, but each institution quotes its own prices adjusted for its order book. In decentralized markets, each dealer provides his own tick data to his clients. As a result, a specific quote on a given financial instrument at any given time may vary from dealer to dealer. Reuters, Telerate, and Knight Ridder, among others, collect quotes from different dealers and disseminate them back, improving the efficiency of the decentralized markets. There are generally thought to be three anomalies in inter-dealer quote discrepancies.

Each dealer's quotes reflect that dealer's own inventory. For example, a dealer that has just sold a customer $100 million of USD/CAD would be eager to diversify the risk of his position and avoid selling any more of USD/CAD. Most dealers are, however, obligated to transact with their clients on tradeable quotes. To incite his clients to place sell orders on USD/CAD, the dealer temporarily raises the bid quote on USD/CAD. At the same time, to encourage his clients to withhold placing buy orders, the dealer raises the ask quote on USD/CAD. Thus, dealers tend to raise both bid and ask prices whenever they are short in a particular financial instrument and lower both bid and ask prices whenever they are disproportionally long in a financial instrument.

In an anonymous marketplace, such as a dark pool, dealers as well as other market makers may "fish" for market information by sending indicative quotes that are much off the previously quoted price to assess the available demand or supply.

Dacorogna et al. (2001) note that some dealers' quotes may lag real market prices. The lag is thought to vary from milliseconds to a minute. Some dealers quote moving averages of quotes of other dealers. The dealers who provide delayed quotes usually do so to advertise their market presence in the data feed. This was particularly true when most order prices were negotiated over the telephone, allowing a considerable delay between quotes and orders. Fast-paced electronic markets discourage lagged quotes, improving the quality of markets.

BID-ASK SPREADS

The difference between the bid quote and the ask quote at any given time is known as the bid-ask spread. The bid-ask spread is the cost of instantaneously buying and selling the security. The higher the bid-ask spread, the higher a gain the security must produce in order to cover the spread along with other transaction costs. Most low-frequency price changes are large enough to make the bid-ask spread negligible in comparison. In tick data, on the other hand, incremental price changes can be comparable or smaller than the bid-ask spread.

Bid-ask spreads usually vary throughout the day. Figure 9.1 illustrates the average bid-ask spread cycles observed in the institutional EUR/USD market for the last two weeks of October 2008. As Figure 9.1 shows, the average spread increases significantly during Tokyo trading hours when the market is quiet. The spread then reaches its lowest levels during the overlap of the London and New York trading sessions when the market has many active buyers and sellers. The spike in the spread over the weekend of October 18–19, 2008, reflects the market concern over the subpoenas

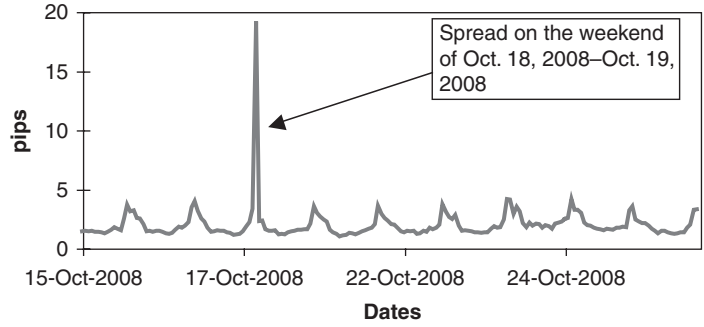

FIGURE 9.1 Average hourly bid-ask spread on EUR/USD spot for the last two weeks of October 2008 on a median transaction size of USD 5 million.

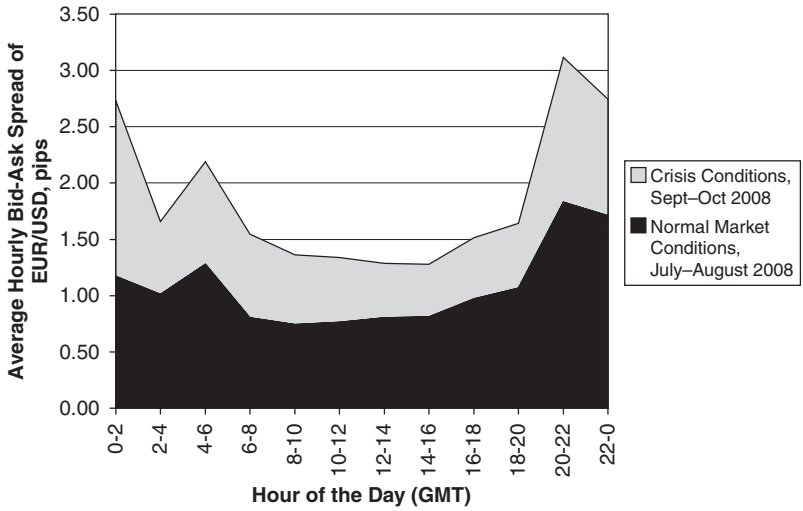

FIGURE 9.2 Comparison of average bid-ask spreads for different hours of the day during normal market conditions and crisis conditions.

issued on October 17, 2009, to senior Lehman executives in a case relating to potential securities fraud at Lehman Brothers.

Bid-ask spreads typically increase during periods of market uncertainty or instability. Figure 9.2, for example, compares average bid-ask spreads on EUR/USD in the stable market conditions of July–August 2008 and the crisis conditions of September–October 2008. As Figure 9.2 shows, the intra-day spread pattern is persistent in both crisis and normal market conditions, but the spreads are significantly higher during crisis months than during normal conditions at all hours of the day. As Figure 9.2 also shows, the spread increase is not uniform at all hours of the day. The average hourly EUR/USD spreads increased by 0.0048% (0.48 basis points or pips) between the hours of 12 GMT and 16 GMT, when the London and New York trading sessions overlap. From 0 to 2 GMT, during the Tokyo trading hours, the spread increased by 0.0156 percent, over three times the average increase during the New York/London hours.

As a result of increasing bid-ask spreads during periods of uncertainty and crises, the profitability of high-frequency strategies decreases during those times. For example, high-frequency EUR/USD strategies running over Asian hours incurred significantly higher costs during September and October 2008 as compared with normal market conditions. A strategy that executed 100 trades during Asian hours alone resulted in 1.56 percent evaporating from daily profits due to the increased spreads, while the same strategy running during London and New York hours resulted in a smaller

but still significant daily profit decrease of 0.48 percent. The situation can be even more severe for high-frequency strategies built for less liquid instruments. For example, bid-ask spreads for NZD/USD (not shown) on average increased thrice during September–October in comparison with market conditions of July–August 2008.

Future realizations of the bid-ask spread can be estimated using the model of Roll (1984), where the price of an asset at time t, p_t, is assumed to equal an unobservable fundamental value, m_t, offset by a value equal to half of the bid-ask spread, s. The price offset is positive when the next market order is a buy, and negative when the trade is a sell, as shown in equation (9.1):

$$p_t = m_t + \frac{s}{2} I_t \tag{9.1}$$

where

$$I_t = \begin{cases} 1, & \text{market buy at ask} \\ -1, & \text{market sell at bid} \end{cases}$$

If either a buy or a sell order can arrive next with equal probability, then $E[I_t] = 0$, and $E[\Delta p_t] = 0$, absent changes in the fundamental asset value, m_t. The covariance of subsequent price changes, however, is different from 0:

$$\operatorname{cov}[\Delta p_t, \Delta p_{t+1}] = E[\Delta p_t \Delta p_{t+1}] = -\frac{s^2}{4} \tag{9.2}$$

As a result, the future expected spread can be estimated as follows:

$$E[s] = 2\sqrt{-\operatorname{cov}[\Delta p_t, \Delta p_{t+1}]} \text{ whenever } \operatorname{cov}[\Delta p_t, \Delta p_{t+1}] < 0.$$

Numerous extensions of Roll's model have been developed to account for contemporary market conditions along with numerous other variables. Hasbrouck (2007) provides a good summary of the models.

BID-ASK BOUNCE

While tick data carries information about market dynamics, it is also distorted by the same processes that make the data so valuable in the first place. Dacorogna et al. (2001) report that sequential trade price bounces between the bid and ask quotes during market execution of orders introduce significant distortions into estimation of high-frequency parameters. Corsi, Zumbach, Müller, and Dacorogna (2001), for example, show that the bid-ask bounce introduces a considerable bias into volatility estimates. The

authors calculate that the bid-ask bounce on average results in –40 percent negative first-order autocorrelation of tick data. Corsi et al. (2001) as well as Voev and Lunde (2007) propose to remedy the bias by filtering the data prior from the bid-ask noise prior to estimation.

MODELING ARRIVALS OF TICK DATA

Unlike low-frequency data, which is recorded at regular time periods, tick data arrives at irregularly spaced intervals. Several researchers have studied whether the time distance between subsequent quote arrivals itself carries information. Most researchers agree that inter-trade intervals indeed carry information on securities for which short sales are disallowed; the lower the inter-trade duration, the more likely the yet-to-be-observed good news and the higher the impending price change.

The process of information arrivals is modeled using so-called duration models. Duration models are used to estimate the factors affecting the duration between any two sequential ticks. Such models are known as quote processes and trade processes, respectively. Duration models are also used to measure the time elapsed between price changes of a prespecified size, as well as the time interval between predetermined trade volume increments. The models working with fixed price are known as price processes; the models estimating variation in duration of fixed volume increments are known as volume processes.

Durations are often modeled using Poisson processes. Poisson processes assume that sequential events, like quote arrivals, occur independently of one another. The number of arrivals between any two time points t and $(t + \tau)$ is assumed to have a Poisson distribution.

In a Poisson process, λ arrivals occur per unit time. In other words, the arrivals occur at an average rate of $(1/\lambda)$. The average arrival rate may be assumed to hold constant, or it may vary with time. If the average arrival rate is constant, the probability of observing exactly k arrivals between times t and $(t + \tau)$ is

$$P[(N(t+\tau) - N(t)) = k] = \frac{1}{k!} e^{-\lambda\tau} (\lambda\tau)^k, k = 0, 1, 2, \ldots \tag{9.3}$$

Diamond and Verrecchia (1987) and Easley and O'Hara (1992) were the first to suggest that the duration between subsequent data arrivals carries information. The models posit that in the presence of short-sale constraints, inter-trade duration can indicate the presence of good news; in markets of securities where short selling is disallowed, the shorter the inter-trade duration, the higher is the likelihood of unobserved good news.

The reverse also holds: in markets with limited short selling and normal liquidity levels, the longer the duration between subsequent trade arrivals, the higher the probability of yet-unobserved bad news. A complete absence of trades, however, indicates a lack of news.

Easley and O'Hara (1992) further point out that trades that are separated by a time interval have a much different information content than trades occurring in close proximity. One of the implications of Easley and O'Hara (1992) is that the entire price sequence conveys information and should be used in its entirety whenever possible, strengthening the argument for high-frequency trading.

Table 9.1 shows summary statistics for a duration measure computed on all trades recorded for S&P 500 Depository Receipts ETF (SPY) on May 13, 2009. As Table 9.1 shows, the average inter-trade duration was the longest outside of regular market hours, and the shortest during the hour preceding the market close (3–4 P.M. ET).

Variation in duration between subsequent trades may be due to several other causes. While the lack of trading may be due to a lack of new information, trading inactivity may also be due to low levels of liquidity, trading halts on exchanges, and strategic motivations of traders. Foucault, Kadan, and Kandel (2005) consider that patiently providing liquidity using limit orders may itself be a profitable trading strategy, as liquidity providers

TABLE 9.1 Hourly Distributions of Inter-Trade Duration Observed on May 13, 2009 for S&P 500 Depository Receipts ETF (SPY)

		Inter-Trade Duration (milliseconds)				
Hour (ET)	Number of Trades	Average	Median	Std Dev	Skewness	Kurtosis
4–5 AM	170	19074.58	5998	47985.39	8.430986	91.11571
5–6 AM	306	11556.95	4781.5	18567.83	3.687372	21.92054
6–7 AM	288	12606.81	4251	20524.15	3.208992	16.64422
7–8 AM	514	7096.512	2995	11706.72	4.288352	29.86546
8–9 AM	767	4690.699	1997	7110.478	3.775796	23.56566
9–10 AM	1089	2113.328	1934	24702.9	3.5185	24.6587
10–11 AM	1421	2531.204	1373	3409.889	3.959082	28.53834
11–12 PM	1145	3148.547	1526	4323.262	3.240606	17.24866
12–1 PM	749	4798.666	1882	7272.774	2.961139	13.63373
1–2 PM	982	3668.247	1739.5	5032.795	2.879833	13.82796
2–3 PM	1056	3408.969	1556	4867.061	3.691909	23.90667
3–4 PM	1721	2094.206	1004	2684.231	2.9568	15.03321
4–5 PM	423	8473.593	1500	24718.41	7.264483	69.82157
5–6 PM	47	73579.23	30763	113747.8	2.281743	7.870699
6–7 PM	3	1077663	19241	1849464	0.707025	1.5

should be compensated for their waiting. The compensation usually comes in the form of a bid-ask spread and is a function of the waiting time until the order limit is "hit" by liquidity takers; lower inter-trade durations induce lower spreads. However, Dufour and Engle (2000) and Saar and Hasbrouck (2002) find that spreads are actually higher when traders observe short durations, contrasting the time-based limit order compensation hypothesis.

In addition to durations between subsequent trades and quotes, researchers have also been modeling durations between fixed changes in security prices and volumes. The time interval between subsequent price changes of a specified magnitude is known as price duration. Price duration has been shown to decrease with increases in volatility. Similarly, the time interval between subsequent volume changes of a prespecified size is known as the volume duration. Volume duration has been shown to decrease with increases in liquidity.

The information content of quote, trade, price, and volume durations introduces biases into the estimation process, however. If the available information determines the time between subsequent trades, time itself ceases to be an independent variable, introducing substantial endogeneity bias into estimation. As a result, traditional estimates of variance of transaction prices are too high in comparison with the true variance of the price series. The variance of the high-frequency data, however, can be consistently estimated using the generalized autoregressive conditional heteroscedasticity (GARCH) process framework that can incorporate inter-trade and inter-quote duration.

APPLYING TRADITIONAL ECONOMETRIC TECHNIQUES TO TICK DATA

Most modern computational techniques have been developed to work with regularly spaced data, presented in monthly, weekly, daily, hourly, or other consistent intervals. The traditional reliance of researchers on fixed time intervals is due to

- Relative availability of daily data (newspapers have published daily quotes since the 1920s)
- Relative ease of processing regularly spaced data
- An outdated view that "whatever drove security prices and returns, it probably did not vary significantly over short time intervals." (Goodhart and O'Hara 1997, pp. 80–81)

The major difference between tick data and traditional, regularly spaced data is that tick-by-tick observations are separated by varying time

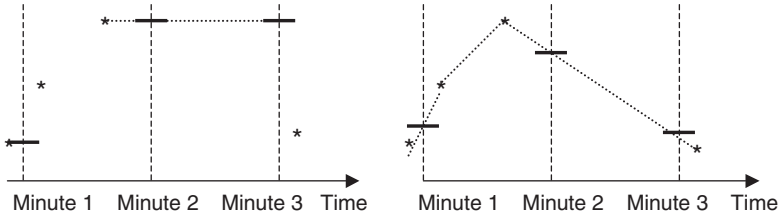

FIGURE 9.3 Data-sampling methodologies.

intervals. One way to overcome the irregularities in the data is to sample it at certain predetermined periods of time—for example, every hour or minute.

Traditional financial literature samples closing prices. For example, if the data is to be converted from tick data to minute "bars," then under the traditional approach, the bid or ask price for any given minute would be determined as the last quote that arrived during that particular minute. If no quotes arrived during a certain minute, then the previous minute's closing prices would be taken as the current minute's closing prices, and so on. Figure 9.3, panel (a) illustrates this idea. This approach implicitly assumes that in the absence of new quotes, the prices stay constant, which does not have to be the case.

Dacorogna et al. (2001) propose a potentially more precise way to sample quotes—linear time-weighted interpolation between adjacent quotes. At the core of the interpolation technique is an assumption that at any given time, unobserved quotes lie on a straight line that connects two neighboring observed quotes. Figure 9.3, panel (b) illustrates linear interpolation sampling.

As shown in Figure 9.3 panels (a) and (b), the two quote-sampling methods produce quite different results. Dacorogna et al. (2001) do not provide or reference any studies that compare the performances of the two sampling methods.

Mathematically, the two sampling methods can be expressed as follows:

$$\text{Quote sampling using closing prices: } \hat{q}_t = q_{t,last} \tag{9.4}$$

Quote sampling using linear interpolation:

$$\hat{q}_t = q_{t,last} + (q_{t,next} - q_{t,last})\frac{t - t_{last}}{t_{next} - t_{last}} \tag{9.5}$$

where $\hat{q}_t$ is the resulting sampled quote, t is the desired sampling time (start of a new minute, for example), t_{last} is the timestamp of the last observed

quote prior to the sampling time t, $q_{t,last}$ is the value of the last quote prior to the sampling time t, t_{next} is the timestamp of the first observed quote after the sampling time t, and $q_{t,next}$ is the value of the first quote after the sampling time t.

Another way to assess the variability of the tick data is through modeling the high-frequency distributions using the mixtures of distributions model (MODM). Tauchen and Pitts (1983), for example, show that if changes in the market prices are normally distributed, then aggregates of price changes and volume of trades approximately form a jointly normal distribution.

CONCLUSION

Tick data differs dramatically from low-frequency data. Utilization of tick data creates a host of opportunities not available at lower frequencies.

CHAPTER 10

Trading on Market Microstructure

Inventory Models

Rational expectations and the efficient markets hypotheses imply that, following a relevant news release, market prices adjust instantaneously. From the perspective of a long-term investor, holding positions for days or months, the adjustment may indeed seem instantaneous. Anyone who has watched the markets surrounding a major news release, however, has observed a different picture—a volatile price that eventually settles within a specific price band. Note that the price eventually settles within a price range and not at a constant price level, because a degree of volatility, however small, accompanies all market conditions. The process of the market finding its optimal post-announcement price band is often referred to as tâtonnement, from French for "trial and error."

Figure 10.1 illustrates price adjustments as viewed at different frequencies. At very high frequencies, the price adjustment process is hardly instantaneous. The tâtonnement toward a new optimal price happens through the implicit negotiation among buyers and sellers that is occurring in the order flow; the market participants develop individual security valuations given the news, which are reflected in their bids and asks. These quotes provide market participants with information about other market participants' valuations. The process repeats until most market participants agree on a range of acceptable prices; the equilibrium price band can then be considered achieved. The process of tâtonnement, therefore, not only incorporates information into prices but also shapes beliefs of market participants through a form of collective bargaining process.

The discipline that studies the price formation process is known as market microstructure. Trading on market microstructure is the holy grail

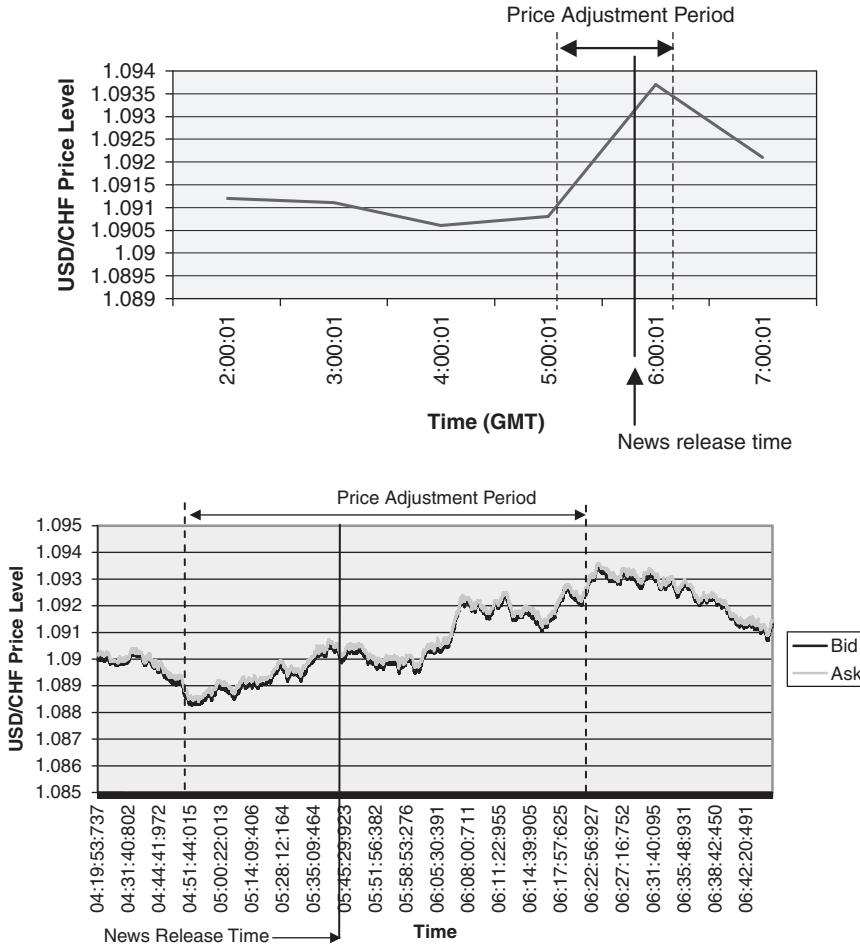

FIGURE 10.1 USD/CHF price adjustments to Swiss unemployment news, recorded on July 8, 2009 at hourly (top panel) and tick-by-tick (bottom panel) frequencies.

of high-frequency trading. The idea of market microstructure trading is to extract information from the observable quote data and trade upon that extracted information in order to obtain gains. Holding periods for positions in market microstructure trading can vary in duration from seconds to hours.

The optimal holding period is influenced by the transaction costs faced by the trader. A gross average gain for a position held just several seconds will likely be in the range of several basis points (1 basis point = 1 bp = 1 pip = 0.01%), at most. To make such trading viable, the expected gain has

to surpass the transaction costs. In an institutional setting (e.g., on a proprietary trading desk of a broker-dealer), a trader will often face transaction costs of 1 bp or less on selected securities, making a seconds-based trading strategy with an expected gain of at least 2 bps per trade quite profitable. Other institutional players, such as hedge funds, can expect their transaction costs to range anywhere from 3 bps to 30 bps per trade, mandating strategies that call for longer holding periods.

According to Lyons (2001), the field of market microstructure encompasses two general types of models—inventory models and information models. Information models are concerned with the process of impounding information into prices in response to news. With information models, order flow carries information that induces price changes. Inventory models, on the other hand, explain transitory variations in prices in the absence of news. As with information models, it is order flow that causes these temporary variations. Unlike information models where order flow is a result of end customers receiving and acting on information, inventory models concern themselves with the order flow resulting from dealer book imbalances.

This chapter reviews inventory models and their applications to high-frequency trading. The following chapter, Chapter 11, discusses information models.

OVERVIEW OF INVENTORY TRADING STRATEGIES

Inventory trading, also known as liquidity provision or market making, concerns itself with profitable management of inventory. Liquidity provision was once performed only by dedicated broker-dealers, known as market makers. Market makers kept enough liquidity on hand to satisfy supply and demand of any arriving traders. During the past decade, this system changed. The proliferation of electronic transacting capability coupled with the 1997 SEC order display rule enabled most traders to place the short-term limit orders required to make markets. Several competitive liquidity provision strategies have emerged to profitably capture liquidity premiums available in the markets.

Inventory trading strategies possess the following key characteristics:

- The strategies are executed predominantly using limit orders, although an occasional market order is warranted to close a position.

- The strategies rely on a multitude of very small realized gains; it is not uncommon for these strategies to move in and out of positions 2,000 times per day.
- As a result, these strategies operate at very high frequencies; short position holding time is what makes it possible to move capital in and out of multiple trades, generating a large surplus at the end of each day.
- High-speed transmission of orders and low-latency execution are required for successful implementation of liquidity provision strategies.

ORDERS, TRADERS, AND LIQUIDITY

Orders Used in Microstructure Trading

Limit orders, first introduced in Chapter 6, are commitments to buy or sell a particular security at a prespecified price. Limit orders can be seen as ex-ante commitments to provide market liquidity. As noted by Demsetz (1968), while limit orders are in queue, the trader who placed limit orders incurs inventory and waiting costs. The inventory costs arise from the uncertainty as to the market price of the securities that the trader may hold in his portfolio while his limit orders are pending. The waiting costs are the opportunity costs associated with the time between placing an order and its execution. In addition, per Copeland and Galai (1983), limit orders suffer from an informational disadvantage, whereby they are picked off by better-informed investors.

Naturally, the probability of limit orders being executed depends on the limit order price proximity to the current market price. Cohen, Maier, Schwartz, and Whitcomb (1981), call this phenomenon a "gravitational pull" of existing quotes. Limit orders placed at current market quotes are likely to be executed, whereas the probability of execution for aggressive limit orders is close to zero.

Trading with limit orders generates nonlinear payoffs; sometimes limit orders execute and sometimes they do not. As a result, limit orders are difficult to model. As Parlour and Seppi (2008) point out, limit orders compete with other limit orders, both existing and those submitted in the future. Further, all limit orders execute against future market orders. Thus, when selecting the price and quantity for a limit order, the trader must take into account the future trading process, including the current limit order book, as well as past histories of orders and trading outcomes.

Early limit order models, known as static equilibrium models, presumed that limit order traders were to be compensated for providing liquidity. Examples of such models include Rock (1996), Glosten (1994), and Seppi (1997). The longer the waiting time until order execution, the

higher was the expected compensation to liquidity providers who did not change their limit order specifications once they submitted the orders. The assumptions behind the static equilibrium models reflected early exchange conditions. Changing the details of a limit order was prohibitively costly, and market makers indeed expected a tidy compensation for bearing the risk of ending up in an adverse position once their limit orders were hit.

Static equilibrium models, however, have found little empirical support in the recent literature. In fact, Sandas (2001) in his study of limit orders on actively traded stocks on the Stockholm Stock Exchange finds that the expected profit on limit orders appears to decrease as the time duration between market orders increases, contradicting previously formulated theory. The implicit outcome of empirical evidence is that the limit orders are submitted by traders with active profit motives, rather than by market makers interested strictly in providing liquidity.

Demand for trading immediacy can be fueled by the traders' need for capital and their risk aversion, among other factors. Traders strapped for cash may choose to set the limit price close to the market to turn their positions into cash as soon as possible. Risk-averse traders may choose to set the limit price close to the market to ensure swift execution and to minimize uncertainty.

An extension to static equilibrium models penalizes aggressive limit orders with a non-execution cost. The cost can be considered a penalty for deviating too far from the trading targets of active limit order traders. Examples of such an approach include Kumar and Seppi (1994), who modeled two types of limit order traders—value traders and liquidity traders. Value traders submit limit orders to exploit undervalued limit orders but have no other reason to trade. Liquidity traders submit limit orders to respond to random liquidity shocks; randomness in their orders leads to price risk for other traders' market orders and execution risk in all limit orders. Cao, Hansch, and Wang (2004) find cointegration of different orders in the limit order book, supporting the existence of value traders.

Trader Types in Market Microstructure Trading

Harris (1998) identifies three types of traders:

1. Informed traders, who possess material information about an impending market move
2. Liquidity traders (also known as uninformed traders), who have no material market insights and aim to profit from providing liquidity and following short-term price momentum

3. Value-motivated traders, who wait for security prices to become cheap relative to their proprietary valuations of security based on fundamental indicators

Informed traders possess private information that allows them to predict future price changes in a particular security. Private information can include analyses from paid-for news sources, like Bloomberg, not yet available to the general public, and superior forecasts based on market microstructure. Informed traders are often high-frequency money managers and other proprietary traders with superior access to information and skill in assessing immediate market situations. The informed traders' private information can make a significant impact on the market. As a result, informed traders are impatient and typically execute their orders at market prices or at prices close to market (see Vega [2007]). Alam and Tkatch (2007) find that institutional orders are more likely to be market orders as opposed to limit orders, potentially reflecting advanced information and skill of institutional money managers.

Liquidity (or uninformed) traders specialize in crafting order submission strategies in an effort to capture best prices for their customers. These traders have little proprietary information about the true value of the security they trade. Executing broker-dealers are examples of liquidity traders.

Value traders run models to determine the fair value of each security, based on the publicly available information. Value traders might be traditional, low-frequency, institutional money managers; individual day-traders; or high-frequency managers running high-frequency fundamental models.

These three types of traders decide when to submit a market order as opposed to a limit order and at what price to submit a limit order. Such decisions are optimized to minimize trading costs and maximize portfolio returns.

According to Harris (1998), market orders and aggressively priced limit orders are placed by impatient traders, those with material market information about to become public. Limit orders that are far away from the current market price are typically placed by value traders, those seeking to obtain bargain prices. The remaining limit orders are placed by uninformed liquidity traders who are attempting to profit from making markets and from detecting and following short-term price momentum. Figure 10.2 illustrates the distributions of order aggressiveness and trader types relative to the market price in the limit order book. Harris (1998) considers a market for a single security, so that no substitute securities can be traded in place of an overpriced or illiquid security.

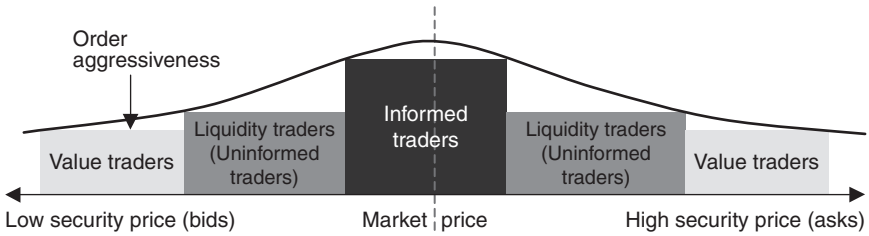

FIGURE 10.2 A graphical representation of order aggressiveness and trader type distributions in the limit order book.

Kaniel and Liu (2006) extend Angel (1992) and Harris (1998) to show that informed investors may use limit orders whenever their private information has a sufficient degree of persistence. Bloomfield, O'Hara, and Saar (2005) discuss how informed investors are natural liquidity providers. Because the informed investors know the true value of an asset, they are the first to know when the prices have adjusted to the levels at which the limit orders cannot be picked off by other traders. Measures of aggressiveness of the order flow may capture informed traders' information and facilitate generation of short-term profits.

Liquidity Provision

Limit orders provide market liquidity. As such, limit orders may fare better in lower-liquidity markets. The extent of market liquidity available can be assessed using one or more of the following measures:

1. **The tightness of the bid-ask spread.** The bid-ask spread indicates the cost of instantaneous reversal of a given position for a standard trading amount, or "clip."
2. **Market depth.** The depth of the market is the size of all limit orders posted at the current market price (the best limit price). Market depth therefore indicates the size of the order that can be processed immediately at the current market price.
3. **Market resilience.** Market resilience is a measure of how quickly the market price mean-reverts to its equilibrium level following a random order flow.
4. **Price sensitivity to block transactions.** The sensitivity is most often measured by what came to be known as "Kyle's λ," which is a

coefficient from OLS estimation of the following regression (Kyle, 1985):

$$\Delta P_t = \alpha + \lambda NVOL_t + \varepsilon_t \tag{10.1}$$

where ΔP_t is the change in market price due to the market impact of orders and $NVOL_t$ is the difference between the buy and sell market depths in period t. The smaller the market sensitivity to transaction size, λ, the larger the market's capacity to absorb orders at the current market price.

5. **Illiquidity ratio of Amihud (2002).** Amihud (2002) notes that illiquid markets are characterized by more drastic relative price changes per trade. Although changes in trade prices in the most liquid securities can be as low as one tick (e.g., 0.005 of 1 percent in most currency markets), in illiquid markets the changes in trade prices can be as large as 20 percent per trade. Amihud (2002), therefore, proposes to measure the degree of market illiquidity as the average ratio of relative price change to quantity traded:

$$\gamma_t = \frac{1}{D_t} \sum_{d=1}^{D_t} \frac{|r_{d,t}|}{v_{d,t}} \tag{10.2}$$

where D_t is the number of trades executed during time period t, $r_{d,t}$ is the relative price change following trade d during trade period t, and $v_{d,t}$ is the trade quantity executed within trade d.

Limit order books are also often characterized by the presence of "holes"—that is, ranges of prices that are skipped by investors when submitting limit orders. Holes were empirically documented by Biais, Hillion and Spatt (1995), among others.

PROFITABLE MARKET MAKING

Harrison and Kreps (1978) showed that the current value of an asset is determined by its resale potential; as a result, high-frequency investors trading in multiple markets arbitrage away price discrepancies among markets. The simplest profitable liquidity provision strategy involves identification of mispricings on the same security across different markets and arbitraging away the difference. This is done by posting a limit order to buy just below the market price in the low-priced market and a limit order to sell just above market in the high-priced market, and then reversing the positions once transaction costs were overcome.

Garman (1976) was the first to investigate the optimal market-making conditions through modeling temporary imbalances between buy and sell orders. These imbalances are due to differences between the individual trader and the way a dealer optimizes his order flow, which may reflect underlying differences in budgets, risk appetite, access to markets, and a host of other idiosyncrasies. The individual trader optimization problems themselves are less important than the aggregated order imbalances that these optimization differences help create. In the Garman (1976) model, the market has one monopolistic market maker (dealer). The market maker is responsible for deciding on and then setting bid and ask prices, receiving all orders, and clearing trades. The market maker's objective is to maximize profits while avoiding bankruptcy or failure. The latter arise whenever the market maker has no inventory or cash. Both buy and sell orders arrive as independent stochastic processes.

The model solution for optimal bid and ask prices lies in the estimation of the rates at which a unit of cash (e.g., a dollar or a "clip" of 10 million in FX) "arrives" to the market maker when a customer comes in to buy securities (pays money to the dealer) and "departs" the market maker when a customer comes in to sell (the dealer pays the customer). Suppose the probability of an arrival, a customer order to buy a security at the market ask price p_a is denoted λ_a. Correspondingly, the probability of departure of a clip from the market maker to the customer, or a customer order to sell securities to the market maker at the bid price p_b, can be denoted λ_b. Garman (1976) proposes a simple but effective model for the minimum bid-ask spreads necessary in order for the market maker to remain viable.

The model's solution is based on the solution to a classical problem known as the Gambler's Ruin Problem. In the dealer's version of the Gambler's Ruin Problem, a gambler, or a dealer, starts out with a certain initial wealth position and wagers (stays in business) until he loses all his money. This version of the Gambler's Ruin Problem is known as an unbounded problem. The bounded problem assumes that the gambler bets until he either loses all his money or reaches a certain level of wealth, at which point he exits.

Under the Gambler's Ruin Problem, the probability that the gambler will lose all his money is

$$\Pr_{Failure} = \left(\frac{\Pr(\text{Loss}) \times \text{Loss}}{\Pr(\text{Gain}) \times \text{Gain}} \right)^{\text{Initial Wealth}} \tag{10.3}$$

where *Initial Wealth* is the gambler's start-up cash, Pr(*Loss*) is the probability of losing an amount (*Loss*) of the initial wealth, and Pr(*Gain*) is the probability of gaining an amount (*Gain*).

From the Gambler's Ruin Problem, we can see that the probability of failure is always positive. It can be further shown that failure is certain whenever the probability of losing exceeds the probability of gaining. In other words, the minimum condition for a positive probability of avoiding failure in the long term is $\Pr(Gain) > \Pr(Loss)$.

Garman (1976) applies the Gambler's Ruin Problem to the market-making business in the following two ways:

1. The market maker fails if he runs out of cash.
2. The market maker fails if he runs out of inventory and is unable to satisfy client demand.

In modeling the Gambler's Ruin Problem for the market maker's ruin through running out of inventory, we assume that both the *Gain* and *Loss* variables are single units of the underlying financial asset. In other words,

$$\begin{aligned} Gain &= 1 \\ Loss &= 1 \end{aligned}$$

In the case of equity, this unit may be a share of stock. In the case of foreign exchange, the unit may be a clip. Then, from the market maker's perspective, the probability of "losing" one unit of inventory is the probability of selling a unit of inventory, and it equals the probability λ_a of a buyer arriving. By the same logic, the probability of gaining one unit of inventory is λ_b, the probability of a seller arriving. The Gambler's Ruin Problem equation (1) now becomes

$$\begin{aligned} \lim_{t\to\infty} \Pr_{Failure}(t) &\approx \left(\frac{\lambda_a}{\lambda_b}\right)^{Initial\ Wealth/E_0(p_a,p_b)}, \quad \text{if } \lambda_b > \lambda_a \\ &= 1, \text{ otherwise.} \end{aligned} \tag{10.4}$$

where $E_0(p_a, p_b)$ is the initial average price of an underlying unit of inventory and

$$\frac{Initial\ Wealth}{E_0(p_a, p_b)}$$

is the initial number of units of the financial instrument in possession of the market maker.

The Gambler's Ruin Problem is further applied to the market maker's probability of failure due to running out of cash. From the market maker's perspective, gaining a unit of cash—say a dollar—happens when a buyer of the security arrives. As before, the arrival of a buyer willing to buy at

price p_a happens with probability λ_a. As a result, the market maker's probability of gaining a dollar is p_a. Similarly, the market maker's probability of "losing" or giving away a dollar to a seller of the security for selling the security at price p_b is λ_b. The Gambler's Ruin Problem now takes the following shape:

$$\lim_{t\to\infty} \Pr_{Failure}(t) \approx \left(\frac{\lambda_b p_b}{\lambda_a p_a}\right)^{Initial\ Wealth}, \quad \text{if } \lambda_a p_a > \lambda_b p_b$$
$$= 1, \text{ otherwise} \tag{10.5}$$

For a market maker to remain in business, the first conditions of equations (10.4) and (10.5) need to be satisfied simultaneously. In other words, the following two inequalities have to hold contemporaneously:

$$\lambda_b > \lambda_a \qquad \text{and} \qquad \lambda_a p_a > \lambda_b p_b$$

For both inequalities to hold at the same time, the following must be true at all times: $p_a > p_b$, defining the bid-ask spread. The bid-ask spread allows the market maker to earn cash while maintaining sufficient inventory positions.

Other inventory models assume more detailed specifications for the market maker's objectives and constraints. For example, Stoll (1978) assumes that the main objective of the dealer is not only to stay in business but to effectively manage his portfolio in the face of market pressures. The bid-ask spread is the market maker's reward for bearing the costs of market making. These costs arise from the following three sources:

1. Inventory costs—the market maker often is left holding a suboptimal position in order to satisfy market demand for liquidity.
2. Order processing costs specific to the market maker's own trading mechanism—these costs may involve exchange fees, settlement and trade clearing costs, and transfer taxes, among other charges.
3. The information asymmetry cost—a market maker trading with well-informed traders may often be forced into a disadvantaged trading position.

As a result, Stoll's (1978) model predicts that the differences in bid-ask spreads between different market makers are a function of the market makers' respective risk tolerances and execution set-ups.

In Ho and Stoll (1981), the market maker determines bid and ask prices so as to maximize wealth while minimizing risk. The market maker controls his starting wealth positions, as well as the amounts of cash and inventory held on the book at any given time. As in Garman (1976), the arrival rates

of bid and ask orders are functions of bid and ask prices, respectively. In the outcome of the Ho and Stoll (1981) model, the market maker's spreads depend on the his time horizon. For example, as the market maker nears the end of the day, the possible changes in positions become smaller, and consequently the market maker's risk of carrying a position decreases. Therefore, the market maker may lower the spread towards the end of the day. On the other hand, when the market maker's time horizon increases, he increases the spread to be compensated for a higher probability of an adverse movement to the market maker's book positions.

Avellaneda and Stoikov (2008) transform Garman's model into a quantitative market-making limit order book strategy that generates persistent positive returns. Furthermore, the strategy outperforms the "best-bid-best-ask" market-making strategy where the trader posts limit orders at the best bid and ask available on the market. For fully rational, "risk-neutral" traders, the strategy of Avellaneda and Stoikov (2008) also outperforms the "symmetric" bid and ask strategy whereby the trader places bid and ask limit orders that are equidistant from the current mid-market price.

Avellaneda and Stoikov (2008) focus on the effects of inventory risk and derive the optimal bid and ask limit prices for the market maker, given the following six parameters:

- **The frequency of new bid quotes,** λ^{b}. For example, λ^{b} can be five per minute. The frequency of bid quotes can be thought of as demand for a given security as it reflects the arrival of new sellers.
- **The frequency of new ask quotes,** λ^{a}. The frequency of ask quotes can be thought of as an indicator of supply of a given security and the probability that new buyers will emerge.
- **The latest change in frequency of new bid quotes,** $\Delta\lambda^{b}$. For example, if 5 bid quotes arrived during the last minute, but 10 bid quotes arrived during the previous minute, then the change in the bid arrival frequency is $\Delta\lambda^{b} = (5 - 10)/10 = -0.5$.
- **The latest change in frequency of new ask quotes,** $\Delta\lambda^{a}$. For example, if 5 ask quotes arrived during the last minute, and 5 ask quotes arrived during the previous minute, then the change in the ask arrival frequency is $\Delta\lambda^{a} = (5 - 5)/5 = 0$.
- **The relative risk aversion of the trader,** γ. A small value of risk aversion, $\gamma \sim 0$, represents a risk-neutral investor. A risk aversion of 0.5, on the other hand, represents a very risk-averse investor.
- **The trader's reservation prices.** These are the highest price at which the trader is willing to buy a given security, r^{b}, and the lowest price at which the trader is willing to sell a given security, r^{a}. Both r^{a} and r^{b} are determined from a partial differential equation with the security price, s, trader's inventory, q, and time, t, as inputs.

The optimal limit bid price, b, and limit ask price, a, are then determined as follows:

$$b = r^b - \frac{1}{\gamma}\ln\left(1 - \gamma\frac{\lambda^b}{\Delta\lambda^b}\right) \quad \text{and} \quad a = r^a - \frac{1}{\gamma}\ln\left(1 - \gamma\frac{\lambda^a}{\Delta\lambda^a}\right)$$

Avellaneda and Stoikov (2008) offer the following comparisons of their inventory strategy with best bid/best ask and symmetric strategies for a reasonable trader risk aversion of 0.1. As Figure 10.3 shows, the inventory strategy proposed by Avellaneda and Stoikov (2008) has a narrow profit distribution, resulting in a high Sharpe ratio trading performance.

DIRECTIONAL LIQUIDITY PROVISION

When the Limit Order Book Is Observable

One of the key observations of inventory models is that the shape of the order book is predictive of impending changes in the market price. Figure 10.4 illustrates the phenomenon identified by Cao, Hansch, and Wang (2004). In panel (a), market price is "pushed" by a large concentration of conservative limit orders.

Cao, Hansch, and Wang (2004) find that the shape of the limit order book is actively exploited by market-making traders. Cao, Hansch, and Wang (2004) also find that the breadth and depth (also known as the length and height) of the limit order book predicts 30 percent of the impending price moves. Furthermore, the asymmetry in the order book generates additional information. Handa, Schwartz, and Tiwari (2003) find that the bid-ask spread is greater in "balanced" markets when the number of buyers and sellers is comparable; conversely, the bid-ask spread is lower whenever the number of traders on one side of the trading equation exceeds the number of traders on the other side. According to Handa, Schwartz, and Tiwari (2003), this imbalance effect stems from the fact that the few traders on the sparse trading side exert greater market power and obtain better prices from the investors on the populous trading side who are desperate to trade.

Rosu (2005) determines that the shape of the limit order book depends on the probability distribution for arriving market orders. High probabilities of large market orders lead to hump-shaped limit order books. Foucault, Kadan, and Kandel (2005) model continuous-time markets as an order-determination process on a multiprice grid with infinitely lived limit orders. Rosu (2005) extends the research with cancelable limit orders.

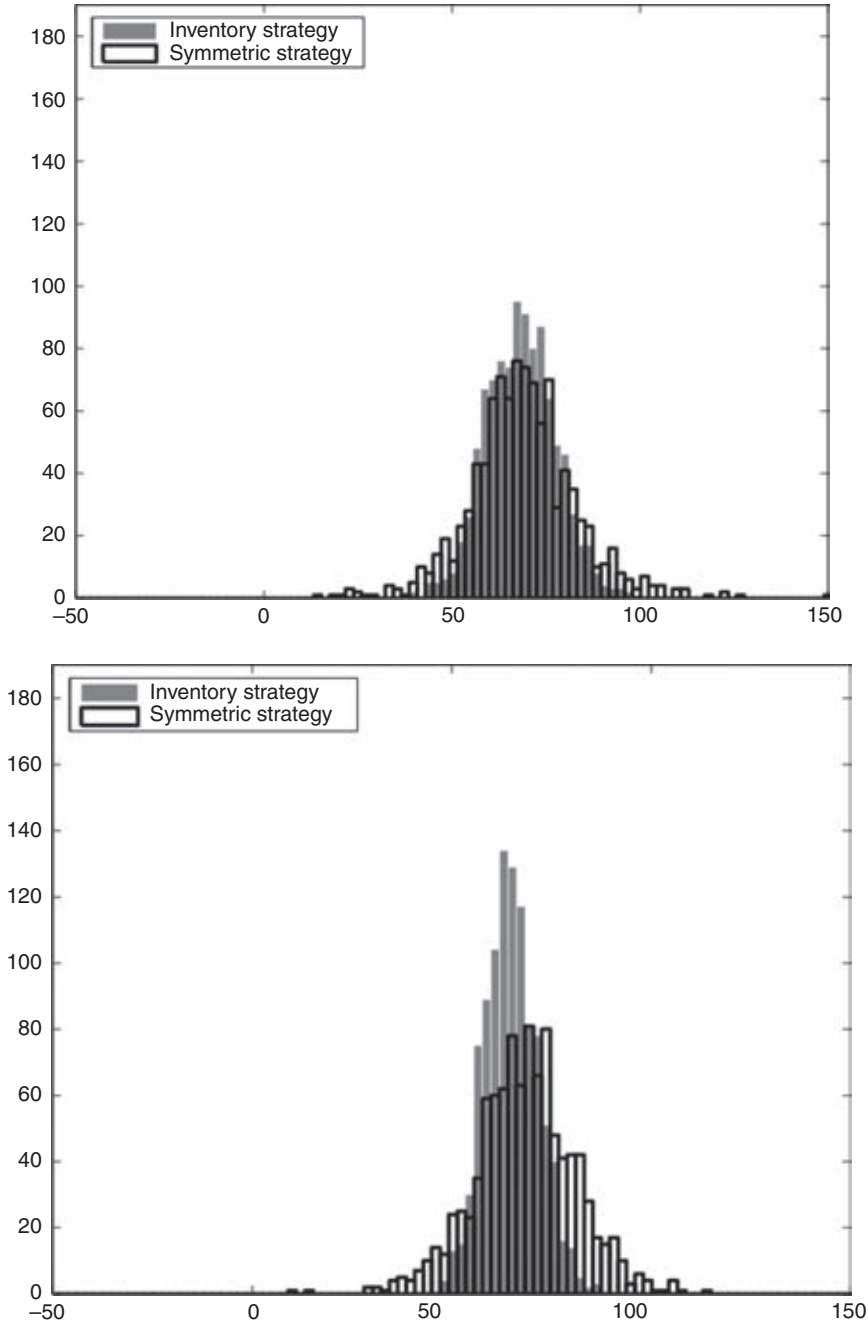

FIGURE 10.3 Comparison of performance of inventory, best bid/best ask, and symmetric strategies per Avellaneda and Stoikov (2008).

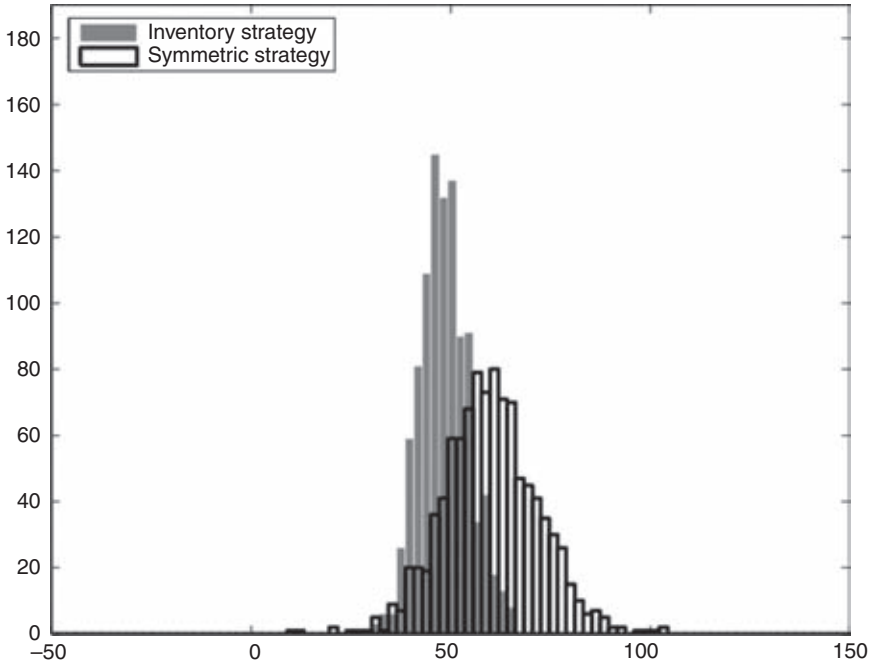

FIGURE 10.3 (*Continued*)

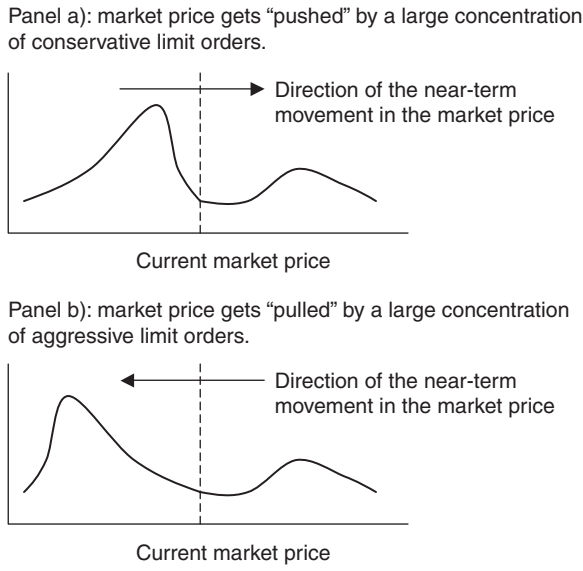

FIGURE 10.4 Limit book distribution and subsequent price moves.

Foucault, Moinas, and Theissen (2005) find that the depth of the limit order book can forecast future volatility of asset prices. In Holden and Subrahmanyam (1992), the more volatile the common valuation of the traded asset, the lower the depth of the information that can be gleaned from the limit order book. As a result, the limit order market multiplies the changes in volatility of the traded asset; small changes in the volatility of the value of the traded asset lead to large changes in volatility of transaction prices, and informed traders are less likely to provide liquidity.

Berber and Caglio (2004) find that limit orders carry private information around events such as earnings announcements.

The ability to observe the limit order book in full, however, can deliver unfair advantage to market makers. Harris and Panchapagesan (2005) show that market makers able to fully observe the information in the limit order book can extract abnormal returns, or "pick off" other limit traders.

When the Limit Order Book Is Not Observable

The directional strategy based on Cao, Hansch, and Wang (2004) requires full transparency of the limit order book for the instrument of interest. In many trading venues (e.g., dark pools), the limit order book is not available. This section discusses approaches for estimating the shape of the order book.

Kavajecz and Odders-White (2004) show limit orders to be indicative of future pockets of liquidity. Technical analysis has long been a friend of traders and a bane of academics. The amount of resources dedicated to technical analysis in the investment management industry, however, continues to puzzle academics, who find little plausible explanation for technical inferences in the science of economics. Most seem to agree that technical analysis is a self-fulfilling prophecy; when enough people believe that a particular pricing move is about to occur, they drive the price to its target location. Yet, technical analysis is more popular in some markets than in others; for example, many foreign exchange traders actively use technical analysis, while proportionally fewer equity traders do.

An interesting new application of technical analysis has been uncovered by Kavajecz and Odders-White (2004). The authors find that technical analysis may provide information about the shape of the limit book. Specifically, Kavajecz and Odders-White (2004) find that traders are more likely to place limit orders at the technical support and resistance levels. Thus, the support and resistance indicators pinpoint the liquidity peaks in the limit order book. This finding may be particularly helpful to ultra–high-frequency traders working in opaque or dark markets.

In addition, Kavajecz and Odders-White (2004) find that indicators based on moving averages help identify the skewness of the order book.

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When a short-run moving average rises above a long-run moving average, the buy-side liquidity pool in the limit order book moves closer to the market price. In this sense, moving average indicators help determine the skewness of the limit order book. Kavajecz and Odders-White (2004) speculate that the popularity of technical analysis in foreign exchange is driven by the absence of a centralized limit order book in the foreign exchange market. The authors believe that technical analysis helps traders reverse-engineer limit order books and deploy profitable liquidity provision strategies.

CONCLUSION

How does one take advantage of the opportunities present at ultra-high frequencies? First, a thorough econometric analysis of past short-term price and order book variability can be used to reveal a set of relationships that can be traded upon. Next, traders can simultaneously submit vectors of market and limit orders to promptly react to random fluctuations in buying and selling interest. The uncertainty in the timing of execution of limit orders, however, must be competently managed because it leads to random slippage in traders' portfolios, introducing a potentially undesirable stochastic dimension to their portfolio holdings.

Liquidity provision is not only profitable but is also an important function. As Parlour and Seppi (2008) note, valuation of publicly traded assets is a "social activity," strengthening the connection between liquidity and asset prices. Thus, trading activity creates value to investors who wish to reallocate their portfolios in response to changes in their personal valuations of assets.

CHAPTER 11

Trading on Market Microstructure

Information Models

Inventory models, discussed in Chapter 10, propose ways in which a market maker can set limit order prices based on characteristics of the market maker such as inventory (limit order book) and risk preferences. As such, inventory models do not account for motivations of other market participants. The dynamics relating to the trading rationale and actions of other market participants, however, can significantly influence the market maker's behavior.

Information models specifically address the intent and future actions of various market participants. Information models include game-theoretic approaches to reverse-engineer quote and trade flows to discover the information a market maker possesses. Information models also use observed or inferred order flow to make informed trading decisions.

At their core, information models describe trading on information flow and possible informational asymmetries arising during the dissemination of information. Differences in information flow persist in different markets. Information flow is comparably faster in transparent centralized markets, such as most equity markets and electronic markets, and slower in the opaque markets, such as foreign exchange and OTC markets in bonds and derivatives.

The main outcome of information models is that the bid-ask spreads persist even when the market maker has unlimited inventory and is able to absorb any trading request instantaneously. In fact, the spread is the way that the market maker stays solvent in the presence of well-informed traders. As the order flow from informed traders to the market maker conveys information from traders to the market maker, the subsequent

changes in the bid-ask spread may also convey information from the market maker to other market participants.

This chapter describes information-based microstructure trading strategies.

MEASURES OF ASYMMETRIC INFORMATION

Asymmetric information present in the markets leads to adverse selection, or the ability of informed traders to "pick off" uninformed market participants. According to Dennis and Weston (2001) and Odders-White and Ready (2006), the following measures of asymmetric information have been proposed over the years:

- Quoted bid-ask spread
- Effective bid-ask spread
- Information-based impact
- Adverse-selection components of the bid-ask spread
- Probability of informed trading

Quoted Bid-Ask Spread

The quoted bid-ask spread is the crudest, yet most readily observable measure of asymmetric information. First suggested by Bagehot (1971) and later developed by numerous researchers, the bid-ask spread reflects the expectations of market movements by the market maker using asymmetric information. When the quoting dealer receives order flow that he suspects may come from an informed trader and may leave the dealer at a disadvantage relative to the market movements, the dealer increases the spread he quotes in order to compensate himself against potentially adverse uncertainty in price movements. As a result, the wider the quoted bid-ask spread, the higher the dealer's estimate of information asymmetry between his clients and the dealer himself. Given that the dealer has the same access to public information as do most of the dealer's clients, the quoted bid-ask spread may serve as a measure of asymmetric information available in the market at large at any given point in time.

Effective Bid-Ask Spread

The effective bid-ask spread is computed as twice the difference between the latest trade price and the midpoint between the quoted bid and ask

prices, divided by the midpoint between the quoted bid and ask prices. The effective spread, therefore, produces a measure that is virtually identical to the quoted bid-ask spread but reflects the actual order book and allows comparison among financial instruments with various price levels.

Information-Based Impact

The information-based impact measure of asymmetric information is attributable to Hasbrouck (1991). Brennan and Subrahmanyam (1996) specify the following vector autoregressive (VAR) model for estimation of the information-based impact measure, λ:

$$V_{i,t} = \theta_{i,0} + \sum_{k=1}^{K} \beta_{i,k} \Delta P_{i,t-k} + \sum_{m=1}^{M} \gamma_{i,m} V_{i,t-m} + \tau_{i,t} \tag{11.1}$$

$$\Delta P_{i,t} = \phi_{i,0} + \phi_{i,1} \text{sign}(\Delta P_{i,t}) + \lambda_i \tau_{i,t} + \varepsilon_{i,t} \tag{11.2}$$

where $\Delta P_{i,t}$ is the change in price of security i from time $t-1$ to time t, $V_{i,t} = \text{sign}(\Delta P_{i,t}) \cdot v_{i,t}$, and $v_{i,t}$ is the volume recorded in trading the security i from time $t-1$ to time t. Brennan and Subrahmanyam (1996) propose five lags in estimation of equation (1): $K = M = 5$.

Adverse Selection Components of the Bid-Ask Spread

The adverse selection components of the bid-ask spread is attributable to Glosten and Harris (1988). The model separates the bid-ask spread into the following three components:

- Adverse selection risk
- Order-processing costs
- Inventory risk

Models in a similar spirit were proposed by Roll (1984); Stoll (1989); and George, Kaul, and Nimalendran (1991). The version of the Glosten and Harris (1988) model popularized by Huang and Stoll (1997) aggregates inventory risk and order-processing costs and is specified as follows:

$$\Delta P_{i,t} = (1 - \lambda_i) \frac{S_{i,t}}{2} \text{sign}(\Delta P_{i,t}) + \lambda_i \frac{S_{i,t}}{2} \text{sign}(\Delta P_{i,t}) \cdot v_{i,t} + \varepsilon_{i,t} \tag{11.3}$$

where $\Delta P_{i,t}$ is the change in price of security i from time $t-1$ to time t, $V_{i,t} = \text{sign}(\Delta P_{i,t}) \cdot v_{i,t}$, $v_{i,t}$ is the volume recorded in trading the security i

from time t-1 to time t, $S_{i,t}$ is the effective bid-ask spread as defined previously, and λ_i is the fraction of the traded spread due to adverse selection.

Probability of Informed Trading

Easley, Kiefer, O'Hara, and Paperman (1996) propose a model to distill the likelihood of informed trading from sequential quote data. The model reverse-engineers the quote sequence provided by a dealer to obtain a probabilistic idea of the order flow seen by the dealer.

The model is built on the following concept: Suppose an event occurs that is bound to impact price levels but is observable only to a select group of investors. Such an event may be a controlled release of selected information or a research finding by a brilliant analyst. The probability of such an event is α. Furthermore, suppose that if the event occurs, the probability of its having a negative effect on prices is δ and the probability of the event having a positive effect on prices is (1-δ). When the event occurs, informed investors know of the impact the event is likely to have on prices; they then place trades according to their knowledge at a rate μ. Thus, all the investors informed of the event will place orders on the same side of the market—either buys or sells. At the same time, investors uninformed of the event will keep placing orders on both sides of the market at a rate ω. The probability of informed trading taking place is then determined as follows:

$$PI = \frac{\alpha\mu}{\alpha\mu + 2\omega} \tag{11.4}$$

The parameters α, μ, and ω are then estimated from the following likelihood function over T periods of time:

$$L(B, S|\alpha, \mu, \omega, \delta) = \prod_{t=1}^{T} \ell(B, S, t|\alpha, \mu, \omega, \delta) \tag{11.5}$$

where $\ell(B, S, t|\alpha, \mu, \omega, \delta)$ is the daily likelihood of observing B buys and S sells:

$$\begin{aligned}\ell(B, S, t|\alpha, \mu, \omega, \delta) &= (1-\alpha)\left[\exp(-\omega T)\frac{(\omega T)^B}{B!}\right]\left[\exp(-\omega T)\frac{(\omega T)^S}{S!}\right] \\ &+ \alpha(1-\delta)\left[\exp(-(\omega+\mu)T)\frac{((\omega+\mu)T)^B}{B!}\right]\left[\exp(-\omega T)\frac{(\omega T)^S}{S!}\right] \\ &+ \alpha\delta\left[\exp(-\omega T)\frac{(\omega T)^B}{B!}\right]\left[\exp(-(\omega+\mu)T)\frac{((\omega+\mu)T)^S}{S!}\right]\end{aligned} \tag{11.6}$$

INFORMATION-BASED TRADING MODELS

Trading on Information Contained in Bid-Ask Spreads

Liquidity-providing market participants (or market makers) use bid-ask spreads as compensation for bearing the costs of market making. These costs arise from the following four sources:

1. **Order-processing costs.** Order-processing costs are specific to the market maker's own trading platform. These costs may involve exchange fees, settlement and trade clearing costs, transfer taxes, and the like. To transfer the order-processing costs to their counterparties, market makers simply increase the bid-ask spread by the amount it costs market makers to process the orders.
2. **Inventory costs.** Market makers often find themselves holding suboptimal positions in order to satisfy market demand for liquidity. They therefore increase the bid-ask spreads they quote to their counterparties to slow down further accumulation of adverse positions, at least until they are able to distribute their inventory among other market participants.
3. **Information asymmetry costs.** A market maker trading with well-informed traders may often be forced into a disadvantageous trading position. For example, if a well-informed trader is able to correctly forecast that EUR/JPY is about to increase by 1 percent, then the well-informed trader buys a certain amount of EUR/JPY from the market maker, leaving the market maker holding a short position in EUR/JPY in the face of rising EUR/JPY. To hedge his risk of ending up in such situations, the market maker widens the bid-ask spreads for all of his counterparties, informed and uninformed alike. The bid-ask spread on average compensates for the market maker's risk of being at a disadvantage.
4. **Time-horizon risk.** Most market makers are evaluated on the basis of their daily performance, with a typical trading day lasting eight hours. At the end of each trading day, the market maker closes his position book or transfers the book to another market maker who takes the responsibility and makes decisions on all open positions. At the beginning of each trading shift, a market maker faces the risk that each of his open market positions may move considerably in the adverse direction by the end of the day if left unattended. As the day progresses, the market maker's time horizon shrinks, and with it shrinks the risk

of a severely adverse move by the traded security. The market maker uses the bid-ask spreads he quotes to his counterparties to hedge the time-horizon risk of his own positions. The bid-ask spreads are greatest at the beginning of the trading day and smallest at the end of each trading day.

Figures 11.1–11.3 illustrate the first three dimensions per Lyons (2001). If bid-ask spreads were to compensate the dealer for order processing costs only, then the mid-price does not change in response to the

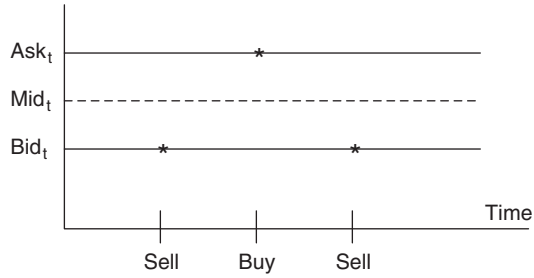


FIGURE 11.1 Order-processing costs.

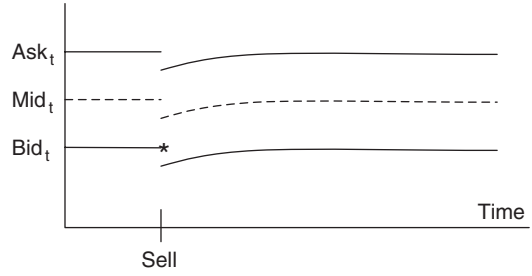


FIGURE 11.2 Inventory costs.

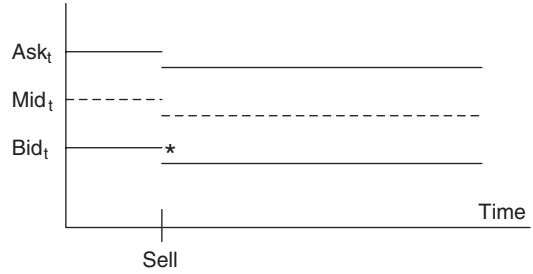


FIGURE 11.3 Asymmetric information (adverse selection).

order. If bid-ask spreads were to compensate the dealer for the risks associated with holding excess inventory, then any changes in prices would be temporary. If all orders were to carry information that led to permanent price changes, the bid-ask spreads would compensate the dealer for the potential risk of encountering adverse asymmetric information.

Analyzing the bid-ask spreads of the market maker gives clues to the position sizes of the market maker's inventory and allows the market maker to estimate the order flow faced by the market maker. Unexpectedly widening spreads may signal that the market maker has received and is processing large positions. These positions may be placed by well-informed institutional traders and may indicate the direction of the security price movement in the near future. Information about future price movements extracted from the bid-ask spreads may then serve as reliable forecasts of direction of upcoming price changes.

Gains achieved in the markets are due either to market activity or to trading activity. Market gains, also referred to as capital gains, are common to most long-term investors. When markets go up, so do the gains of most investors long in the markets; the opposite occurs when the markets go down. Over time, as markets rise and fall, market investors expect to receive the market rate of return on average. As first noted by Bagehot (1971), however, the presence of short-term speculative traders may skew the realized return values.

If a market maker knew for sure that a specific trader had superior information, that market maker could raise the spread for that trader alone. However, most traders trade on probabilities of specific events occurring and cannot be distinguished ahead of closing their positions from uninformed market participants. To compensate for the informed trader–related losses, the market maker will extract higher margins from all of his clients by raising the bid-ask spread. As a result, in the presence of well-informed traders, both well-informed and less-informed market participants bear higher bid-ask spreads.

Important research on the topic is documented in Glosten and Milgrom (1985). The authors model how informed traders' orders incorporate and distribute information within a competitive market. At the fundamental level, traders who have bad news about a particular financial security ahead of the market will sell that security, and traders with good news will buy the security. However, well-informed traders may also buy and sell to generate liquidity. Depending on the type of the order the market maker receives (either a buy or a sell), the market maker will adjust his beliefs about the impending direction of the market for a particular financial security. As a result, the market maker's expected value of the security changes, and the market maker subsequently adjusts the bid and the ask prices he is willing to trade upon with his clients.

Glosten and Milgrom (1985) model the trading process as a sequence of actions. First, some traders obtain superior information about the true value of the financial security, V, while other market participants do not. Informed traders probabilistically profit more often than do uninformed traders and consequently are interested in trading as often as possible. Because informed traders know the true value V, they will always gain at the expense of the uninformed traders. The uninformed traders may choose to trade for reasons other than short-term profits. For example, market participants such as long-term investors uninformed of the true value of the security one minute from now may have a pretty good idea of the true value of the security one day from now. These uninformed investors are therefore willing to trade with an informed trader now even though in just one minute the investors could get a better price, unbeknownst to them at present. In the foreign exchange market, uninformed market participants such as multinational corporations may choose to trade to hedge their foreign exchange exposure.

The informed traders' information gets impounded into prices by the market maker. When a market maker receives an order, the market maker reevaluates his beliefs about the true value of the financial security based on the parameters of the order. The order parameters may be the action (buy or sell), limit price if any, order quantity, leverage or margin, and the trader's prior success rate, among other order characteristics. The process of incorporating new information into prior beliefs that the market maker undergoes with every order is often modeled according to the Bayes rule. Updating beliefs according to the Bayes rule is known as Bayesian learning. Computer algorithms employing Bayesian learning are often referred to as genetic algorithms.

"In Praise of Bayes," an article in the *The Economist* from September 40, 2000, describes Bayesian learning as follows:

> *The essence of the Bayesian approach is to provide a mathematical rule explaining how you should change your existing beliefs in the light of new evidence. In other words, it allows scientists to combine new data with their existing knowledge or expertise. The canonical example is to imagine that a precocious newborn observes his first sunset, and wonders whether the sun will rise again or not. He assigns equal prior probabilities to both possible outcomes, and represents this by placing one white and one black marble into a bag. The following day, when the sun rises, the child places another white marble in the bag. The probability that a marble plucked randomly from the bag will be white (i.e., the child's degree of belief in future sunrises) has thus gone from a half to two-thirds. After sunrise the next day, the child adds another white marble, and the probability*

(and thus the degree of belief) goes from two-thirds to three-quarters. And so on. Gradually, the initial belief that the sun is just as likely as not to rise each morning is modified to become a near-certainty that the sun will always rise.

Mathematically, the Bayes rule can be specified as follows:

$$\begin{aligned}\Pr(\text{seeing data}) = {} & \Pr(\text{seeing data} \mid \text{event occurred})\ \Pr(\text{event occurred}) \\ & + \Pr(\text{seeing data} \mid \text{no event})\Pr(\text{no event})\end{aligned} \tag{11.7}$$

where Pr(event) may refer to the probability of the sun rising again or the probability of security prices rising, while seeing may refer to registering a buy order or actually observing the sunrise. No event may refer to a lack of buy orders or inability to observe a sunrise on a particular day—for example, due to a cloudy sky. The probability of seeing data and registering an event at the same time has the following symmetric property:

$$\begin{aligned}&\Pr(\text{seeing data, event}) = \Pr(\text{event}|\text{seeing data})\ \Pr(\text{seeing data}) \\ &= \Pr(\text{seeing data}|\text{event})\Pr(\text{event})\end{aligned} \tag{11.8}$$

Rearranging equation (11.8) to obtain expression for Pr(event|seeing data) and then substituting Pr (seeing data) from equation (1) produces the following result:

$$\Pr(\text{event}|\text{seeing data}) = \frac{\Pr(\text{seeing data}|\text{event})\Pr(\text{event})}{\Pr(\text{seeing data}|\text{event})\Pr(\text{event}) + \Pr(\text{seeing data}|\text{no event})\Pr(\text{no event})} \tag{11.9}$$

Equation (11.9) is known as the Bayes rule, and it can be rewritten as follows:

$$\begin{aligned}&\text{Posterior belief} = \Pr(\text{event}|\text{seeing data}) \\ &= \frac{\textit{Prior belief} \times \Pr(\text{seeing data}|\text{event})}{\text{Marginal likelihood of the data}}\end{aligned} \tag{11.10}$$

Market makers apply the Bayes rule after each order event, whether they consciously calculate probabilities or unconsciously use their trading experience. For example, suppose that the market maker is in charge of providing liquidity for the Australian dollar, AUD/USD. The current mid-price, V_{mid}, for AUD/USD is 0.6731. Consequently, the market maker's

initial belief about the true price for AUD/USD is 0.6731. At what level should the market maker set bid and ask prices?

According to the Bayes rule, the market maker should set the new ask price to the expected V_{ask}, given the buy order: $E[V_{\text{ask}}|\text{buy order}]$. Suppose that the market maker cannot distinguish between informed and uninformed traders and assigns a 50 percent probability to the event that a market buy order will arrive from an informed trader and a 50 percent probability to the event that a market buy order will arrive from an uninformed trader. In addition, suppose that any informed trader would place a buy order only if the trader was certain he could make at least 5 pips on each trade in excess of the bid-ask spread and that there are no other transaction costs. If the average bid-ask spread on AUD/USD quoted by the market maker is 2 pips, then an informed trader would place a buy order only if he believes that the true value of AUD/USD is at least 0.6738. From the market maker's perspective, the Bayesian probability of the true value of AUD/USD, V_{ask}, being worth 0.6738 after observing a buy order is calculated as follows:

$$\Pr(V_{\text{ask}} = 0.6738|\text{buy order}) = \frac{\Pr(V_{\text{ask}} = 0.6738)\Pr(\text{buy order}|V_{\text{ask}} = 0.6738)}{\Pr(V_{\text{ask}} = 0.6731)\Pr(\text{buy order}|V_{\text{ask}} = 0.6731) + \Pr(V_{\text{ask}} = 0.6738)\Pr(\text{buy order}|V_{\text{ask}} = 0.6738)} \tag{11.11}$$

If the true value V_{ask} is indeed 0.6738, then an informed trader places the buy order with certainty (a probability of 100 percent), while the uninformed trader may place a buy order with a probability of 50 percent (also with a probability of 50 percent, the uninformed trader may place a sell order instead). Thus, if the true price $V_{\text{ask}} = 0.6738$,

$$\Pr(\text{buy order}|V_{\text{ask}} = 0.6738) = \Pr(\text{informed trader}) * 100 \text{ percent} + \Pr(\text{uninformed trader}) * 50 \text{ percent} \tag{11.12}$$

Since we previously assumed that the market maker cannot distinguish between informed and uninformed traders and assigns equal probability to either,

$$\Pr(\text{informed trader}) = \Pr(\text{uninformed trader}) = 50 \text{ percent} \tag{11.13}$$

Combining equations (11.12) and (11.13), we obtain the following probability of the buy order being an indication of the buy order resulting from higher true value V_{ask}:

$$\begin{aligned}&\Pr(\text{buy order}|V_{\text{ask}} = 0.6738) = 50 \text{ percent}^* 100 \text{ percent}\\&+50 \text{ percent}^* \ 50 \text{ percent}\\&= 75 \text{ percent}.\end{aligned} \tag{11.14}$$

The probability of the buy order resulting from the lower, unchanged, true value V_{ask} is then

$$\begin{aligned}&\Pr(\text{buy order}|V_{\text{ask}} = 0.6731) = 1- \ \Pr(\text{buy order}|V_{\text{ask}} = 0.6738)\\&= 25 \text{ percent}.\end{aligned} \tag{11.15}$$

Assuming that the market maker has no indication where the market is going—in other words, from the market maker's perspective at the given moment,

$$\Pr(V_{\text{ask}} = 0.6738) = \Pr(V_{\text{ask}} = 0.6731) = 50 \text{ percent} \tag{11.16}$$

and substituting equations (11.13), (11.14), (11.15), and (11.16) into equation (11.11), we obtain the following probability of the true value of AUD/USD being at least 0.6738 given that the buy order arrived:

$$\Pr(V_{\text{ask}} = 0.6738|\text{buy order}) = \frac{50\% \times 75\%}{50\% \times 25\% + 50\% \times 75\%} = 75\%$$

By the same logic, $\Pr(V_{\text{ask}} = 0.6731|\text{buy order}) = 25\%$.

Having calculated these probabilities, the market maker is now ready to set market prices. He sets the price equal to the conditional expected value as follows:

$$\begin{aligned}&\text{New ask price} = E[V|\text{buy order}] = 0.6731 \times \Pr(V_{\text{ask}} = 0.6731)\\&+0.6738 \times \Pr(V_{\text{ask}} = 0.6738)\\&= 0.6731 \times 25\% + 0.6738 \times 75\% = 0.6736\end{aligned} \tag{11.17}$$

Similarly, a new bid price can be calculated as $E[V_{\text{bid}}|\text{sellorder}]$ after observing a sell order. The resulting bid and ask values are tradable "regret-free" quotes. When the market maker sells a unit of his inventory known as a clip to the buyer at 0.6736, the market maker is protected against the loss because of the buyer's potentially superior information.

If a buy order at ask of 0.6738 actually arrives, the market maker then recalculates his bid and ask prices as new conditional expected values. The market maker's new posterior probability of a buy coming from an informed trader is 75 percent by the foregoing calculation. Furthermore, an

informed buyer would buy at 0.6738 only if the true value of AUD/USD less the ask price exceeded the average spread and the trader's minimum desired gain. Suppose that, as before, the trader expects the minimum desired gain to be 5 pips and the average spread to be 2 pips, making him willing to buy at 0.6738 only if the true value of AUD/USD is at least 0.6745. The market maker will now once again adjust his bid and ask prices conditional on such expectation.

One outcome of Glosten and Milgrom (1985) is that in the presence of a large number of informed traders, a market maker will set unreasonably high spreads in order to break even. As a result, no trades will occur and the market will shut down.

In Glosten and Milgrom (1985) all trades are done on one unit of the financial security. Glosten and Milgrom (1985) do not consider the impact that the trade size has on price. Easley and O'Hara (1987) extend the Glosten and Milgrom model to incorporate varying trade sizes. Easley and O'Hara (1987) further add one more level of complexity—information arrives with probability α. Still, both Glosten and Milgrom (1985) and Easley and O'Hara are models in which informed traders simply submit orders at every trading opportunity until prices adjust to their new full-information value. The traders do not strategically consider the optimal actions of market makers and how market makers may act to reduce traders' profitability.

By contrast, the class of models known as strategic models develop conjectures about pricing policies of the market maker and incorporate those conjectures into their trading actions. One such strategic model by Kyle (1985) analyzes how a single informed trader could best take advantage of his information in order to maximize his profits.

Kyle (1985) describes how information is incorporated into security prices over time. A trader with exclusive information (e.g., a good proprietary quantitative model) hides his orders among those of uninformed traders to avoid provoking the market maker into increasing the spread or otherwise adjusting the price in any adverse manner.

Mende, Menkhoff, and Osler (2006) note that the process of embedding information into foreign exchange prices differs from the process of other asset classes, say equities. Traditional microstructure theory observes four components contributing to the bid-ask spread: adverse selection, inventory risk, operating costs, and occasional monopoly power. Foreign exchange literature often excludes the possibility of monopolistic pricing in the foreign exchange markets due to decentralization of competitive foreign exchange dealers. Some literature suggests that most bid-ask spreads arise as a function of adverse selection; dealers charge the bid-ask spread to neutralize the effects of losing trades in which the counterparties are better informed than the dealer himself. As a result, dealers that can differentiate between informed and uninformed customers charge higher spreads

on trades with informed customers and lower spreads on trades with uninformed customers.

Mende, Menkhoff, and Osler (2006) note that in foreign exchange markets the reverse is true: uninformed customers, such as corporate and commercial entities that transact foreign exchange as part of their operations, receive higher bid-ask spreads from dealers than do institutional customers that transact foreign exchange for investment and speculative purposes. Mende, Menkhoff, and Osler (2006) further suggest that the dealers may be simply enjoying higher margins on corporate and commercial entities than on institutional customers due to competitive pressures from the electronic marketplace in the latter markets. Mende, Menkhoff, and Osler (2006) attribute this phenomenon to the relative market power of foreign exchange dealers among corporate and commercial enterprises.

In addition, Mende, Menkhoff, and Osler (2006) suggest that the dealers may strategically subsidize the trades that carry information, as first noted by Leach and Madhavan (1992, 1993) and Naik, Neuberkert, and Viswanathan (1999). For example, the dealers may provide lower spreads on large block orders in an effort to gather information and use it in their own proprietary trades.

Mende, Menkhoff, and Osler (2006), however, emphasize that the majority of price variations in response to customer orders occurs through dealer inventory management. When the dealer transacts with an informed customer, the dealer immediately needs to diversify the risk of ending up on the adverse side of the transaction. For example, if a dealer receives a buy order from an informed customer, there is a high probability that the market price is about to rise; still, the dealer has just sold his inventory to the customer. To diversify his exposure, the dealer places a buy in the interdealer markets. When the dealer receives a buy order from an uninformed customer, on the other hand, the probability that the market price will rise is low, and the dealer has no immediate need to diversify the exposure that results from his trading with the uninformed customer.

Trading on Order Aggressiveness

Much of the success of microstructure trading is based on the trader's ability to retrieve information from observed market data. The market data can be publicly observed, as is real-time price and volume information. The data can also be private, such as the information about client order flow that can be seen only by the client's broker.

To extract the market information from the publicly available data, Vega (2007) proposes monitoring the aggressiveness of trades. Aggressiveness refers to the percentage of orders that are submitted at market prices, as opposed to limit prices. The higher the percentage of market orders, the

more aggressive the trader in his bid to capture the best available price and the more likely the trader is to believe that the price of the security is about to move away from the market price.

The results of Vega (2007) are based on those of Foster and Viswanathan (1996), who evaluate the average response of prices in a situation where different market participants are informed to a different degree. For example, before an expected economic announcement is made, it is common to see "a consensus forecast" that is developed by averaging forecasts of several market analysts. The consensus number is typically accompanied by a range of forecasts that measures the dispersion of forecasts by all analysts under consideration. For example, prior to the announcement of the January 2009 month-to-month change in retail sales in the United States, Bloomberg LP reported the analysts' consensus to be –0.8 percent, while all the analysts' estimates for the number ranged from –2.2 percent to 0.3 percent (the actual number revealed at 8:30 A.M. on February 12, 2009, happened to be +1.0 percent).

Foster and Viswanathan (1996) show that the correlation in the degree of informativeness of various market participants affects the speed with which information is impounded into prices, impacts profits of traders possessing information, and also determines the ability of the market participants to learn from each other. In other words, the narrower the analysts' forecast range, the faster the market arrives at fair market prices of securities following a scheduled news release. The actual announcement information enters prices through active trading. Limit orders result in more favorable execution prices than market orders; the price advantage, however, comes at a cost—the wait and the associated risk of non-execution. Market orders, on the other hand, are executed immediately but can be subject to adverse pricing. Market orders are used in aggressive trading, when prices are moving rapidly and quick execution must be achieved to capture and preserve trading gains. The better the trader's information and the more aggressive his trading, the faster the information enters prices.

As a result, aggressive orders may themselves convey information about the impending direction of the security price move. If a trader executes immediately instead of waiting for a more favorable price, the trader may convey information about his beliefs about where the market is going. Vega (2007) shows that better-informed market participants trade more aggressively. Mimicking aggressive trades, therefore, may result in a consistently profitable trading strategy. Measures of aggressiveness of the order flow may further capture informed traders' information and facilitate generation of short-term profits.

Anand, Chakravarty, and Martell (2005) find that on the NYSE, institutional limit orders perform better than limit orders placed by

individuals, orders at or better than market price perform better than limit orders placed inside the bid-ask spread, and larger orders outperform smaller orders. To evaluate the orders, Anand, Chakravarty, and Martell (2005) sampled all orders and the execution details of a 3-month trading audit trail on the NYSE, spanning November 1990 through January 1991.

Anand, Chakravarty, and Martell (2005) use the following regression equation to estimate the impact of various order characteristics on the price changes measured as $Diff_t$, the difference between the bid-ask midpoints at times t and $t + n$:

$$Diff_t = \beta_0 + \beta_1 Size_t + \beta_2 Aggressiveness_t + \beta_3 Institutional_t + \gamma_1 D_{1t} + \cdots + \gamma_k D_{k-1,t} + \varepsilon_t$$

where t is the time of the order submission, n equals 5 and then 60 minutes after order submission. *Size* is the number of shares in the particular order divided by the mean daily volume of shares traded in the particular stock over the sample period. For buy orders, *Aggressiveness* is a dummy that takes the value 1 if the order is placed at or better than the standing quote and zero otherwise. *Institutional* is a dummy variable that takes the value 1 for institutional orders and 0 for individual orders. D_1 to $D_{k\text{-}1}$ are stock-specific dummies associated with the particular stock that was traded.

TABLE 11.1 Difference in the Performance of Institutional and Individual Orders

	Intercept	Size	Aggressiveness	Institutional
Panel A: 97 stocks				
5 min after order placement	0.005	0.010*	0.016*	0.004*
60 min after order placement	0.020**	0.020*	0.012*	0.006*
Panel B: 144 stocks				
5 min after order placement	0.006	0.012*	0.014*	0.004*
60 min after order placement	0.021**	0.023*	0.012*	0.004*

This table, from Anand, Chakravarty, and Martell (2005), summarizes the results of robustness regressions testing for a difference in the performance of institutional and individual orders. The regression equation controls for stock selection by institutional and individual traders. The dependent variable in the regression is the change in the bid-ask midpoint 5 and then 60 minutes after order submission.
*t-test significant at 1 percent.
**t-test significant at 5 percent.
Reprinted from *Journal of Financial Markets*, 8/3 2005, Amber Anand, Sugato Chakravarty, and Terrence Martell, "Empirical Evidence on the Evolution of Liquidity: Choice of Market versus Limit Orders by Informed and Uninformed Traders," page 21, with permission from Elsevier.

According to several researchers, market aggressiveness exhibits autocorrelation that can be used to forecast future realizations of market aggressiveness. The autocorrelation of market aggressiveness is thought to originate from either of the following sources:

- Large institutional orders that are transmitted in smaller slices over an extended period of time at comparable levels of market aggressiveness
- Simple price momentum

Research into detecting autocorrelation of market aggressiveness was performed by Biais, Hillion, and Spatt (1995), who separated orders observed on the Paris Bourse by the degree of aggressiveness—from the least aggressive market orders that move prices to the most aggressive limit orders outside the current book. The authors found that the distribution of orders in terms of aggressiveness depends on the state of the market and that order submissions are autocorrelated. The authors detected a "diagonal effect" whereby initial orders of a certain level of aggressiveness are followed by other orders of the same level of aggressiveness. Subsequent empirical research confirmed the findings for different stock exchanges. See, for example, Griffiths, Smith, Turnbull, and White (2000) for the Toronto Stock Exchange; Ranaldo (2004) for the Swiss Stock Exchange; Cao, Hansch, and Wang (2004) for the Australian Stock Exchange; Ahn, Bae, and Chan (2001) for the Stock Exchange of Hong Kong; and Handa, Schwartz, and Tiwari (2003) for the CAC40 stocks traded on the Paris Bourse.

Trading on Order Flow

Order Flow Overview Order flow is the difference between buyer-initiated and seller-initiated trading volume. Order flow has lately been of particular interest to both academics and practitioners studying the flow's informational content. According to Lyons (2001), order flow is informative for three reasons:

1. Order flow can be thought of as market participants exposing their equity to their own forecasts. A decision to send an order can be costly to market participants. Order flow therefore reflects market participants' honest beliefs about the upcoming direction of the market.
2. Order flow data is decentralized with limited distribution; brokers see the order flow of their clients and inter-dealer networks only. Clients seldom see any direct order flow at all, but can partially infer the order flow information from market data provided by their brokers using a

complex and costly mechanism. Because the order flow is not available to everyone, those who possess full order flow information are in a unique position to exploit it before the information is impounded into market prices.

3. Order flow shows large and nontrivial positions that will temporarily move the market regardless of whether the originator of the trades possesses any superior information. Once again, an entity observing the order flow is best positioned to capitalize on the market movements surrounding the transaction.

Lyons (2001) further distinguishes between transparent and opaque order flows, with transparent order flows providing immediate information, and opaque order flows failing to produce useful data or subjective analysis to extract market beliefs. According to Lyons (2001), order flow transparency encompasses the following three dimensions:

- Pre-trade versus post-trade information
- Price versus quantity information
- Public versus dealer information

Brokers observing the customer and inter-dealer flow firsthand have access to the information pre-trade, can observe both the price and the quantity of the trade, and can see both public and dealer information. On the other hand, end customers can generally see only the post-trade price information by the time it becomes public or available to all customers. Undoubtedly, dealers are much better positioned to use the wealth of information embedded in the order flow to obtain superior returns, given the appropriate resources to use the information efficiently.

Order flow information is easy to trade profitably. A disproportionately large number of buy orders will inevitably push the price of the traded security higher; placing a buy order at the time a large buy volume is observed will result in positive gains. Similarly, a large number of sell orders will depress prices, and a timely sell order placed when the sell order flow is observed will generate positive results.

Order Flow Is Directly Observable As noted by Lyons (1995), Perraudin and Vitale (1996), and Evans and Lyons (2002), among others, order flow is a centralized measure of information that was previously dispersed among market participants. Order flow for a particular financial security at any given time is formally measured as the difference between buyer-initiated and seller-initiated trading interest. Order flow is sometimes referred to as buying or selling pressures. When the trade sizes are

observable, the order flow can be computed as the difference between the cumulative size of buyer-initiated trades and the cumulative size of seller-initiated trades. When trade quantities are not directly observable, order flow can be measured as the difference between the number of buyer-initiated trades and seller-initiated trades in each specific time interval.

Both trade-size–based and number-of-trades–based measures of order flow have been used in the empirical literature. The measures are comparable since most orders are transmitted in "clips," or parcels of a standard size, primarily to avoid undue attention and price run-ups that would accompany larger trades. Jones, Kaul, and Lipson (1994) actually found that order flow measured in number of trades predicts prices and volatility better than order flow measured in aggregate size of trades.

The importance of order flow in arriving at a new price level following a news announcement has been verified empirically. Love and Payne (2008), for example, examine the order flow in foreign exchange surrounding macroeconomic news announcements and find that order flow directly accounts for at least half of all the information impounded into market prices.

Love and Payne (2008) studied the impact of order flow on three currency pairs: USD/EUR, GBP/EUR, and USD/GBP. The impact of the order flow on the respective rates found by Love and Payne (2008) is shown in Table 11.2. The authors measure order flow as the difference between the number of buyer-initiated and the number of seller-initiated trades in each 1-minute interval. Love and Payne (2008) document that at the time of news release from Eurozone, each additional buyer-initiated trade in excess of seller-initiated trades causes USD/EUR to increase by 0.00626 or 0.626 percent.

TABLE 11.2 Average Changes in 1-Minute Currency Returns Following a Single Trade Increase in the Number of Buyer-Initiated Trades in Excess of Seller-Initiated Trades

	USD/EUR Return	GBP/EUR Return	USD/GBP Return
$Flow_t$ at a time coinciding with a news release from Eurozone	0.00626*	0.000544	0.00206
$Flow_t$ at a time coinciding with a news release from the UK	0.000531	0.00339***	0.00322***
$Flow_t$ at a time coinciding with a news release from the U.S.	0.00701***	0.00204	0.00342**

***, ** and * denote 99.9 percent, 95 percent, and 90 percent statistical significance, respectively.

Order Flow Is Not Directly Observable Order flow is not necessarily transparent to all market participants. For example, executing brokers can directly observe buy-and-sell orders coming from their customers, but generally the customers can see only the bid and offer prices, and, possibly, the depth of the market.

As a result, various models have sprung up to extract order flow information from the observable data. Most of these models are based on the following principle: the aggregate number of buy orders dominates the aggregate number of sell orders for a particular security whenever the price of that security rises, and vice versa. Hasbrouck (1991) proposes the following identification of order flow, adjusted for orders placed in previous time periods and conditioned on the time of day:

$$x_{i,t} = \alpha_x + \sum_{k=1}^{K} \beta_k r_{i,t-k} + \sum_{m=1}^{M} \gamma_m x_{t-m} + \sum_{t=1}^{T} \delta D_t + \varepsilon_{i,t} \tag{11.18}$$

where $x_{i,t}$ is the aggregate order flow for a security i at time t, equal to $+1$ for buys and -1 for sells; $r_{i,t}$ is a one-period return on the security i from time t-1 to time t; and D_t is the dummy indicator controlling for the time of day into which time t falls. Hasbrouck (1991) considers nineteen D_t operators corresponding to half-hour periods between 7:30 A.M. and 5:00 P.M. EST.

Autocorrelation of Order Flows Like market aggressiveness, order flows exhibit autocorrelation, according to a number of articles including those by Biais, Hillion, and Spatt (1995); Foucault (1999); Parlour (1998); Foucault, Kadan, and Kandel (2005); Goettler, Parlour, and Rajan (2005, 2007); and Rosu (2005).

Ellul, Holden, Jain, and Jennings (2007) interpret short-term autocorrelation in high-frequency order flows as waves of competing order flows responding to current market events within liquidity depletion and replenishment. Ellul, Holden, Jain, and Jennings (2007) confirm strong positive serial correlation in order flow at high frequencies, but find negative order firm correlation at lower frequencies on the New York Stock Exchange. Hollifield, Miller, and Sandas (2004) test the relationship of the limit order fill rate at different profitability conditions on a single Swedish stock. Like Hedvall, Niemeyer, and Rosenqvist (1997) and Ranaldo (2004), Hollifield, Miller, and Sandas (2004) find asymmetries in investor behavior on the two sides of the market of the Finnish Stock Exchange. Foucault, Kadan, and Kandel (2005) and Rosu (2005) make predictions about order flow autocorrelations that support the diagonal autocorrelation effect first documented in Biais, Hillion, and Spatt (1995).

CONCLUSION

Understanding the type and motivation of each market participant can unlock profitable trading strategies. For example, understanding whether a particular market participant possesses information about impending market movement may result in immediate profitability from either engaging the trader if he is uninformed or following his moves if he has superior information.

CHAPTER 12

Event Arbitrage

With news reported instantly and trades placed on a tick-by-tick basis, high-frequency strategies are now ideally positioned to profit from the impact of announcements on markets. These high-frequency strategies, which trade on the market movements surrounding news announcements, are collectively referred to as event arbitrage. This chapter investigates the mechanics of event arbitrage in the following order:

- Overview of the development process
- Generating a price forecast through statistical modeling of
  - Directional forecasts
  - Point forecasts
- Applying event arbitrage to corporate announcements, industry news, and macroeconomic news
- Documented effects of events on foreign exchange, equities, fixed income, futures, emerging economies, commodities, and REIT markets

DEVELOPING EVENT ARBITRAGE TRADING STRATEGIES

Event arbitrage refers to the group of trading strategies that place trades on the basis of the markets' reaction to events. The events may be economic or industry-specific occurrences that consistently affect the securities of interest time and time again. For example, unexpected increases in the Fed

Funds rates consistently raise the value of the U.S. dollar, simultaneously raising the rate for USD/CAD and lowering the rate for AUD/USD. The announcements of the U.S. Fed Funds decisions, therefore, are events that can be consistently and profitably arbitraged.

The goal of event arbitrage strategies is to identify portfolios that make positive profit over the time window surrounding each event. The time window is typically a time period beginning just before the event and ending shortly afterwards. For events anticipated ex-ante, such as scheduled economic announcements, the portfolio positions may be opened ahead of the announcement or just after the announcement. The portfolio is then fully liquidated shortly after the announcement.

Trading positions can be held anywhere from a few seconds to several hours and can result in consistently profitable outcomes with low volatilities. The speed of response to an event often determines the trade gain; the faster the response, the higher the probability that the strategy will be able to profitably ride the momentum wave to the post-announcement equilibrium price level. As a result, event arbitrage strategies are well suited for high-frequency applications and are most profitably executed in fully automated trading environments.

Developing an event arbitrage trading strategy harnesses research on equilibrium pricing and leverages statistical tools that assess tick-by-tick trading data and events the instant they are released. Further along in this chapter, we will survey academic research on the impact of events on prices; now we investigate the mechanics of developing an event arbitrage strategy.

Most event arbitrage strategies follow a three-stage development process:

1. For each event type, identify dates and times of past events in historical data.
2. Compute historical price changes at desired frequencies pertaining to securities of interest and surrounding the events identified in Step 1.
3. Estimate expected price responses based on historical price behavior surrounding past events.

The sources of dates and times for specified events that occurred in the past can be collected from various Internet sites. Most announcements recur at the same time of day and make the job of collecting the data much easier. U.S. unemployment announcements, for example, are always released at 8:30 A.M. Eastern time. Some announcements, such as those of the U.S. Federal Open Markets Committee interest rate changes, occur at irregular times during the day and require greater diligence in collecting the data.

WHAT CONSTITUTES AN EVENT?

The events used in event arbitrage strategies can be any releases of news about economic activity, market disruptions, and other events that consistently impact market prices. Classic financial theory tells us that in efficient markets, the price adjusts to new information instantaneously following a news release. In practice, market participants form expectations about inflation figures well before the formal statistics are announced. Many financial economists are tasked with forecasting inflation figures based on other continuously observed market variables, such as prices on commodity futures and other market securities. When such forecasts become available, market participants trade securities on the basis of the forecasts, impounding their expectations into prices well before the formal announcements occur.

All events do not have the same magnitude. Some events may have positive and negative impacts on prices, and some events may have more severe consequences than others. The magnitude of an event can be measured as a deviation of the realized event figures from the expectations of the event. The price of a particular stock, for example, should adjust to the net present value of its future cash flows following a higher- or lower-than-expected earnings announcement. However, if earnings are in line with investor expectations, the price should not move. Similarly, in the foreign exchange market, the level of a foreign exchange pair should change in response to an unexpected change—for example, in the level of the consumer price index (CPI) of the domestic country. If, however, the domestic CPI turns out to be in line with market expectations, little change should occur.

The key objective in the estimation of news impact is the determination of what actually constitutes the unexpected change, or news. The earliest macroeconomic event studies, such as those of Frenkel (1981) and Edwards (1982), considered news to be an out-of-sample error based on the one-step-ahead autoregressive forecasts of the macroeconomic variable in question. The thinking went that most economic news develops slowly over time, and the trend observed during the past several months or quarters is the best predictor of the value to be released on the next scheduled news release day. The news, or the unexpected component of the news release, is then the difference between the value released in the announcement and the expectation formed on the basis of autoregressive analysis.

Researchers such as Eichenbaum and Evans (1993) and Grilli and Roubini (1993) have been using the autoregressive framework to predict the decisions of the central bankers, including the U.S. Federal Reserve.

Once again, the main rationale behind the autoregressive predictability of the central bankers' actions is that the central bankers are not at liberty to make drastic changes to economic variables under their control, given that major changes may trigger large-scale market disruptions. Instead, the central bankers adopt and follow a longer-term course of action, gradually adjusting the figures in their control, such as interest rates and money supply, to lead the economy in the intended direction.

The empirical evidence of the impact of news defined in the autoregressive fashion shows that the framework indeed can be used to predict future movements of securities. Yet the impact is best seen in shorter terms—for example, on intra-day data. Almeida, Goodhart, and Payne (1998) documented a significant effect of macroeconomic news announcements on the USD/DEM exchange rate sampled at five-minute intervals. The authors found that news announcements pertaining to the U.S. employment and trade balance were particularly significant predictors of exchange rates, but only within two hours following the announcement. On the other hand, U.S. non-farm payroll and consumer confidence news announcements caused price momentum lasting 12 hours or more following an announcement.

Lately, surprises in macroeconomic announcements have been measured relative to published averages of economists' forecasts. For example, every week *Barron's* and the *Wall Street Journal* publish consensus forecasts for the coming week's announcements. The forecasts are developed from a survey of field economists.

FORECASTING METHODOLOGIES

Directional and point forecasts are the two approaches to estimating the price response to an announcement. A directional forecast predicts whether the price of a particular security will go up or down, whereas a point forecast predicts the level to which the new price will go. The following two sections consider directional and point forecast methodologies in detail. The last section of the chapter discusses event study results that have been documented in the academic literature to date.

Directional Forecasts

Directional forecasts of the post-event price movement of the security price can be created using the sign test. The sign test answers the following question: does the security under consideration consistently move up or down in response to announcements of a certain kind?

The sign test assumes that in the absence of the event, the price change, or the return, is equally likely to be positive or negative. When an event occurs, however, the return can be persistently positive or negative, depending on the event. The sign test aims to estimate whether a persistently positive or negative sign of the response to a specific event exists and whether the response is statistically significant. If the sign test produces a statistically significant result, an event arbitrage trading strategy is feasible.

MacKinlay (1997) specifies the following test hypotheses for the sign test:

- The null hypothesis, $H_0 : p \leq 0.5$, states that the event does *not* cause consistent behavior in the price of interest—that is, the probability p of the price moving consistently in one direction in response to the event is less than or equal to 50 percent.
- The alternative hypothesis, $H_A : p > 0.5$, is that the event *does* cause consistent behavior in the price of the security of interest—in other words, the probability p of the price moving consistently in one direction in response to the event is greater than 50 percent.

We next define N to be the total number of events and let N^+ denote the number of events that were accompanied by positive return of the security under our consideration. The null hypothesis is rejected, and the price of the security is determined to respond consistently to the event with statistical confidence of $(1-\alpha)$ if the asymptotic test statistic $\theta > \Phi^{-1}(\alpha)$, where $\theta = \left[\frac{N^+}{N} - 0.5\right] \frac{\sqrt{N}}{0.5} \sim N(0,1)$.

Example: Trading USD/CAD on U.S. Inflation Announcements

The latest figures tracking U.S. inflation are released monthly at 8:30 A.M. on prespecified dates. On release, USD/CAD spot and other USD crosses undergo an instantaneous one-time adjustment, at least in theory. Identifying when and how quickly the adjustments happen in practice, we can construct profitable trading strategies that capture changes in price levels following announcements of the latest inflation figures.

Even to a casual market observer, the movement of USD/CAD at the time inflation figures are announced suggests that the price adjustment may not be instantaneous and that profitable trading opportunities may exist surrounding U.S. inflation announcements. When the sign test is applied to intra-day USD/CAD spot data, it indeed shows that profitable trading opportunities are plentiful. These opportunities, however, exist only at high frequencies.

The first step in identification of profitable trading opportunities is to define the time period from the announcement to the end of the trading

opportunity, known as the "event window." We select data sample windows surrounding the recent U.S. inflation announcements in the tick-level data from January 2002 through August 2008. As all U.S. inflation announcements occur at 8:30 A.M. EST, we define 8 A.M. to 9 A.M. as the trading window and download all of the quotes and trades recorded during that time. We partition the data further into 5-minute, 1-minute, 30-second, and 15-second blocks. We then measure the impact of the announcement on the corresponding 5-minute, 1-minute, 30-second, and 15-second returns of USD/CAD spot.

According to the purchasing power parity (PPP), a spot exchange rate between domestic and foreign currencies is the ratio of the domestic and foreign inflation rates. When the U.S. inflation rate changes, the deviation disturbs the PPP equilibrium and the USD-based exchange rates adjust to new levels. When the U.S. inflation rate rises, USD/CAD is expected to increase instantaneously, and vice versa. To keep matters simple, in this example we will consider the inflation news in the same fashion as it is announced, ignoring the market's pre-announcement adjustment to expectations of inflation figures.

The sign test then tells us during which time intervals, if any, the market properly and consistently responds to announcements during our "trading window" from 8 to 9 A.M. The sample includes only days when inflation rates were announced. The summary of the results is presented in Table 12.1.

Looking at 5-minute intervals surrounding the U.S. inflation announcements, it appears that USD/CAD reacts persistently only to decreases in the U.S. inflation rate and that reaction is indeed instantaneous. USD/CAD decreases during the 5-minute interval from 8:25 A.M. to 8:30 A.M. in response to announcements of lower inflation with 95 percent statistical confidence. The response may potentially support the instantaneous adjustment hypothesis; after all, the U.S. inflation news is released at 8:30 A.M., at which point the adjustment to drops in inflation appears to be completed. No statistically significant response appears to occur following rises in inflation.

TABLE 12.1 Number of Persistent Trading Opportunities in USD/CAD Following the U.S. Inflation Rate Announcements

Estimation Frequency	U.S. Inflation Up	U.S. Inflation Down
5 minutes	0	0
1 minute	1	0
30 seconds	4	1
15 seconds	5	6

Higher-frequency intervals tell us a different story—the adjustments occur in short-term bursts. At 1-minute intervals, for example, the adjustment to increases in inflation can now be seen to consistently occur from 8:34 to 8:35 A.M. This post-announcement adjustment, therefore, presents a consistent profit-taking opportunity.

Splitting the data into 30-second intervals, we observe that the number of tradable opportunities increases further. For announcements of rising inflation, the price adjustment now occurs in four 30-second post-announcement intervals. For the announcements showing a decrease in inflation, the price adjustment occurs in one 30-second post-announcement time interval.

Examining 15-second intervals, we note an even higher number of time-persistent trading opportunities. For rising inflation announcements, there are five 15-second periods during which USD/CAD consistently increased in response to the inflation announcement between 8:30 and 9:00 A.M., presenting ready tradable opportunities. Six 15-second intervals consistently accompany falling inflation announcements during the same 8:30 to 9:00 A.M. time frame.

In summary, as we look at shorter time intervals, we detect a larger number of statistically significant currency movements in response to the announcements. The short-term nature of the opportunities makes them conducive to a systematic (i.e., black-box) approach, which, if implemented knowledgeably, reduces risk of execution delays, carrying costs, and expensive errors in human judgment.

Point Forecasts

Whereas directional forecasts provide insight about direction of trends, point forecasts estimate the future value of price in equilibrium following an announcement. Development of point forecasts involves performing event studies on very specific trading data surrounding event announcements of interest.

Event studies measure the quantitative impact of announcements on the returns surrounding the news event and are usually conducted as follows:

1. The announcement dates, times, and "surprise" changes are identified and recorded. To create useful simulations, the database of events and the prices of securities traded before and after the event should be very detailed, with events categorized carefully and quotes and trades captured at high frequencies. The surprise component can be measured in following ways:
   - As the difference between the realized value and the prediction based on autoregressive analysis

- As the difference between the realized value and the analyst forecast consensus obtained from Bloomberg or Thomson Reuters.

2. The returns corresponding to the times of interest surrounding the announcements are calculated for the securities under consideration. For example, if the researcher is interested in evaluating the impact of CPI announcements on the 5-minute change in USD/CAD, the 5-minute change in USD/CAD is calculated from 8:30 A.M. to 8:35 A.M. on historical data on past CPI announcement days. (The 8:30 to 8:35 A.M. interval is chosen for the 5-minute effect of CPI announcements, because the U.S. CPI announcements are always released at 8:30 A.M. ET.)
3. The impact of the announcements is then estimated in a simple linear regression:

$$R_t = \alpha + \beta \Delta X_t + \varepsilon_t$$

where R_t is the vector of returns surrounding the announcement for the security of interest arranged in the order of announcements; ΔX_t is the vector of "surprise" changes in the announcements arranged in the order of announcements; ε_t is the idiosyncratic error pertaining to news announcements; α is the estimated intercept of the regression that captures changes in returns due to factors other than announcement surprises; and, finally, β measures the average impact of the announcement on the security under consideration.

Changes in equity prices are adjusted by changes in the overall market prices to account for the impact of broader market influences on equity values. The adjustment is often performed using the market model of Sharpe (1964):

$$R_t^a = R_t - \hat{R}_t \tag{12.1}$$

where $\hat{R}_t$ is the expected equity return estimated over historical data using the market model:

$$R_t = \alpha + \beta R_{m,t} + \varepsilon_t \tag{12.2}$$

The methodology was first developed by Ball and Brown (1968), and the estimation method to this day delivers statistically significant trading opportunities.

Event arbitrage trading strategies may track macroeconomic news announcements, earnings releases, and other recurring changes in the economic information. During a typical trading day, numerous economic announcements are made around the world. The news announcements may

be related to a particular company, industry, or country; or, like macroeconomic news, they may have global consequences. Company news usually includes quarterly and annual earnings releases, mergers and acquisitions announcements, new product launch announcements, and the like. Industry news comprises industry regulation in a particular country, the introduction of tariffs, and economic conditions particular to the industry. Macroeconomic news contains interest rate announcements by major central banks, economic indicators determined from government-collected data, and regional gauges of economic performance.

With the development of information technology such as RSS feeds, alerts, press wires, and news aggregation engines such as Google, it is now feasible to capture announcements the instant they are released. A well-developed automated event arbitrage system captures news, categorizes events, and matches events to securities based on historical analysis.

TRADABLE NEWS

Corporate News

Corporate activity such as earnings announcements, both quarterly and annual, significantly impacts equity prices of the firms releasing the announcements. Unexpectedly positive earnings typically lift equity prices, and unexpectedly negative earnings often depress corporate stock valuation.

Earnings announcements are preceded by analyst forecasts. The announcement that is materially different from the economists' consensus forecast results in a rapid adjustment of the security price to its new equilibrium level. The unexpected component of the announcements is computed as the difference between the announced value and the mean or median economists' forecast. The unexpected component is the key variable used in estimation of the impact of an event on prices.

Theoretically, equities are priced as present values of future cash flows of the company, discounted at the appropriate interest rate determined by Capital Asset Pricing Model (CAPM), the arbitrage pricing theory of Ross (1976), or the investor-specific opportunity cost:

$$\text{equity price} = \sum_{t=1}^{\infty} \frac{E[\text{Earnings}_t]}{(1+R_t)^t} \tag{12.3}$$

where $E[Earnings_t]$ are the expected cash flows of the company at a future time t, and R_t is the discount rate found appropriate for discounting time t dividends to present. Unexpected changes to earnings generate rapid price

responses whereby equity prices quickly adjust to new information about earnings.

Significant deviations of earnings from forecasted values can cause large market movements and can even result in market disruptions. To prevent large-scale impacts of earnings releases on the overall market, most earnings announcements are made after the markets close.

Other firm-level news also affects equity prices. The effect of stock splits, for example, has been documented by Fama, Fisher, Jensen, and Roll (1969), who show that the share prices typically increase following a split relative to their equilibrium price levels.

Event arbitrage models incorporate the observation that earnings announcements affect each company differently. The most widely documented firm-level factors for evaluation include the size of the firm market capitalization (for details, see Atiase, 1985; Freeman, 1987; and Fan-fah, Mohd, and Nasir, 2008).

Industry News

Industry news consists mostly of legal and regulatory decisions along with announcements of new products. These announcements reverberate throughout the entire sector and tend to move all securities in that market in the same direction. Unlike macroeconomic news that is collected and disseminated in a systematic fashion, industry news usually emerges in an erratic fashion.

Empirical evidence on regulatory decisions suggests that decisions relaxing rules governing activity of a particular industry result in higher equity values, whereas the introduction of rules constricting activity pushes equity values down. The evidence includes the findings of Navissi, Bowman, and Emanuel (1999), who ascertained that announcements of relaxation or elimination of price controls resulted in an upswing in equity values and that the introduction of price controls depressed equity prices. Boscaljon (2005) found that the relaxation of advertising rules by the U.S. Food and Drug Administration was accompanied by rising equity values.

Macroeconomic News

Macroeconomic decisions and some observations are made by government agencies on a predetermined schedule. Interest rates, for example, are set by economists at the central banks, such as the U.S. Federal Reserve or the Bank of England. On the other hand, variables such as consumer price indices (CPIs) are typically not set but are observed and reported by statistics agencies affiliated with the countries' central banks.

Other macroeconomic indices are developed by research departments of both for-profit and nonprofit private companies. The ICSC Goldman store sales index, for example, is calculated by the International Council of Shopping Centers (ICSC) and is actively supported and promoted by the Goldman Sachs Group. The index tracks weekly sales at sample retailers and serves as an indicator of consumer confidence: the more confident consumers are about the economy and their future earnings potential, the higher their retail spending and the higher the value of the index. Other indices measure different aspects of economic activity ranging from relative prices of McDonalds' hamburgers in different countries to oil supplies to industry-specific employment levels.

Table 12.2 shows an ex-ante schedule of macroeconomic news announcements for Tuesday, March 3, 2009, a typical trading day. European news is most often released in the morning of the European trading session while North American markets are closed. Most macroeconomic announcements of the U.S. and Canadian governments are distributed in the morning of the North American session that coincides with afternoon trading in Europe. Most announcements from the Asia Pacific region, which includes Australia and New Zealand, are released during the morning trading hours in Asia.

Many announcements are accompanied by "consensus forecasts," which are aggregates of forecasts made by economists of various financial institutions. The consensus figures are usually produced by major media and data companies, such as Bloomberg LP, that poll various economists every week and calculate average industry expectations.

Macroeconomic news arrives from every corner of the world. The impact on currencies, commodities, equities, and fixed-income and derivative instruments is usually estimated using event studies, a technique that measures the persistent impact of news on the prices of securities of interest.

APPLICATION OF EVENT ARBITRAGE

Foreign Exchange Markets

Market responses to macroeconomic announcements in foreign exchange were studied by Almeida, Goodhart, and Payne (1998); Edison (1996); Andersen, Bollerslev, Diebold, and Vega (2003); and Love and Payne (2008), among many others.

Edison (1996) studied macroeconomic news impact on daily changes in the USD-based foreign exchange rates and selected fixed-income securities, and finds that foreign exchange reacts most significantly to news about real economic activity, such as non-farm payroll employment figures.

TABLE 12.2 Ex-Ante Schedule of Macroeconomic Announcements for March 3, 2009

Time (ET)	Event	Prior Value	Consensus Forecast	Country
1:00 A.M.	Norway Consumer Confidence	−13.3		Norway
1:45 A.M.	GDP Q/Q	0.0 percent	−0.8 percent	Switzerland
1:45 A.M.	GDP Y/Y	1.6 percent	−0.1 percent	Switzerland
2:00 A.M.	Wholesale Price Index M/M	−3.0 percent	−2.0 percent	Germany
2:00 A.M.	Wholesale Price Index Y/Y	−3.3 percent	−6.3 percent	Germany
3:00 A.M.	Norway PMI SA	40.8	40.2	Norway
4:30 A.M.	PMI Construction	34.5	34.2	UK
7:45 A.M.	ICSC Goldman Store Sales			U.S.
8:55 A.M.	Redbook			U.S.
9:00 A.M.	Bank of Canada Rate	1.0 percent	0.5 percent	Canada
10:00 A.M.	Pending Home Sales	6.3 percent	−3.0 percent	U.S.
1:00 P.M.	Four-Week Bill Auction			U.S.
2:00 P.M.	Total Car Sales	9.6M	9.6M	U.S.
2:00 P.M.	Domestic Car Sales	6.8M	6.9M	U.S.
5:00 P.M.	ABC/*Washington Post* Consumer Confidence	v48	−47	U.S.
5:30 P.M.	AIG Performance of Service Index	41		Australia
7:00 P.M.	Nationwide Consumer Confidence	40	38	UK
7:30 P.M.	GDP Q/Q	0.1 percent	0.1 percent	Australia
7:30 P.M.	GDP Y/Y	1.9 percent	1.1 percent	Australia
9:00 P.M.	ANZ Commodity Prices	−4.3 percent		New Zealand

"SA" stands for "seasonally adjusted"; "NSA" indicates non-seasonally adjusted data.

In particular, Edison (1996) shows that for every 100,000 surprise increases in non-farm payroll employment, USD appreciates by 0.2 percent on average. At the same time, the author documents little impact of inflation on foreign exchange rates.

Andersen, Bollerslev, Diebold, and Vega (2003) conducted their analysis on foreign exchange quotes interpolated based on timestamps to create exact 5-minute intervals—the procedure outlined in Chapter 9 of

this book. The authors show that average exchange rate levels adjust quickly and efficiently to new levels according to the information releases. Volatility, however, takes longer to taper off after the spike surrounding most news announcements. The authors also document that bad news usually has a more pronounced effect than good news.

Andersen, Bollerslev, Diebold, and Vega (2003) use the consensus forecasts compiled by the International Money Market Services (MMS) as the expected value for estimation of surprise component of news announcements. The authors then model the 5-minute changes in spot foreign exchange rate R_t as follows:

$$R_t = \beta_0 + \sum_{i=1}^{I} \beta_i R_{t-i} + \sum_{k=1}^{K} \sum_{j=0}^{J} \beta_{kj} S_{k,t-j} + \varepsilon_t,\ t = 1, \ldots, T \qquad (12.4)$$

where R_{t-i} is i-period lagged value of the 5-minute spot rate, $S_{k,t-j}$ is the surprise component of the k^{th} fundamental variable lagged j periods, and ε_t is the time-varying volatility that incorporates intra-day seasonalities. Andersen, Bollerslev, Diebold, and Vega (2003) estimate the impact of the following variables:

- GDP (advance, preliminary, and final figures)
- Non-farm payroll
- Retail sales
- Industrial production
- Capacity utilization
- Personal income
- Consumer credit
- Personal consumption expenditures
- New home sales
- Durable goods orders
- Construction spending
- Factory orders
- Business inventories
- Government budget deficit
- Trade balance
- Producer price index
- Consumer price index
- Consumer confidence index
- Institute for Supply Management (ISM) index (formerly, the National Association of Purchasing Managers [NAPM] index)
- Housing starts
- Index of leading indicators
- Target Fed Funds rate
- Initial unemployment claims

- Money supply (M1, M2, M3)
- Employment
- Manufacturing orders
- Manufacturing output
- Trade balance
- Current account
- Producer prices
- Wholesale price index
- Import prices
- Money stock M3

Andersen, Bollerslev, Diebold, and Vega (2003) considered the following currency pairs: GBP/USD, USD/JPY, DEM/USD, CHF/USD, and EUR/USD from January 3, 1992 through December 30, 1998. The authors document that all currency pairs responded positively, with 99 percent significance, to surprise increases in the following variables: non-farm payroll employment, industrial production, durable goods orders, trade balance, consumer confidence index, and NAPM index. All the currency pairs considered responded negatively to surprise increases in the initial unemployment claims and money stock M3.

Love and Payne (2008) document that macroeconomic news from different countries affects different currency pairs. Love and Payne (2008) studied the impact of the macroeconomic news originating in the United States, the Eurozone, and the UK on the EUR/USD, GBP/USD, and EUR/GBP exchange-rate pairs. The authors find that the U.S. news has the largest effect on the EUR/USD, while GBP/USD is most affected by the news originating in the UK. Love and Payne (2008) also document the specific impact of the type of news from the three regions on their respective currencies; their findings are shown in Table 12.3.

TABLE 12.3 Effect of Region-Specific News Announcements on the Respective Currency, per Love and Payne (2008)

	News Announcement Type		
Region of News Origination	Increase in Prices or Money	Increase of Output	Increase in Trade Balance
Eurozone, Effect on EUR	Appreciation	Appreciation	
UK, Effect on GBP	Appreciation	Appreciation	Appreciation
U.S., Effect on USD	Depreciation	Appreciation	Appreciation

Equity Markets

A typical trading day is filled with macroeconomic announcements, both domestic and foreign. How does the macroeconomic news impact equity markets?

According to classical financial theory, changes in equity prices are due to two factors: changes in expected earnings of publicly traded firms, and changes in the discount rates associated with those firms. Expected earnings may be affected by changes in market conditions. For example, increasing consumer confidence and consumer spending are likely to boost retail sales, uplifting earnings prospects for retail outfits. Rising labor costs, on the other hand, may signal tough business conditions and decrease earnings expectations as a result.

The discount rate in classical finance is, at its bare minimum, determined by the level of the risk-free rate and the idiosyncratic riskiness of a particular equity share. The risk-free rate pertinent to U.S. equities is often proxied by the 3-month bill issued by the U.S. Treasury; the risk-free rate significant to equities in another country is taken as the short-term target interest rate announced by that country's central bank. The lower the risk-free rate, the lower the discount rate of equity earnings and the higher the theoretical prices of equities.

How does macroeconomic news affect equities in practice? Ample empirical evidence shows that equity prices respond strongly to interest rate announcements and, in a less pronounced manner, to other macroeconomic news. Decreases in both long-term and short-term interest rates indeed positively affect monthly stock returns with 90 percent statistical confidence for long-term rates and 99 percent confidence for short-term rates. Cutler, Poterba, and Summers (1989) analyzed monthly NYSE stock returns and found that, specifically, for every 1 percent decrease in the yield on 3-month Treasury bills, monthly equity returns on the NYSE increased by 1.23 percent on average in the 1946–1985 sample.

Stock reaction to nonmonetary macroeconomic news is usually mixed. Positive inflation shocks tend to induce lower stock returns independent of other market conditions (see Pearce and Roley, 1983, 1985 for details). Several other macroeconomic variables produce reactions conditional on the contemporary state of the business cycle. Higher-than-expected industrial production figures are good news for the stock market during recessions but bad news during periods of high economic activity, according to McQueen and Roley (1993).

Similarly, unexpected changes in unemployment statistics were found to cause reactions dependent on the state of the economy. For example, Orphanides (1992) finds that returns increase when unemployment rises, but only during economic expansions. During economic contractions, returns drop following news of rising unemployment. Orphanides (1992)

attributes the asymmetric response of equities to the overheating hypothesis: when the economy is overheated, increase in unemployment actually presents good news. The findings have been confirmed by Boyd, Hu, and Jagannathan (2005). The asymmetric response to macroeconomic news is not limited to the U.S. markets. Löflund and Nummelin (1997), for instance, observe the asymmetric response to surprises in industrial production figures in the Finnish equity market; they found that higher-than-expected production growth bolsters stocks in sluggish states of the economy.

Whether or not macroeconomic announcements move stock prices, the announcements are usually surrounded by increases in market volatility. While Schwert (1989) pointed out that stock market volatility is not necessarily related to volatility of other macroeconomic factors, surprises in macroeconomic news have been shown to significantly increase market volatility. Bernanke and Kuttner (2005), for example, show that an unexpected component in the interest rate announcements of the U.S. Federal Open Market Committee (FOMC) increase equity return volatility. Connolly and Stivers (2005) document spikes in the volatility of equities constituting the Dow Jones Industrial Average (DJIA) in response to U.S. macroeconomic news. Higher volatility implies higher risk, and financial theory tells us that higher risk should be accompanied by higher returns. Indeed, Savor and Wilson (2008) show that equity returns on days with major U.S. macroeconomic news announcements are higher than on days when no major announcements are made. Savor and Wilson (2008) consider news announcements to be "major" if they are announcements of Consumer Price Index (CPI), Producer Price Index (PPI), employment figures, or interest rate decisions of the FOMC. Veronesi (1999) shows that investors are more sensitive to macroeconomic news during periods of higher uncertainty, which drives asset price volatility. In the European markets, Errunza and Hogan (1998) found that monetary and real macroeconomic news has considerable impact on the volatility of the largest European stock markets.

Different sources of information appear to affect equities at different frequencies. The macroeconomic impact on equity data appears to increase with the increase in frequency of equity data. Chan, Karceski, and Lakonishok (1998), for example, analyzed monthly returns for U.S. and Japanese equities in an arbitrage pricing theory setting and found that idiosyncratic characteristics of individual equities are most predictive of future returns at low frequencies. By using factor-mimicking portfolios, Chan, Karceski, and Lakonishok (1998) show that size, past return, book-to-market ratio, and dividend yield of individual equities are the factors that move in tandem ("covary") most with returns of corresponding equities. However, Chan, Karceski, and Lakonishok (1998, p. 182) document that

"the macroeconomic factors do a poor job in explaining return covariation" at monthly return frequencies. Wasserfallen (1989) finds no impact of macroeconomic news on quarterly equities data.

Flannery and Protopapadakis (2002) found that daily returns on the U.S. equities are significantly impacted by several types of macroeconomic news. The authors estimate a GARCH return model with independent variables and found that the following macroeconomic announcements have significant influence on both equity returns and volatility: consumer price index (CPI), producer price index (PPI), monetary aggregate, balance of trade, employment report, and housing starts figures.

Ajayi and Mehdian (1995) document that foreign stock markets in developed countries typically overreact to the macroeconomic news announcements from the United States. As a result, foreign equity markets tend to be sensitive to the USD-based exchange rates and domestic account balances. Sadeghi (1992), for example, notes that in the Australian markets, equity returns increased in response to increases in the current account deficit, the AUD/USD exchange rate, and the real GDP; equity returns decreased following news of rising domestic inflation or interest rates.

Stocks of companies from different industries have been shown to react differently to macroeconomic announcements. Hardouvelis (1987), for example, pointed out that stocks of financial institutions exhibited higher sensitivity to announcements of monetary adjustments. The extent of market capitalization appears to matter as well. Li and Hu (1998) show that stocks with large market capitalization are more sensitive to macroeconomic surprises than are small-cap stocks.

The size of the surprise component of the macroeconomic news impacts equity prices. Aggarwal and Schirm (1992), for example, document that small surprises, those within one standard deviation of the average, caused larger changes in equities and foreign exchange markets than did large surprises.

Fixed-Income Markets

Jones, Lamont, and Lumsdaine (1998) studied the effect of employment and producer price index data on U.S. Treasury bonds. The authors find that while the volatility of the bond prices increased markedly on the days of the announcements, the volatility did not persist beyond the announcement day, indicating that the announcement information is incorporated promptly into prices.

Hardouvelis (1987) and Edison (1996) note that employment figures, producer price index (PPI), and consumer price index (CPI) move bond prices. Krueger (1996) documents that a decline in U.S. unemployment causes higher yields in bills and bonds issued by the U.S. Treasury.

High-frequency studies of the bond market responses to macroeconomic announcements include those by Ederington and Lee (1993); Fleming and Remolona (1997, 1999); and Balduzzi, Elton and Green (2001). Ederington and Lee (1993) and Fleming and Remolona (1999) show that new information is fully incorporated in bond prices just two minutes following its announcement. Fleming and Remolona (1999) estimate the high-frequency impact of macroeconomic announcements on the entire U.S. Treasury yield curve. Fleming and Remolona (1999) measure the impact of 10 distinct announcement classes: consumer price index (CPI), durable goods orders, gross domestic product (GDP), housing starts, jobless rate, leading indicators, non-farm payrolls, producer price index (PPI), retail sales, and trade balance. Fleming and Remolona (1999) define the macroeconomic surprise to be the actual number released less the Thomson Reuters consensus forecast for the same news release.

All of the 10 macroeconomic news announcements studied by Fleming and Remolona (1999) were released at 8:30 A.M. The authors then measure the significance of the impact of the news releases on the entire yield curve from 8:30 A.M. to 8:35 A.M., and document statistically significant average changes in yields in response to a 1 percent positive surprise change in the macro variable. The results are reproduced in Table 12.4. As Table 12.4 shows, a 1 percent "surprise" increase in the jobless rate led on average to a 0.9 percent drop in the yield of the 3-month bill with 95 percent

TABLE 12.4 Effects of Macroeconomic News Announcements Documented by Fleming and Remolona (1999)

Announcement	3-Month Bill	2-Year Note	30-Year Bond
CPI	0.593*	1.472†	1.296†
Durable Goods Orders	1.275†	2.180†	1.170†
GDP	0.277	0.379	0.167
Housing Starts	0.670†	1.406†	0.731†
Jobless Rate	−0.939*	−1.318†	−0.158
Leading Indicators	0.411†	0.525*	0.271*
Non-Farm Payrolls	3.831†	6.124†	2.679*
PPI	0.768†	1.879†	1.738
Retail Sales	0.582*	1.428†	0.766†
Trade Balance	−0.138	0.027	−0.062

The table shows the average change in percent in the yields of the 3-month U.S. Treasury bill, the 2-year U.S. Treasury note, and the 30-year U.S. Treasury bond, corresponding to a 1 percent "surprise" in each macroeconomic announcement. * and † indicate statistical significance at the 95 percent and 99 percent confidence levels, respectively. The estimates were conducted on data from July 1,1991 to September 29,1995.

statistical confidence and a 1.3 percent drop in the yield of the 2-year note with 99 percent confidence. The corresponding average drop in the yield of the 30-year bond was not statistically significant.

Futures Markets

The impact of the macroeconomic announcements on the futures market has been studied by Becker, Finnerty, and Kopecky (1996); Ederington and Lee (1993); and Simpson and Ramchander (2004). Becker, Finnerty, and Kopecky (1996) and Simpson and Ramchander (2004) document that news announcements regarding the PPI, merchandise trade, non-farm payrolls, and the CPI move prices of bond futures. Ederington and Lee (1993) find that news-induced price adjustment of interest rate and foreign exchange futures happens within the first minute after the news is released. News-related volatility, however, may often persist for the following 15 minutes.

Emerging Economies

Several authors have considered the impact of macroeconomic news on emerging economies. For example, Andritzky, Bannister, and Tamirisa (2007) study how macroeconomic announcements affect bond spreads. The authors found that the U.S. news had a major impact, whereas domestic announcements did not generate much effect. On the other hand, Nikkinen, Omran, Sahlström, and Äijö (2006) conducted similar analysis on equity markets and found that while mature equity markets respond almost instantaneously to U.S. macroeconomic announcements, emerging equity markets are not affected. Kandir (2008) estimated macroeconomic impact on monthly returns of equities trading on the Istambul Stock Exchange, and found that the Turkish Lira/USD exchange rate, the Turkish interest rate, and the world market returns significantly affect Turkish equities, while domestic variables such as industrial production and money supply had little effect. Muradoglu, Taskin, and Bigan (2000) found that emerging markets were influenced by global macroeconomic variables, depending on the size of the emerging market under consideration and the degree of the market's integration with the world economy.

ASEAN countries, however, appear to be influenced predominantly by their domestic variables. Wongbangpo and Sharma (2002) find that local GNPs, CPIs, money supplies, interest rates, and the USD-based exchange rates of ASEAN countries (Indonesia, Malaysia, Philippines, Singapore, and Thailand) significantly influence local stock markets. At the same time, Bailey (1990) found no causal relation between the U.S. money supply and stock returns of Asian Pacific markets.

Commodity Markets

Empirical evidence in the commodity markets includes the findings of Gorton and Rouwenhorst (2006), who document that both real activity and inflation affect commodity prices. The effect of the news announcements, however, can be mixed; higher-than-expected real activity and inflation generally have a positive effect on commodity prices, except when accompanied by rising interest rates, which have a cooling impact on commodity valuations. See Bond (1984), Chambers (1985), and Frankel (2006) for more details on the relation between commodity prices and interest rates.

Real Estate Investment Trusts (REITS)

Equity real estate investment trusts (REITs) are fairly novel publicly traded securities, established by the U.S. Congress in 1960. The market capitalization of all U.S.-based REITs was about $9 million in 1991 and steadily grew to $300 billion by 2006. A REIT is traded like an ordinary equity, but it is required to have the following peculiar structure: at least 75 percent of the REIT's assets should be invested in real estate, and the REIT must pay out at least 90 percent of its taxable earnings as dividends. Because of their high payout ratios, REITs may respond differently to macroeconomic news announcements than would ordinary equities.

The impact of inflation on REIT performance has been documented by Simpson, Ramchander, and Webb (2007). The authors found that the returns on REITs increase when inflation unexpectedly falls as well as when inflation unexpectedly rises. Bredin, O'Reilly, and Stevenson (2007) examine the response of REIT returns to unanticipated changes in U.S. monetary policy. The authors find that the response of REITs is comparable to that of equities—increase in the Federal Funds rates increases the volatility of REIT prices while depressing the REIT prices themselves.

CONCLUSION

Event arbitrage strategies utilize high-frequency trading since price equilibrium is reached only after market participants have reacted to the news. Short trading windows and estimation of the impact of historical announcements enable profitable trading decisions surrounding market announcements.

CHAPTER 13

Statistical Arbitrage in High-Frequency Settings

Statistical arbitrage (stat-arb) exploded on the trading scene in the late 1990s, with PhDs in physics and other "hard" sciences reaping double-digit returns using simple statistical phenomena. Since then, statistical arbitrage has been both hailed and derided. The advanced returns generated before 2007 by many stat-arb shops popularized the technique. Yet some blame stat-arb traders for destabilizing the markets in the 2007 and 2008 crises. Stat-arb can lead to a boon in competent hands and a bust in semi-proficient applications.

The technique is a modern cousin of a technical analysis strategy utilizing "Bollinger Bands" that was used to indicate maximum highs and lows at any given point in time by plotting a two-standard deviation envelope around the simple moving average of the price. Despite the recent explosive popularity of stat-arb strategies, many misconceptions about the technique are prevalent. This chapter examines the stat-arb technique in detail.

At its core, stat-arb rests squarely on data mining. To begin with, stat-arb analysts sift through volumes of historical data with the objective of identifying a pervasive statistical relationship. Such a relationship may be between the current price level of the security and the price level of the same security in the recent past. The relationship may also be between price levels of two different securities, or the price level of one security and the volatility of another. The critical point in the identification process is that the relationship has to hold with at least 90 percent statistical confidence, 90 percent being the lowest acceptable confidence threshold in most statistical analyses.

Once a statistically significant relationship is detected, a stat-arb trading model is built around the following assumption: if at any point in time the statistical relationship is violated, the relationship will mean-revert to its natural historical level and the trade should be placed in the mean-reverting direction. The tendency to mean-revert is assumed to increase whenever the relationship is violated to a large extent.

The degree of violation of the historical relationship can be measured by the number of standard deviations the relationship has moved away from the historical mean of values characterizing the relationship. For example, if the variable of interest is price and the price level of USD/CAD rises by two or more standard deviations above its average historical price difference with the level of USD/CHF in a short period of time, the stat-arb strategy assumes that the unusually large move of USD/CAD is likely to reverse in the near future, and the trading strategy enters into a short position in USD/CAD. If the mean-reversion indeed materializes, the strategy captures a gain. Otherwise, a stop loss is triggered, and the strategy books a loss.

MATHEMATICAL FOUNDATIONS

Mathematically, the steps involved in the development of stat-arb trading signals are based on a relationship between price levels or other variables characterizing any two securities. A relationship based on price levels $S_{i,t}$ and $S_{j,t}$ for any two securities i and j can be can be arrived at through the following procedure:

1. Identify the universe of liquid securities—that is, securities that trade at least once within the desired trading frequency unit. For example, for hourly trading frequency choose securities that trade at least once every hour.
2. Measure the difference between prices of every two securities, i and j, identified in step (1) across time t:

$$\Delta S_{ij,t} = S_{i,t} - S_{j,t},\ t \in [1, T] \tag{13.1}$$

where T is a sufficiently large number of daily observations. According to the central limit theorem (CLT) of statistics, 30 observations at selected trading frequency constitute the bare minimum. The intra-day data, however, has high seasonality—that is, persistent relationships can be observed at specific hours of the day. Thus, a larger T of at least 30 daily observations is strongly recommended. For robust inferences, a T of 500 daily observations (two years) is desirable.

3. For each pair of securities, select the ones with the most stable relationship—security pairs that move together. To do this, Gatev, Goetzmann, and Rouwenhorst (2006) perform a simple minimization of the historical differences in returns between every two liquid securities:

$$\min_{i,j} \sum_{t=1}^{T} (\Delta S_{ij,t})^2 \tag{13.2}$$

   The stability of the relationship can also be assessed using cointegration and other statistical techniques.

   Next, for each security i, select the security j with the minimum sum of squares obtained in equation (13.2).

4. Estimate basic distributional properties of the difference as follows.

   Mean or average of the difference:

$$E[\Delta S_t] = \frac{1}{T} \sum_{t=1}^{T} \Delta S_t$$

   Standard deviation:

$$\sigma[\Delta S_t] = \frac{1}{T-1} \sum_{t=1}^{T} (\Delta S_t - E[\Delta S_t])^2$$

5. Monitor and act upon differences in security prices:

   At a particular time τ, if

$$\Delta S_\tau = S_{i,\tau} - S_{j,\tau} > E[\Delta S_\tau] + 2\sigma[\Delta S_\tau]$$

   sell security i and buy security j. On the other hand, if

$$\Delta S_\tau = S_{i,\tau} - S_{j,\tau} < E[\Delta S_\tau] - 2\sigma[\Delta S_\tau]$$

   buy security i and sell security j.

6. Once the gap in security prices reverses to achieve a desirable gain, close out the positions. If the prices move against the predicted direction, activate stop loss.

Instead of detecting statistical anomalies in price levels, statistical arbitrage can be applied to other variables, such as correlation between two securities and traditional fundamental relationships. The details of implementation of statistical arbitrage based on fundamental factors are discussed in detail in the following text.

Stat-arb strategies can be trained to dynamically adjust to changing market conditions. The mean of the variable under consideration, to which the identified statistical relationships are assumed to tend, can be computed as a moving weighted average with the latest observations being

given more weight than the earliest observations in the computation window. Similarly, the standard deviation used in computations can be computed using a limited number of the most recent observations, reflecting the latest economic environment.

The shortcomings of statistical arbitrage strategies are easy to see; often enough, detected statistical relationships are random or "spurious" and have little predictive staying power. Yet other statistical relationships, those validated by academic research in economics and finance, have consistently produced positive results for many traders. Thorough understanding of economic theory helps quantitative analysts distinguish between solid and arbitrary relationships and, in turn, improves the profitability of trading operations that use stat-arb methodology.

In addition to the issues embedded in the estimation of statistical relationships, statistical arbitrage strategies are influenced by numerous adverse market conditions.

- The strategies face a positive probability of bankruptcy of the parties issuing one or both of the selected financial instruments. Tough market conditions, an unexpected change in regulation, or terrorist events can destroy credible public companies overnight.
- Transaction costs may wipe out all the profitability of stat-arb trading, particularly for investors deploying high leverage or limited capital.
- The bid-ask spread may be wide enough to cancel any gains obtained from the strategy.
- Finally, the pair's performance may be determined by the sizes of the chosen stocks along with other market frictions—for example, price jumps in response to earnings announcements.

Careful measurement and management of risks, however, can deliver high stat-arb profitability. Gatev, Goetzmann, and Rouwenhorst (2006) document that the out-of-sample back tests conducted on the daily equity data from 1967 to 1997 using their stat-arb strategy delivered Sharpe ratios well in excess of 4. High-frequency stat-arb delivers even higher performance numbers.

PRACTICAL APPLICATIONS OF STATISTICAL ARBITRAGE

General Considerations

Most common statistical arbitrage strategies relying solely on statistical relationships with no economic background produce fair results, but these

relationships often prove to be random or spurious. A classic example of a spurious relationship is the relationship between time as a continuous variable and the return of a particular stock; all publicly listed firms are expected to grow with time, and while the relationship produces a highly significant statistical dependency, it can hardly be used to make meaningful predictions about future values of equities. Another extreme example of a potentially spurious statistical relationship is shown by Challe (2003), who documents statistical significance between the occurrence of sunspots and the predictability of asset returns.

High-frequency statistical arbitrage based on economic models has ex-ante longer staying power, because it is based on solid economic principles. The stat-arb strategies arbitraging deviations in economic equations can be called fundamental arbitrage models in that they exploit deviations from fundamental economic principles.

The prices of the pair of securities traded often will be related in some fashion or other, but they can nevertheless span a variety of asset classes and individual names. In equities, the companies issuing securities may belong to the same industry and will therefore respond similarly to changes in the broad market. Alternatively, the securities may actually be issued by the same company. Companies often issue more than one class of shares, and the shares typically differ by voting rights. Even shares of the same class issued by the same company but trading on different exchanges may have profitable intra-day deviations in prices. In foreign exchange, the pair of securities chosen can be a foreign exchange rate and a derivative (e.g., a futures contract) on the same foreign exchange rate. The same underlying derivative trading strategy may well apply to equities and fixed-income securities. Passive indexes, such as infrequently rebalanced ETFs, can be part of profitable trades when the index and its constituents exhibit temporary price deviations from equilibrium. In options, the pair of securities may be two options on the same underlying asset but with different times to expiration.

This section discusses numerous examples of statistical arbitrage applied to various securities. Table 13.1 itemizes the strategies discussed subsequently. The selected strategies are intended to illustrate the ideas of fundamental arbitrage. The list is by no means exhaustive, and many additional fundamental arbitrage opportunities can be found.

Foreign Exchange

Foreign exchange has a number of classic models that have been shown to work in the short term. This section summarizes statistical arbitrage applied to triangular arbitrage and uncovered interest rate parity models. Other fundamental foreign exchange models, such as the flexible price

TABLE 13.1 Summary of Fundamental Arbitrage Strategies by Asset Class Presented in This Section

Asset Class	Fundamental Arbitrage Strategy
Foreign Exchange	Triangular Arbitrage
Foreign Exchange	Uncovered Interest Parity (UIP) Arbitrage
Equities	Different Equity Classes of the Same Issuer
Equities	Market Neutral Arbitrage
Equities	Liquidity Arbitrage
Equities	Large-to-Small Information Spillovers
Futures and the Underlying Asset	Basis Trading
Indexes and ETFs	Index Composition Arbitrage
Options	Volatility Curve Arbitrage

monetary model, the sticky price monetary model, and the portfolio model can be used to generate consistently profitable trades in the statistical arbitrage framework.

Triangular Arbitrage Triangular arbitrage exploits temporary deviations from fair prices in three foreign exchange crosses. The following example illustrates triangular arbitrage of EUR/CAD, following a triangular arbitrage example described by Dacorogna et al. (2001). The strategy arbitrages mispricings between the market prices on EUR/CAD and "synthetic" prices on EUR/CAD that are computed as follows:

$$\text{EUR/CAD}_{\text{Synthetic,bid}} = \text{EUR/USD}_{\text{Market,bid}} \times \text{USD/CAD}_{\text{Market,bid}} \tag{13.3}$$

$$\text{EUR/CAD}_{\text{Synthetic,ask}} = \text{EUR/USD}_{\text{Market,ask}} \times \text{USD/CAD}_{\text{Market,ask}} \tag{13.4}$$

If market ask for EUR/CAD is lower than synthetic bid for EUR/CAD, the strategy is to buy market EUR/CAD, sell synthetic EUR/CAD, and wait for the market and synthetic prices to align, then reverse the position, capturing the profit. The difference between the market ask and the synthetic bid should be high enough to at least overcome two spreads—on EUR/USD and on USD/CAD. The USD-rate prices used to compute the synthetic rate should be sampled simultaneously. Even a delay as small as one second in price measurement can significantly distort the relationship as a result of unobserved trades that affect the prices in the background; by the time the dealer receives the order, the prices may have adjusted to their no-arbitrage equilibrium levels.

Uncovered Interest Parity Arbitrage The uncovered interest parity (UIP) is just one such relation. Chaboud and Wright (2005) find that the UIP best predicts changes in foreign exchange rates at high frequencies and daily rates when the computation is run between 4 P.M. ET and 9 P.M. ET. The UIP is specified as follows:

$$1 + i_t = (1 + i_t^*)\frac{E_t\left[S_{t+1}\right]}{S_t} \tag{13.5}$$

where i_t is the one-period interest rate on the domestic currency deposits, i_t^* is the one-period interest rate on deposits denominated in a foreign currency, and S_t is the spot foreign exchange price of one unit of foreign currency in units of domestic currency. Thus, for example, if domestic means United States–based and foreign means Swiss, the UIP equation, equation (13.5), can be used to calculate the equilibrium CHF/USD rate as follows:

$$1 + i_{t,USD} = (1 + i_{t,CHF}^*)\frac{E_t\left[S_{t+1,CHF/USD}\right]}{S_{t,CHF/USD}} \tag{13.6}$$

The expression can be conveniently transformed to the following regression form suitable for linear estimation:

$$\begin{aligned}\ln(S_{t+1,CHF/USD}) - \ln(S_{t,CHF/USD}) = \alpha + \beta(\ln(1 + i_{t,USD}) \\ - \ln(1 + i_{t,CHF}^*)) + \varepsilon_{t+1}\end{aligned} \tag{13.7}$$

A statistical arbitrage of this relationship would look into the statistical deviations of the two sides of equation (13.7) and make trading decisions accordingly.

Equities

Examples of successful statistical arbitrage strategies involving fundamental equities models abound. This section reviews the following popular trading pair trading strategies: different equity classes of the same issuer, market-neutral pairs trading, liquidity arbitrage, and large-to-small information spillovers.

Arbitraging Different Equity Classes of the Same Issuer It is reasonable to expect stocks corresponding to two common equity classes issued by the same company to be trading within a relatively constant price range from each other. Different classes of common equity issued by the same company typically diverge in the following two characteristics only: voting rights and number of shares outstanding.

Shares with superior voting rights are usually worth more than the shares with inferior voting rights or non-voting shares, given that shares with wider voting privileges allow the shareholders to exercise a degree of control over the direction of the company—see Horner (1988) and Smith and Amoako-Adu (1995), for example. Nenova (2003) shows that the stock price premium for voting privileges exists in most countries. The premium varies substantially from country to country and depends on the legal environment, the degree of investor protection, and takeover regulations, among other factors. In countries with the greatest transparency, such as Finland, the voting premium is worth close to 0, whereas in South Korea, the voting premium can be worth close to 50 percent of the voting stock's market value.

Stocks with a higher number of shares outstanding are usually more liquid, prompting actively trading investors to value them more highly; see Amihud and Mendelson (1986, 1989); Amihud (2002); Brennan and Subrahmanyam (1996); Brennan, Chordia and Subrahmanyam (1998); and Eleswarapu (1997). At the same time, the more liquid class of shares is likely to incorporate market information significantly faster than the less liquid share class, creating the potential for information arbitrage.

A typical trade may work as follows: if the price range widens to more than two standard deviations of the average daily range without a sufficiently good reason, it may be a fair bet that the range will narrow within the following few hours.

The dual-class share strategy suffers from two main shortcomings and may not work for funds with substantial assets under management (AUM).

1. The number of public companies that have dual share classes trading in the open markets is severely limited, restricting the applicability of the strategy. In January 2009, for example, Yahoo! Finance carried historical data for two equity classes for just eight companies trading on the NYSE: Blockbuster, Inc.; Chipotle; Forest City Entertainment; Greif, Inc.; John Wiley & Sons; K V Pharma; Lennar Corp.; and Moog, Inc.
2. The daily volume for the less liquid share class is often small, further restricting the capacity of the strategy. Table 13.2 shows the closing price and daily volume for dual-class shares registered on the NYSE on January 6, 2009. For all names, Class B daily volume on January 6, 2009 does not reach even one million in shares and is too small to sustain a trading strategy of any reasonable trading size.

Market-Neutral Arbitrage Market arbitrage refers to a class of trading models that are based on classical equilibrium finance literature. At core, most market arbitrage models are built on the capital asset pricing

TABLE 13.2 Closing Price and Daily Volume of Dual-Class Shares on NYSE on January 6, 2009

Company Name	Ticker Class A	Class A Close	Class A Volume (MM Shares)	Ticker Class B	Class B Close	Class B Volume (MM Shares)
Blockbuster, Inc.	BBI	1.59	2.947	BBI-B	0.88	0.423
Chipotle	CMG	60.38	0.659	CMG-B	55.87	0.156
Forest City Entertainment	FCE-A	8.49	1.573	FCE-B	8.41	0.008
Greif, Inc.	GEF	35.42	0.378	GEF-B	35.15	0.016
John Wiley & Sons	JW-A	36.82	0.237	JW-B	36.63	0.005
K V Pharma	KV-A	3.68	0.973	KV-B	3.78	0.007
Lennar Corp.	LEN	11.17	8.743	LEN-B	8.5	0.074
Moog, Inc.	MOG-A	37.52	0.242	MOG-B	37.9	0.000

model (CAPM) developed by Sharpe (1964), Lintner (1965), and Black (1972).

The CAPM is based on the idea that returns on all securities are influenced by the broad market returns. The degree of the co-movement that a particular security may experience with the market is different for each individual security and can vary through time. For example, stocks of luxury companies have been shown to produce positive returns whenever the broad market produces positive returns as well, whereas breweries and movie companies tend to produce higher positive returns whenever the overall market returns are downward sloping.

The CAPM equation is specified as follows:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{M,t} - r_{f,t}) + \varepsilon_t \tag{13.8}$$

where $r_{i,t}$ is the return on security i at time t, $r_{M,t}$ is the return on a broad market index achieved in time period t, and $r_{f,t}$ is the risk-free interest rate, such as Fed Funds rate, valid in time period t. The equation can be estimated using Ordinary Least Squares (OLS) regression. The resulting parameter estimates, $\hat{\alpha}$ and $\hat{\beta}$, measure the abnormal return that is intrinsic to the security ($\hat{\alpha}$) and the security's co-movement with the market ($\hat{\beta}$).

The simplest example of CAPM-based pair arbitrage in equities is trading pairs with the same response to the changes in the broader market conditions, or beta, but different intrinsic returns, or alpha. This type of strategy is often referred to as a market-neutral strategy, with the idea that going long and short, respectively, in two securities with similar beta would neutralize the resulting portfolio from broad market exposure.

Often, the two securities used belong to the same or a similar industry, although this is not mandatory. The alpha and beta for two securities i and j are determined from the CAPM equation (13.8). Once the point estimates for alphas and betas of the two securities are produced, along with standard deviations of those point estimates, the statistical significance of difference in alphas and betas is then determined using the difference in the means test, described here for betas only:

$$\Delta\hat{\beta} = \hat{\beta}_i - \hat{\beta}_j \tag{13.9}$$

$$\hat{\sigma}_{\Delta\beta} = \sqrt{\frac{\sigma_{\beta i}^2}{n_i} + \frac{\sigma_{\beta j}^2}{n_j}} \tag{13.10}$$

where n_i and n_j are the numbers of observations used in the estimation of equation (13.8) for security i and security j, respectively.

The standard t-ratio statistic is then determined as follows:

$$\text{Student}_\beta = \frac{\Delta\hat{\beta}}{\hat{\sigma}_{\Delta\beta}} \tag{13.11}$$

The difference test for alphas follows the same procedure as the one outlined for betas in equations (13.9)–(13.11).

As with other t-test estimations, betas can be deemed to be statistically similar if the t statistic falls within one standard deviation interval:

$$t_\beta \in [\Delta\hat{\beta} - \hat{\sigma}_{\Delta\beta}, \Delta\hat{\beta} + \hat{\sigma}_{\Delta\beta}] \tag{13.12}$$

At the same time, the difference in alphas has to be both economically and statistically significant. The difference in alphas has to exceed trading costs, TC, and the t-ratio has to indicate a solid statistical significance, with 95 percent typically considered the minimum:

$$\Delta\hat{\alpha} > TC \tag{13.13}$$

$$|t_\alpha| > [\Delta\hat{\alpha} + 2\hat{\sigma}_{\Delta\alpha}] \tag{13.14}$$

Once a pair of securities satisfying equations (13.12)–(13.14) is identified, the trader goes long in the security with the higher alpha and shorts the security with the lower alpha. The position is held for the predetermined horizon used in the forecast.

Variations on the basic market-neutral pair trading strategy include strategies accounting for other security-specific factors, such as equity

fundamentals. For example, Fama and French (1993) show that the following three-factor model can be successfully used in equity pair trading:

$$r_{i,t} = \alpha_i + \beta_i^{MKT} MKT_t + \beta_i^{SMB} SMB_t + \beta_i^{HML} HML_t + \varepsilon_t \tag{13.15}$$

where $r_{i,t}$ is the return on stock i at time t, MKT_t is the time-t return on a broad market index, SMB_t (small minus big) is the time-t difference in returns between market indices or portfolios of small and big capitalization stocks, and HML_t (high minus low) is the return on a portfolio constructed by going long in stocks with comparatively high book-to-market ratios and going short in stocks with comparatively low book-to-market ratios.

Liquidity Arbitrage In classical asset pricing literature, a financial security that offers some inconvenience to the prospective investors should offer higher returns to compensate investors for the inconvenience. Limited liquidity is one such inconvenience; lower liquidity levels make it more difficult for individual investors to unwind their positions, potentially leading to costly outcomes. On the flipside, if liquidity is indeed priced in asset returns, then periods of limited liquidity may offer nimble investors highly profitable trading opportunities.

In fact, several studies have documented that less liquid stocks have higher average returns: see Amihud and Mendelson (1986); Brennan and Subrahmanyam (1996); Brennan, Chordia, and Subrahmanyam (1998); and Datar, Naik, and Radcliffe (1998). Trading the illiquid stocks based exclusively on the information that they are illiquid, however, delivers no positive abnormal returns. The relatively high average returns simply compensate prospective investors for the risks involved in holding these less liquid securities.

Pástor and Stambaugh (2003), however, recognize that at least a portion of the observed illiquidity of financial securities may be attributed to market-wide causes. If the market-wide liquidity is priced into individual asset returns, then market illiquidity arbitrage strategies may well deliver consistent positive abnormal returns on the risk-adjusted basis.

Pástor and Stambaugh (2003) find that in equities, stocks whose returns have higher exposure to variability in the market-wide liquidity indeed deliver higher returns than stocks that are insulated from the market-wide liquidity. To measure sensitivity of stock i to market liquidity, Pástor and Stambaugh (2003) devise a metric γ that is estimated in the following OLS specification:

$$r_{i,t+1}^e = \theta + \beta r_{i,t} + \gamma \; sign(r_{i,t}^e) \cdot v_{i,t} + \tau_{t+1} \tag{13.16}$$

where $r_{i,t}$ is the return on stock i at time t, $v_{i,t}$ is the dollar volume for stock i at time t, and $r_{i,t}^e$ is the return on stock i at time t in excess of the

market return at time t: $r^e_{i,t} = r_{i,t} - r_{m,t}$. The sign of the excess return $r^e_{i,t}$ proxies for the direction of the order flow at time t; when stock returns are positive, it is reasonable to assume that the number of buy orders in the market outweighs the number of sell orders, and vice versa. The prior time-period return $r_{i,t}$ is included to capture the first-order autocorrelation effects shown to be persistent in the return time series of most financial securities.

Large-to-Small Information Spillovers Equity shares and other securities with relatively limited market capitalization are considered to be "small." The precise cutoff for "smallness" varies from exchange to exchange. On the NYSE in 2002, for example, "small" stocks were those with market capitalization below $1 billion; stocks with market capitalization of $1 billion to $10 billion were considered to be "medium," and "large" stocks were those with market cap in excess of $10 billion.

Small stocks are known to react to news significantly more slowly than large stocks. Lo and MacKinlay (1990), for example, found that returns on smaller stocks follow returns on large stocks. One interpretation of this phenomenon is that large stocks are traded more actively and absorb information more efficiently than small stocks. Hvidkjaer (2006) further documents "an extremely sluggish reaction" of small stocks to past returns of large stocks and attributes this underreaction to the inefficient behavior of small investors.

A proposed reason for the delay in the response of small stocks is their relative unattractiveness to institutional investors who are the primary source of the information that gets impounded into market prices. The small stocks are unattractive to institutional investors because of their size. A typical size of a portfolio of a mid-career institutional manager is $200 million; if a portfolio manager decides to invest into small stocks, even a well-diversified share of an institutional portfolio will end up moving the market for any small stock significantly, cutting into profitability and raising the liquidity risk of the position. In addition, ownership of 5 percent or more of a particular U.S. stock must be reported to the SEC, further complicating institutional investing in small stocks. As a result, small stocks are traded mostly by small investors, many of whom use daily data and traditional "low-tech" technical analysis to make trading decisions.

The market features of small stocks make the stocks illiquid and highly inefficient, enabling profitable trading. Llorente, Michaely, Saar, and Wang (2002) studied further informational content of trade volume and found that stocks of smaller firms and stocks with large bid-ask spreads exhibit momentum following high-volume periods. Stocks of large firms and firms with small bid-ask spread, however, exhibit no momentum and sometimes exhibit reversals following high-volume time periods. Profitable trading

strategies, therefore, involve trading small stocks based on the results of correlation or cointegration with lagged returns of large stocks as well as the volume of large and small stocks' records during preceding periods.

Futures

Statistical arbitrage can also be applied to pairs consisting of a security and its derivative. The derivative of choice is often a futures contract since futures prices are linear functions of the underlying asset:

$$F_t = S_t \exp[r_t(T - t)]$$

where F_t is the price of a futures contract at time t, S_t is the price of the underlying asset (e.g., equity share, foreign exchange rate, or interest rate) also at time t, T is the time the futures contract expires, and r_t is the interest rate at time t. For foreign exchange futures, r_t is the differential between domestic and foreign interest rates.

Basis Trading The statistical arbitrage between a futures contract and the underlying asset is known as "basis trading." As with equity pairs trading, the basis-trading process follows the following steps: estimation of the distribution of the contemporaneous price differences, ongoing monitoring of the price differences, and acting upon those differences.

Lyons (2001) documents results of a basis-trading strategy involving six currency pairs: DEM/USD, USD/JPY, GBP/USD, USD/CHF, FRF/USD, and USD/CAD. The strategy bets that the difference between the spot and futures prices reverts to its mean or median values. The strategy works as follows: sell foreign currency futures whenever the futures price exceeds the spot price by a certain predetermined level or more, and buy foreign currency futures whenever the futures price falls short of the spot price by at least a prespecified difference. Lyons (2001) reports that when the predetermined strategy trigger levels are computed as median basis values, the strategy obtains a Sharpe ratio of 0.4–0.5.

Futures/Equity Arbitrage In response to macroeconomic news announcements, futures markets have been shown to adjust more quickly than spot markets. Kawaller, Koch, and Koch (1993), for example, show that prices of the S&P 500 futures react to news faster than prices of the S&P 500 index itself, in the Granger causality specification. A similar effect was documented by Stoll and Whaley (1990): for returns measured in 5-minute intervals, both S&P 500 and money market index futures led stock market returns by 5 to 10 minutes.

The quicker adjustment of the futures markets relative to the equities markets is likely due to the historical development of the futures and

equities markets. The Chicago Mercantile Exchange, the central clearinghouse for futures contracts in North America, rolled out a fully functional electronic trading platform during the early 1990s; most equity exchanges still relied on a hybrid clearing mechanism that involved both human traders and machines up to the year 2005. As a result, faster information-arbitraging strategies have been perfected for the futures market, while systematic equity strategies remain underdeveloped to this day. By the time this book was written, the lead-lag effect between futures and spot markets had decreased from the 5- to 10-minute period documented by Stoll and Whaley (1990) to a 1- to 2-second advantage. However, profit-taking opportunities still exist for powerful high-frequency trading systems with low transaction costs.

Indexes and ETFs

Index arbitrage is driven by the relative mispricings of indexes and their underlying components. Under the Law of One Price, index price should be equal to the price of a portfolio of individual securities composing the index, weighted according to their weights within the index. Occasionally, relative prices of the index and the underlying securities deviate from the Law of One Price and present the following arbitrage opportunities. If the price of the index-mimicking portfolio net of transaction costs exceeds the price of the index itself, also net of transaction costs, sell the index-mimicking portfolio, buy index, hold until the market corrects its index pricing, then realize gain. Similarly, if the price of the index-mimicking portfolio is lower than that of the index itself, sell index, buy portfolio, and close the position when the gains have been realized.

Alexander (1999) shows that cointegration-based index arbitrage strategies deliver consistent positive returns and sets forth a cointegration-based portfolio management technique step by step:

1. A portfolio manager selects or is assigned a benchmark. For a portfolio manager investing in international equities, for example, the benchmark can be a European, Asian, or Far East (EAFE) Morgan Stanley index and its constituent indexes. Outperforming the EAFE becomes the objective of the portfolio manager.
2. The manager next determines which countries lead EAFE by running the error-correcting model (ECM) with log(EAFE) as a dependent variable and log prices of constituent indexes in local currencies as independent (explanatory) variables:

$$EAFE_t = \alpha + \beta_1 x_{1,t} + \ldots + \beta_n x_{n,t} + \varepsilon_t \tag{13.17}$$

where the statistically significant $\beta_1 \ldots \beta_n$ coefficients indicate optimal allocations pertaining to their respective country indices $x_1 \ldots x_n$, and α represents the expected outperformance of the EAFE benchmark if the residual from the cointegrating regression is stationary. $\beta_1 \ldots \beta_n$ can be constrained in estimation, depending on investor preferences.

An absolute return strategy can further be obtained by going long in the indexes in proportions identified in step 2 and shorting EAFE.

Options

In options and other derivative instruments with a nonlinear payoff structure, statistical arbitrage usually works between a pair of instruments written on the same underlying asset but having one different characteristic. The different characteristic is most often either the expiration date or the strike price of the derivative. The strategy development proceeds along the steps noted in the previous sections.

CONCLUSION

Statistical arbitrage is powerful in high-frequency settings as it provides a simple set of clearly defined conditions that are easy to implement in a systematic fashion in high-frequency settings. Statistical arbitrage based on solid economic theories is likely to have longer staying power than strategies based purely on statistical phenomena.

CHAPTER 14

Creating and Managing Portfolios of High-Frequency Strategies

The portfolio management process allocates trading capital among the best available trading strategies. These allocation decisions are made with a two-pronged goal in mind:

1. Maximize returns on total capital deployed in the trading operation.
2. Minimize the overall risk.

High-frequency portfolio management tasks can range from instantaneous decisions to allocate capital among individual trading strategies to weekly or monthly portfolio rebalancing among groups of trading strategies. The groups of trading strategies can be formed on the basis of the methodology deployed (e.g., event arbitrage), common underlying instruments (e.g., equity strategies), trading frequency (e.g., one hour), or other common strategy factors. One investment consultant estimates that most successful funds run close to 25 trading strategies at any given time; fewer strategies provide insufficient risk diversification, and managing a greater number of strategies becomes unwieldy. Each strategy can, in turn, simultaneously trade anywhere from one to several thousands of financial securities.

This chapter reviews modern academic and practitioner approaches to high-frequency portfolio optimization. As usual, effective management begins with careful measurement of underlying performance; distributions of returns of strategies composing the overall portfolio are the key inputs into the portfolio optimization. This chapter discusses the theoretical underpinnings of portfolio optimization once the distributions of returns of

the underlying strategies have been estimated. It begins with a review of classical portfolio theory and proceeds to consider the latest applications in portfolio management.

ANALYTICAL FOUNDATIONS OF PORTFOLIO OPTIMIZATION

Graphical Representation of the Portfolio Optimization Problem

The dominant focus of any portfolio management exercise is minimizing risks while maximizing returns. The discipline of portfolio optimization originated from the seminal work of Markowitz (1952). The two dimensions of a portfolio that he reviewed are the average return and risk of the individual securities that compose the portfolio and of the portfolio as a whole. Optimization is conducted by constructing an "efficient frontier," a set of optimal risk-return portfolio combinations for the various instruments under consideration. In the absence of leveraging opportunities (opportunities to borrow and increase the total capital available as well as opportunities to lend to facilitate leverage of others), the efficient frontier is constructed as follows:

1. For every possible combination of security allocations, the risk and return are plotted on a two-dimensional chart, as shown in Figure 14.1. Due to the quadratic nature of the risk function, the resulting chart takes the form of a hyperbola.

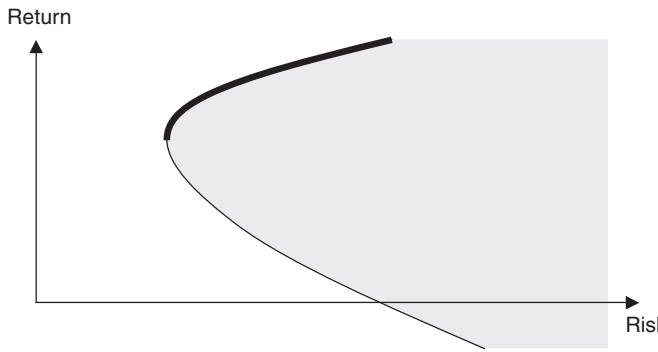


FIGURE 14.1 Graphical representation of the risk-return optimization constructed in the absence of leveraging opportunities. The bold line indicates the efficient frontier.

2. The points with the highest level of return for every given level of risk are selected as the efficient frontier. The same result is obtained if the frontier is selected as the set of points with the lowest level of risk for every given level of return. The bold segment highlights the efficient frontier.
3. An individual investor then selects a portfolio on the efficient frontier that corresponds to the investor's risk appetite.

In the presence of leveraging, the efficient frontier shifts dramatically upward to a straight line between the lending rate, approximated to be risk free for the purposes of high-level analysis, and the "market" portfolio, which is a portfolio lying on a line tangent to the efficient frontier of Figure 14.2. Figure 14.2 shows the resulting efficient frontier.

An interpretation of the efficient frontier in the presence of the leverage rate R_F proceeds as follows. If an investor can lend a portion of his wealth at the rate R_F to high-grade borrowers, he can reduce the risk of his overall portfolio by reducing his risk exposure. The lending investor then ends up on the bold line between R_F and the market portfolio point (σ_M, R_M). The investor incurs two advantages by lending compared with selecting a portfolio from the efficient set with no lending as represented in Figure 14.1:

1. The investor may be able to attain lower risk than ever possible in the no-lending situation.

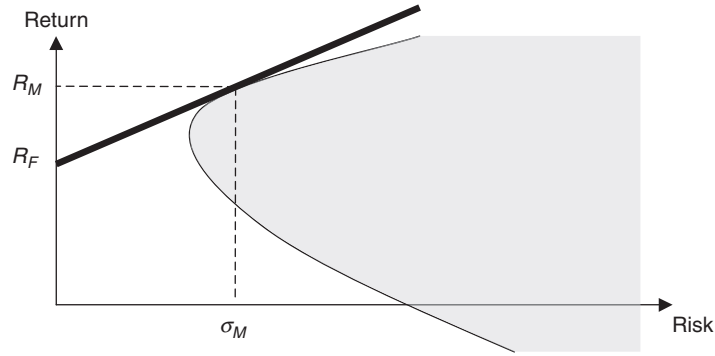


FIGURE 14.2 Graphical representation of the risk-return optimization constructed in the presence of leveraging opportunities. All leveraging is assumed to be conducted at the risk-free rate R_F. The bold line indicates the efficient frontier. The point (σ_M, R_M) corresponds to the "market portfolio" for the given R_F and the portfolio set.

2. With lending capabilities, the investor's return gets scaled linearly to his scaling of risk. In the no-lending situation, the investor's return decreases faster than the investor's decrease in risk.

Similarly, an investor who decides to borrow to increase the capital of his portfolio ends up on the efficient frontier but to the right of the market portfolio. The borrowing investor, too, enjoys the return, which increases linearly with risk and is above the no-borrowing opportunity set.

Core Portfolio Optimization Framework

Analytical estimation of the efficient frontier requires an understanding of the returns delivered by the strategies making up the portfolio. The return of each strategy i is measured as a simple average return over the time period $t \in [1, \ldots, T]$,

$$\overline{R}_i = \frac{1}{T} \sum_{t=1}^{T} R_{it} \tag{14.1}$$

where R_{it} is the return of strategy i in time period t, $t \in [1, \ldots, T]$. The annualized risk of each strategy i, ${\sigma_i}^2$ is often measured as a variance V_i, a square of the standard deviation:

$$V_i = \frac{1}{T-1} \sum_{t=1}^{T} (R_{it} - \overline{R}_i)^2 \tag{14.2}$$

Note that in computation of the average return, $\overline{R}_i$, the sum of returns is divided by the total number of returns, T, whereas in computation of the risk in equation (14.2), the sum of squared deviations from the mean is divided by $T-1$ instead. The $T-1$ factor reflects the number of "degrees of freedom" used in the computation of V_i. Every statistical equation counts every independent variable (a raw number) as a degree of freedom; at the same time, every estimate used in the statistical equation reduces the number of degrees of freedom by 1. Thus, in the estimation of $\overline{R}_i$, the number of independent variables is T, while in the estimation of V_i, the number of independent variables is reduced by 1 since the equation (14.2) uses $\overline{R}_i$, an estimate itself.

The sample frequency of time period t should match the frequency intended for the analysis. In developing high-frequency trading frameworks, it may be desirable to make all the inferences from returns at very high intra-day frequencies—for example, a minute or a second. For investor

relations purposes, daily or even monthly frequency of returns is often sufficient.

If the portfolio comprises I strategies, each represented by a proportion x_i within the portfolio, and each with the average annualized return of $\overline{R}_i$ and risk of V_i, the total risk and return of the portfolio can be determined as follows:

$$E[R_p] = \sum_{i=1}^{I} x_i E[R_i] \tag{14.3}$$

$$V[R_p] = \sum_{i=1}^{I} \sum_{j=1}^{i} x_i x_j \operatorname{cov}[R_i, R_j] \tag{14.4}$$

where x_i is the proportion of the portfolio capital allocated to the strategy i at any given time, $E[R_p]$ and $E[R_i]$ represent respective average annualized returns of the combined portfolio and of the individual strategy i, and $\operatorname{cov}[R_i, R_j]$ is the covariance between returns of strategy i and returns of strategy j:

$$\operatorname{cov}[R_i, R_j] = \rho_{ij} V_i^{0.5} V_j^{0.5} = E[R_i]E[R_j] - E[R_i R_j] \tag{14.5}$$

Additionally, the optimal portfolio should satisfy the following constraint: the sum of all allocations x_i in the portfolio should add up to 100 percent of the portfolio:

$$\sum_{i=1}^{I} x_i = 1 \tag{14.6}$$

Note that the formulation (14.6) allows portfolio weights of individual securities $\{x_i\}$ to be all real numbers, both positive and negative. Positive numbers denote long positions, while negative numbers denote short positions.

The basic portfolio optimization problem is then specified as follows:

$$\min V[R_p], \text{ s.t. } E[R_p] \geq \mu, \sum_{i=1}^{I} x_i = 1 \tag{14.7}$$

where μ is the minimal acceptable average return.

For a trading operation with the coefficient of risk aversion of λ, the mean-variance optimization framework becomes the one shown in equation (14.8):

$$\max \sum_{t=1}^{T} (E[R_{p,t}] - \lambda V[R_{p,t}]), \ \sum_{i=1}^{I} x_i = 1 \tag{14.8}$$

The value of the objective function of equation (14.8) resulting from the optimization can be interpreted as "value added" to the particular investor with risk aversion of λ. The risk aversion parameter λ is taken to be about 0.5 for very risk-averse investors, 0 for risk-neutral investors, and negative for risk-loving investors.

Furthermore, when the trading operation is tasked with outperforming a particular benchmark, μ, the optimization problem is reformulated as follows:

$$\max \sum_{t=1}^{T} (E[R_{p,t}] - \lambda V[R_{p,t}]), \ \text{s.t.} \ \sum_{t=1}^{T} E[R_{p,t}] \geq \mu, \ \sum_{i=1}^{I} x_i = 1 \tag{14.9}$$

Portfolio Optimization in the Presence of Transaction Costs

The portfolio optimization model considered in the previous section did not account for transaction costs. Transaction costs, analyzed in detail in Chapter 19, decrease returns and distort the portfolio risk profile; depending on the transaction costs' correlation with the portfolio returns, transaction costs may increase overall portfolio risk. This section addresses the portfolio optimization solution in the presence of transaction costs.

The trading cost minimization problem can be specified as follows:

$$\min_{s.t. V[TC] \leq K} E[TC] \tag{14.10}$$

where $E[TC]$ is the average of observed trading costs, $V[TC]$ is the variance of observed trading costs, and K is the parameter that specifies the maximum trading cost variance. Changing the parameter K allows us to trace out the "efficient trading frontier," a collection of minimum trading costs for each level of dispersion of trading costs.

Alternatively, given the risk-aversion coefficient λ of the investor or portfolio manager, the target trading cost strategy can be determined from the following optimization:

$$\min E[TC] + \lambda V[TC] \tag{14.11}$$

Both the efficient trading frontier and the target trading cost scenario can be used as benchmarks to compare execution performance of individual traders and executing broker-dealers. However, the cost optimization by itself does not answer the question of portfolio optimization in the presence of trading costs.

Engle and Ferstenberg (2007) further propose an integrative framework for portfolio and execution risk decisions. Using x_{it} to denote the proportion of the total portfolio value allocated to the security i at the end of period t, p_{it} to denote the price of security i at the end of period t, and c_t to denote the cash holdings in the portfolio at the end of period t, Engle and Ferstenberg (2007) specify the portfolio value at the end of period t as follows:

$$y_t = \sum_{i=1}^{I} x_{it} p_{it} + c_t \tag{14.12}$$

If the portfolio rebalancing happens at the end of each period, the one-period change in the portfolio value from time t to time $t+1$ is then

$$\begin{aligned}\Delta y_{t+1} = y_{t+1} - y_t &= \sum_{i=1}^{I} x_{i,t}(p_{i,t+1} - p_{it}) + \sum_{i=1}^{I} (x_{i,t+1} - x_{it}) p_{i,t+1} + (c_{t+1} - c_t) \\ &= \sum_{i=1}^{I} x_{i,t} \Delta p_{i,t+1} + \sum_{i=1}^{I} \Delta x_{i,t+1} p_{i,t+1} + \Delta c_{t+1}\end{aligned} \tag{14.13}$$

If the cash position bears no interest and there are no dividends, the change in the cash position is strictly due to changes in portfolio composition executed at time t at transaction prices $\tilde{p}_{it}$ for each security i:

$$\Delta c_{i,t+1} = -\sum_{i=1}^{I} \Delta x_{i,t+1} \tilde{p}_{i,t+1} \tag{14.14}$$

The negative sign on the right-hand side of equation (14.14) reflects the fact that the increase in the holding position of security i, Δx_{it} results in a decrease of cash available in the portfolio. Combining equations (14.13) and (14.14) produces the following specification for the changes in the portfolio at time t:

$$\Delta y_{t+1} = \sum_{i=1}^{I} x_{it} \Delta p_{i,t+1} + \sum_{i=1}^{I} \Delta x_{i,t+1}(p_{i,t+1} - \tilde{p}_{i,t+1}) = \sum_{i=1}^{I} x_{it} \Delta p_{i,t+1} - TC_{t+1} \tag{14.15}$$

where $\sum_{i=1}^{I} x_{it} \Delta p_{i,t+1}$ is the change in portfolio value due to the active portfolio management and $\sum_{i=1}^{I} \Delta x_{i,t+1}(p_{i,t+1} - \tilde{p}_{i,t+1})$ is due to trading costs.

Specifically, $\sum_{i=1}^{I} \Delta x_{i,t+1}(p_{i,t+1} - \tilde{p}_{i,t+1})$ would equal 0 if all the trades were executed at their target prices, $p_{i,t+1}$.

The combined portfolio optimization problem in the presence of risk aversion λ, $\max E[\Delta y_{t+1}] - \lambda V[\Delta y_{t+1}]$, can then be rewritten as follows for each period $t+1$:

$$\max E\left[\sum_{i=1}^{I} x_{it}\Delta p_{i,t+1} - TC_{t+1}\right] - \lambda V\left[\sum_{i=1}^{I} x_{it}\Delta p_{i,t+1} - TC_{t+1}\right] \tag{14.16}$$

where $TC_t = \sum_{i=1}^{I} \Delta x_{it}(p_{it} - \tilde{p}_{it})$, Δx_{it} is the one-period change in portfolio weight of security i, p_{it} is the target execution price for trading of security i at the end of period t, and $\tilde{p}_{it}$ is the realized execution price for security i at the end of period t.

In addition, the interaction between transaction costs and portfolio allocations can be captured as follows:

$$V\left[\sum_{i=1}^{I} x_{it}\Delta p_{i,t+1} - TC_{t+1}\right] = V\left[\sum_{i=1}^{I} x_{it}\Delta p_{i,t+1}\right] + V\left[TC_{t+1}\right] - 2\,\mathrm{cov}\left[\sum_{i=1}^{I} x_{it}\Delta p_{i,t+1}, TC_{t+1}\right] \tag{14.17}$$

The resulting Sharpe ratio of the portfolio can be ex-ante computed as follows:

$$Sharpe\ ratio = \frac{E\left[\sum_{i=1}^{I} x_i \Delta p_i - TC\right] - R^f}{\sqrt{T}\sqrt{V\left[\sum_{i=1}^{I} x_{it}\Delta p_{i,t+1} - TC_{t+1}\right]}} \tag{14.18}$$

Portfolio Diversification with Asymmetric Correlations

The portfolio optimization frameworks discussed previously assume that the correlations between trading strategies behave comparably in rising and falling markets. Ang and Chen (2002), however, show that this does not have to be the case; the authors document that correlation of equity returns often increases in falling markets, likely distorting correlations of trading strategies used in trading portfolios. Explicit modeling of time-varying correlations of portfolio strategies may refine portfolio estimation and generate more consistent results.

One way to model correlations is to follow the methodology developed by Ang and Chen (2002). The authors' methodology is based on examining the distribution of correlations of returns: if correlations behave normally, they are symmetrical; the correlations accompanying extreme negative returns are equal to the correlations accompanying extreme positive returns. Any asymmetry in correlations of extreme returns should be incorporated in portfolio management solutions.

Dealing with Estimation Errors in Portfolio Optimization

All portfolio optimization exercises involve estimates of average returns, return variances, and correlations. The classic Markowitz (1952) methodology takes the estimated parameters as true distributional values and ignores the estimation error. Frankfurter, Phillips, and Seagle (1971); Dickenson (1979); and Best and Grauer (1991), among others, point out that the estimation errors distort the portfolio selection process and result in poor out-of-sample performance of the complete portfolio.

A common way to overcome estimation errors is to learn from them. A mechanism known as the Bayesian approach proposes that the system learns from its own estimation mistakes by comparing its realized performance with its forecasts. The portfolio optimization system then corrects its future estimates based on its own learnings. In a purely systematic environment, the self-correction process happens without any human intervention. The Bayesian self-correction mechanism is often referred to as a "genetic algorithm."

In the Bayesian approach, the average return estimate of a particular security is considered to be a random variable and is viewed probabilistically in the context of previously obtained information, or priors. All expectations are subsequently developed with respect to the distribution obtained for the estimate. Multiple priors, potentially representing multiple investors or analysts, increase the accuracy of the distribution for the estimate.

Under the Bayesian specification, all mean and variance-covariance estimates are associated with a confidence interval that measures the accuracy of the forecast. An accurate forecast has a tight confidence interval, while the inaccurate forecast has a wide confidence interval. After the accuracy of the previous forecast has been determined, the portfolio weight of a security is scaled depending on the width of the confidence intervals of these securities. The wider the confidence intervals for parameter estimates, the smaller the portfolio weight for that security. When the confidence intervals approach 0, the weights are similar to those of the classic mean-variance optimization.

The traditional Bayesian approach, applied to mean-variance optimization by Jorion (1986), works as follows: both mean and variance estimates computed on a contemporary data sample are adjusted by lessons gleaned from historical (prior) observations.

The dispersion of the distributions of the true mean and variance of the distributions shrink as more observations are collected and analyzed with time. If $R_{p,t}$ is the portfolio return following the mean-variance optimization of equation (14.7) from time $t-1$ to time t, and $\hat{E}[R_{i,t}]$ is the average return estimate for security i, $\hat{E}[R_{i,t}] = \frac{1}{t}\sum_{\tau=1}^{t} R_{i,\tau}$, the "Bayes-Stein shrinkage estimators" for expected return and variance of an individual security i to be used in the mean-variance optimization for the next period $t+1$, are computed as follows:

$$E[R_{i,t+1}]_{BS} = (1-\phi_{i,BS})\hat{E}[R_{i,t}] + \phi_{i,BS}R_{p,t}$$

$$V[R_{i,t+1}]_{BS} = V[R_{i,t}]\left[1+\frac{1}{t+v}\right] + \frac{v}{t(t+1+v)}V[R_{i,t}]$$

where v is the precision of the mean estimates: $v = \frac{(N-2)}{t}\frac{V[R_{i,t}]}{(R_{p,t}-\hat{E}[R_{i,t}])^2}$, N is the number of observations in the sample at time t, and ϕ_{BS} is the shrinkage factor for the mean: $\phi_{BS} = \frac{v}{t+v}$. The case of zero precision ($v = 0$) corresponds to completely diffuse estimates.

Some investors feel particularly strongly about the accuracy of a forecast and would prefer to exclude systems generating inaccurate or ambiguous forecasts from their trading tool belt. Garlappi, Uppal, and Wang (2007) propose a Bayesian portfolio allocation methodology for such investors. An ambiguity-averse investor may be one who relies on multiple information sources and prefers to trade a particular financial security only when those information sources are in agreement about the future movement of that security. This ambiguity aversion is different from risk aversion. Risk aversion measures the investor's tolerance for variance in returns measured after the trades have been executed, or ex-post, whereas ambiguity aversion measures the investor's tolerance for dispersion in the trade outcome forecasts before any trades have been executed, or ex-ante.

To specify the ambiguity aversion, Garlappi, Uppal, and Wang (2007) add the following constraint to the standard mean-variance optimization: $f(E[R], \hat{E}[R], V[R]) \leq \varepsilon$, where f is the uncertainty about forecasts of expected returns, and ε is the investor's maximum tolerance for such uncertainty:

$$f(E[R], \hat{E}[R], V[R]) = \frac{(E[R]-\hat{E}[R])^2}{V[R]/T} \tag{14.19}$$

where T is the number of observations in the sample.

The optimization problem of equation (14.7) over a one-period horizon and multiple assets now becomes:

$$\max(E[R] - \lambda V[R]), \text{ s.t. } \sum_{i=1}^{I} x_i = 1,\ E[R] \geq \mu,\ f(E[R], \hat{E}[R], V[R]) \leq \varepsilon \tag{14.20}$$

Garlappi, Uppal, and Wang (2007) show that the optimization of equation (14.20) can be rewritten as follows:

$$\max(E[R] - \lambda V[R] - \sqrt{\xi V[R]}), \text{ s.t. } \sum_{i=1}^{I} x_i = 1 \tag{14.21}$$

where ξ specifies the multi-asset aversion to ambiguity:

$$(\hat{E}[R] - E[R])' V[R](\hat{E}[R] - E[R]) \leq \xi \tag{14.22}$$

The methodology presented here documents analytical approaches to portfolio optimization. The following sections discuss practical approaches to the estimation of portfolio optimization problems defined previously as well as to other aspects of effective portfolio management in high-frequency trading operations.

EFFECTIVE PORTFOLIO MANAGEMENT PRACTICES

Effective practical portfolio management involves making the following key decisions:

1. How much leverage is appropriate within the portfolio?
2. What proportion of the portfolio should be invested into which trading strategy?

This section presents best-practices answers for each of these questions.

How Much Leverage Is Appropriate within the Portfolio?

Two methodologies, option-based portfolio insurance (OBPI) and constant proportion portfolio insurance (CPPI), address the leverage component of

the portfolio optimization process. Both methodologies require that a substantial proportion of the portfolio at any time be left in cash or invested in risk-free bonds. Each methodology determines exactly what proportion of the portfolio should be left in cash and what proportion should be levered and then invested into risky securities. The OBPI method is static in nature, while the CPPI allocation changes with changes in the market value of the overall portfolio.

1. The OBPI methodology suggests that only a fixed proportion of the portfolio (e.g., X percent, $X < 100$ percent) be invested in risky instruments. The technique was first developed by Leland and Rubenstein (1976), who introduced the concept as options-based insurance. In Leland and Rubenstein (1976), the portfolio is structured to preserve $(100 - X)$ percent of the original portfolio capital, while allowing the portfolio to benefit from the potential upside of the X percent of the portfolio invested in risky securities, such as options. Such portfolios are now commonly securitized and marketed as "structured products." The proportion of the portfolio X can be determined through the cost-benefit analysis of option and bond prices versus expected probabilities of option payouts or, in high-frequency trading cases, the selling price of the option.
2. CPPI is another popular portfolio allocation strategy that calls for dynamic adjustment of portfolio breakdown into cash and risky securities, unlike OBPI, in which the breakdown is static. Black and Jones (1987) and Perold and Sharpe (1988) created CPPI as an extension of OBPI that morphed into an automated method widely used by industry practitioners today.

   CPPI works in accordance with the following steps:

1. Management sets the absolute worst-case maximum drawdown, a floor for the market value of the portfolio. If the market value of the portfolio reaches the floor, the portfolio is fully liquidated into cash. Suppose that the maximum allowable drawdown is *L percent*.
2. The "cushion" is the difference between the market value of the portfolio and the floor. A proportion of the cushion is levered and invested in risky securities. The exact proportion of the cushion invested in the risky instruments is determined by a "multiplier," M, set by management. Common multipliers range from 3 to 6.
3. The risk capital allocated to the risky securities then becomes $M \times Cushion$. As an illustration, suppose that the total capital allocated to a particular portfolio is \$100 million, with 10 percent being the absolute

maximum drawdown. The value of the cushion at this point is \$10 million. If the multiplier is set to 5 ($M = 5$), the maximum actively invested at-risk capital can be \$50 million. However, this \$50 million can be levered. Black and Jones (1987) and Perold and Sharpe (1988) assumed that the leverage ratio on the cushion stays constant during the whole life of the portfolio, leading to the "constant" term in CPPI. Modern CPPI strategies allow for dynamic leverage strategies that scale leverage down in adverse market conditions.

The CPPI allocation ensures that the portfolio always has enough cash to mitigate portfolio risks and to safeguard the investment principal. The portfolio is periodically rebalanced to reflect the current market value of the portfolio. The higher the market value of the portfolio, the more of its proportion is allocated to risky assets. Conversely, the lower the market value of the portfolio, the higher the proportion of the portfolio that is held in cash or in nearly risk-free fixed-income securities.

What Proportion of the Portfolio Should Be Invested into Which Trading Strategy?

After the performance of individual securities and trading strategies has been assessed and the best performers identified, the composition of the master portfolio is determined from the best-performing strategies. This step of the process is known as asset allocation and involves determining the relative weights of strategies within the master portfolio.

The easiest approach to portfolio optimization is to create an equally weighted portfolio of the best-performing strategies. Although the equally weighted framework diversifies the risk of the overall portfolio, it may not diversify the risk as well as a thorough portfolio optimization process. As the number of securities in the portfolio increases, however, determining the optimal weights for each security becomes increasingly complex and time-consuming—a real challenge in the high-frequency environment.

Several classes of algorithms have been proposed to simplify and speed up setting the optimal portfolio weights. Optimization algorithms fall into three classes:

1. The simultaneous equations framework is the algorithm that directly follows the Markowitz (1952) specification. It has been shown to be inefficient for optimization if the portfolio exceeds 10 strategies, and it may produce highly erroneous forecasts when 20 or more assets are involved. The forecast errors are due to the estimation errors that occur when the average returns and variances are computed. The Bayesian error-correction framework, discussed previously in this chapter,

alleviates some of the input estimation errors. Still, in addition to the issues of forecast errors, the estimation time of this algorithm grows exponentially with the number of trading strategies involved, making this method hardly suitable for high-frequency trading of many assets.

2. Nonlinear programming is a class of optimizers popular in commercial software. The nonlinear algorithms employ a variety of techniques with the objective of maximizing or minimizing the target portfolio optimization function given specified parameters such as portfolio allocation weights. Some of these algorithms employ a gradient technique whereby they analyze the slope of the objective function at any given point and select the fastest increasing or decreasing path to the target maximum or minimum, respectively. The nonlinear programming algorithms are equally sensitive to the estimation errors of the input means and variances of the returns. Most often, the algorithms are too computationally complex to be feasible in the high-frequency environments. A recent example of a nonlinear optimizer is provided by Steuer, Qi, and Hirschberger (2006).
3. The critical line–optimizing algorithm was developed by Markowitz (1959) to facilitate the computation of his own portfolio theory. The algorithm is fast and comparatively easy to implement. Instead of providing point weights for each individual security considered in the portfolio allocation, the critical line optimizer delivers a set of portfolios on the efficient frontier, a drawback that has precluded many commercial companies from adapting this method. A recent algorithm by Markowitz and Todd (2000) addresses some of the issues. According to Niedermayer and Niedermayer (2007), the Markowitz and Todd (2000) algorithm outperforms the algorithm designed by Steuer, Qi, and Hirschberger (2006) by a factor of 10,000 for at least 2,000 assets considered simultaneously.

The existing algorithms, whatever the complexity and accuracy of their portfolio allocation outputs, may not be perfectly suited to the high-frequency trading environment. First, in environments where a delay of 1 second can result in a million-dollar loss, the optimization algorithms in their current form still consume too much time and system power. Second, these algorithms ignore the liquidity considerations pertinent to the contemporary trading settings; most of the transactions occur in blocks or "clips" of a prespecified size. Trades of larger-than-normal sizes as well as trades of smaller blocks incur higher transaction costs that in the high-frequency environment can put a serious strain on the system's profitability.

A simple high-frequency alternative to the complex optimization solutions is a discrete pair-wise (DPW) optimization developed by Aldridge

(2009c). The DPW algorithm is a fast compromise between the equally weighted portfolio setting and a full-fledged optimization machine that outputs portfolio weights in discrete clips of the prespecified sizes. No fractional weights are allowed. The algorithm works as follows:

1. Candidates for selection into the overall portfolio are ranked using Sharpe ratios and sorted from the highest Sharpe ratio to the lowest. This step of the estimation utilizes the fact that the Sharpe ratio itself is a measure of where each individual strategy lies on the efficient frontier.
2. An even number of strategies with the highest Sharpe ratios are selected for inclusion into the portfolio. Half of the selected strategies should have historically positive correlations with the market, and half should have historically negative correlations with the market.
3. After the universe of financial instruments is selected on the basis of the Sharpe ratio characteristics, all selected strategies are ranked according to their current liquidity. The current liquidity can be measured as the number of quotes or trades that have been recorded over the past 10 minutes of trading activity, for example.
4. After all the strategies have been ranked on the basis of their liquidity, the pairs are formed through the following process: the two strategies within each pair have opposite historical correlation with the market. Thus, strategies historically positively correlated with the market are matched with strategies historically negatively correlated with the market. Furthermore, the matching should occur according to the strategy liquidity rank. The most liquid strategy positively correlated with the market should be matched with the most liquid strategy negatively correlated with the market, and so on until the least liquid strategy positively correlated with the market is matched with the least liquid strategy negatively correlated with the market. The liquidity-based matching ensures that the high-frequency dynamic captured by correlation is due to idiosyncratic movements of the strategy rather than the illiquidity conditions of one strategy.
5. Next, for each pair of strategies, the high-frequency volatility of a portfolio of just the two strategies is computed for discrete position sizes in either strategy. For example, in foreign exchange, where a standard transactional clip is $20 million, the discrete position sizes considered for the pair-wise optimization may be −$60 million, −$40 million, −$20 million, 0, $20 million, $40 million, and $60 million, where the minus sign indicates the short position. Once the volatility for the various portfolio combinations is selected within each pair of strategies, the positions with the lowest portfolio volatility are selected.

6. The resulting pair portfolios are subsequently executed given the maximum allowable allocation constraints for each strategy. The maximum long and short allocation is predetermined and constrained as follows: the cumulative gross position in each strategy cannot exceed a certain size, and the cumulative net position cannot exceed another, separately set, limit that is smaller than the aggregate of the gross limits for all strategies. The smaller net position clause ensures a degree of market neutrality.

The DWP algorithm is particularly well suited to high-frequency environments because it has the following properties:

- The DPW algorithm avoids the brunt of the impact of input estimation errors by reducing the number of strategies in each portfolio allocation decision.
- The negative historical correlation of input securities ensures that within each pair of matched strategies, the minimum variance will result in long positions in both strategies most of the time. Long positions in the strategies are shown to historically produce the highest returns per unit of risk, as is determined during the Sharpe ratio ranking phase. The times that the system results in short positions for one or more strategy are likely due to idiosyncratic market events.
- The algorithm is very fast in comparison with other portfolio optimization algorithms. The speed of the algorithm comes from the following "savings" in computational time:
  - If the total number of strategies selected in the Sharpe ratio ranking phase is $2K$, the DPW algorithm computes only K correlations. Most other portfolio optimization algorithms compute correlation among every pair of strategies among the $2K$ securities, requiring $2K(K-1)$ correlation computations instead.
  - The grid search employed in seeking the optimal portfolio size for each strategy within each portfolio pair optimizes only between two strategies, or in two dimensions. A standard algorithm requires a $2K$-dimensional optimization.
  - Finally, the grid search allows only a few discrete portfolio weight values. In the main example presented here, there are seven allowable portfolio weights: −$60 MM; −$40 MM; −$20 MM; 0; $20 MM; $40 MM; and $60 MM. This limits the number of iterations and resulting computations from, potentially, infinity, to $7^2 = 49$.

Alexander (1999) notes that correlation and volatility are not sufficient to ensure long-term portfolio stability; both correlation and volatility are typically computed using short-term returns, which only partially reflect

dynamics in prices and necessitate frequent portfolio rebalancing. Instead, Alexander (1999) suggests that in portfolio optimization more attention should be paid to cointegration of constituent strategies. Auxiliary securities, such as options and futures, can be added into the portfolio mix based on cointegration analysis to further strengthen the risk-return characteristics of the trading operation. The cointegration-enhanced portfolios can work particularly well in trading operations that are tasked with outperforming specific financial benchmarks.

CONCLUSION

Competent portfolio management enhances the performance of high-frequency strategies. Ultra-fast execution of portfolio optimization decisions is difficult to achieve but is critical in high-frequency settings.

CHAPTER 15

Back-Testing Trading Models

Once a trading idea is formed, it needs to be tested on historical data. The testing process is known as a back test. This chapter describes the key considerations for a successful and meaningful back test.

The purpose of back tests is twofold. First, a back test validates the performance of the trading model on large volumes of historical data before being used for trading live capital. Second, the back test shows how accurately the strategies capture available profit opportunities and whether the strategies can be incrementally improved to capture higher revenues.

Optimally, the trading idea itself is developed on a small set of historical data. The performance from this sample is known as "in-sample" performance. One month of data can be perfectly sufficient for in-sample estimation, depending on the chosen strategy. To draw any statistically significant inferences about the properties of the trading model at hand, the trading idea should be verified on large amounts of data that was not used in developing the trading model itself. Having a large reserve of historical data (at least two years of continuous tick data) ensures that the model minimizes the data-snooping bias, a condition that occurs when the model overfits to a nonrecurring aberration in the data. Running the back test on a fresh set of historical data is known as making "out-of-sample" inferences.

Once the out-of-sample back-test results have been obtained, they must be evaluated. At a minimum, the evaluation process should compute

basic statistical parameters of the trading idea's performance: cumulative and average returns, Sharpe ratio, and maximum drawdown, as explained in Chapter 5.

For the purposes of accuracy analyses, trading systems can be grouped into those that generate point forecasts and those that generate directional forecasts. Point forecasts predict that the price of a security will reach a certain level, or point. For example, a system that determines that the S&P 500 index will rise from the current 787 level to 800 within the following week is a point forecast system; the point forecast in this case is the 800 number predicted for the S&P 500. Directional systems make decisions to enter into positions based on expectations of the system going up or down, without specific target forecasts. A directional system may predict that USD/CAD will rise from its current level without making a specific prediction about how far USD/CAD will rise.

EVALUATING POINT FORECASTS

The simplest way to evaluate the validity of point forecasts is to run a regression of realized values from the historical data against the out-of-sample forecasts. For example, suppose that the trading model predicts future price levels of equities. Regressing the realized equity prices on the forecasted ones shows the degree of usefulness of the forecast.

Specifically, the model evaluation regression is specified as follows:

$$Y_t = \alpha + \beta X_t + \varepsilon_t \tag{15.1}$$

where Y is the realized price level, X is the forecasted price level, α and β are parameters estimated by the regression, and ε is a normally distributed error term. Whenever the forecast perfectly predicts the realized values, $\beta = 1$ and $\alpha = 0$. The deviation of the α and β parameters from the optimal $\beta = 1$ and $\alpha = 0$ itself indicates the reliability and usefulness of the forecasting model. In addition, the R^2 coefficient obtained from the regression shows the percentage of realized observations explained by the forecasts. The higher the realized R^2, the greater the accuracy of the forecasting model.

The accuracy of point forecasts can also be evaluated by comparing the forecasts with the realized values. Methods for forecast comparisons include:

- Mean squared error (MSE)
- Mean absolute deviation (MAD)

- Mean absolute percentage error (MAPE)
- Distributional performance
- Cumulative accuracy profiling

If the value of a financial security is forecasted to be $x_{F,t}$ at some future time t and the realized value of the same security at time t is $x_{R,t}$, the forecast error for the given forecast, $\varepsilon_{F,t}$, is computed as follows:

$$\varepsilon_{F,t} = x_{F,t} - x_{R,t} \tag{15.2}$$

The mean squared error (MSE) is then computed as the average of squared forecast errors over T estimation periods, analogously to volatility computation:

$$MSE = \frac{1}{T}\sum_{\tau=1}^{T} \varepsilon_{F,\tau}^2 \tag{15.3}$$

The mean absolute deviation (MAD) and the mean absolute percentage error (MAPE) also summarize properties of forecast errors:

$$MAD = \frac{1}{T}\sum_{\tau=1}^{T} |\varepsilon_{F,\tau}| \tag{15.4}$$

$$MAPE = \frac{1}{T}\sum_{\tau=1}^{T} \left|\frac{\varepsilon_{F,\tau}}{x_{R,\tau}}\right| \tag{15.5}$$

Naturally, the lower each of the three metrics (MSE, MAD, and MAPE), the better the forecasting performance of the trading system.

The distributional evaluation of forecast performance also examines forecast errors $\varepsilon_{F,t}$ normalized by the realized value, $x_{R,t}$. Unlike MSE, MAD, and MAPE metrics, however, the distributional performance metric seeks to establish whether the forecast errors are random. If the errors are indeed random, there exists no consistent bias in either direction of price movement, and the distribution of normalized errors $\left\{\frac{\varepsilon_{F,t}}{x_{R,t}}\right\}$ should fall on the uniform [0, 1] distribution. If the errors are nonrandom, the forecast can be improved. One test that can be used to determine whether the errors are random is a comparison of errors with the uniform distribution using the Kolmogorov-Smirnov statistic.

The accuracy of models can be further considered in asymmetric situations. For example, does the MSE of negative forecast errors exceed the MSE of positive errors? If so, the model tends to err by underestimating

the subsequently realized value and needs to be fine-tuned to address the asymmetric nature of forecast accuracy. Similarly, the accuracy of forecast errors can be examined when the errors are grouped based on various market factors:

- Market volatility at the time the errors were measured
- Magnitude of the errors
- Utilization rate of computer power in generating the forecasts, among other possible factors

The objective of the exercise is to identify the conditions under which the system persistently errs and to fix the error-generating issue.

EVALUATING DIRECTIONAL FORECASTS

Testing the accuracy of directional systems presents a greater challenge. Yet, accuracy evaluation of the directional systems can be similar to that of the point forecast systems, with binary values of 1 and 0 indicating whether the direction of the forecast matches the direction of the realized market movement or not. As with the forecasts themselves, directional accuracy estimates are much less accurate than the accuracy estimates of the point forecasts.

Aldridge (2009a) proposes the trading strategy accuracy (TSA) method to measure the ability of a trading strategy to exploit the gain that opportunities present to the strategy in the market. As such, the method evaluates not only the market value of trading opportunities realized by the system but the market value of trading opportunities that the system missed. The methodology of the test is based on that of the cumulative accuracy profile, also known as Gini curve or power curve. To the author's best knowledge, the cumulative accuracy profile has not been applied to the field of trading strategy evaluation to date.

The TSA methodology evaluates trading strategies in back-testing—that is, in observing the strategy run on historical data. The methodology comprises the following three steps:

1. Determination of model-driven trade signals in the historical data
2. Ex-ante identification of successful and unsuccessful trades in the historical data
3. Computation of the marginal probabilities of the trade signals obtained in Step 2 predicting trading outcomes obtained in Step 1

TABLE 15.1 Model-Generated Trading Behavior

Date	Time	Buy a Unit of Security?	Sell a Unit of Security?
March 9, 2009	6:00 A.M.	1	0
March 9, 2009	7:00 A.M.	0	0
March 9, 2009	8:00 A.M.	1	0
March 9, 2009	9:00 A.M.	0	0
March 9, 2009	10:00 A.M.	0	0
March 9, 2009	11:00 A.M.	0	1
March 9, 2009	12:00 P.M.	0	0
March 9, 2009	1:00 P.M.	0	0

Determination of Model-Driven Trade Signals

This step is similar to a standard back test for a trading strategy on a single security. The trading model is run on data of the selected frequency. The buy and sell trading signals that the model generates are recorded in Table 15.1, where 1 corresponds to a decision to execute a trade and 0 denotes the absence of such a decision.

Ex-Ante Identification of Successful and Unsuccessful Trades in the Historical Data

This step involves dividing all trading opportunities in the historical data into profitable and unprofitable buys and sells. At each trade evaluation time, the evaluation process looks ahead in the historical data of a given security to determine whether a buy or a sell entered into for the security at that point in time is a success—that is, a profitable trade.

The frequency of the buy or sell decision times corresponds to the frequency of the portfolio-rebalancing decisions in the trading strategy being evaluated. Some strategies are designed to make portfolio rebalancing decisions at the end of each day; other higher-frequency strategies make decisions on whether to place a buy or a sell on the given security following each quote tick. The ex-ante identification of successful and unsuccessful trades proceeds in tandem with the frequency of the trading strategy studied.

The trade's profitability is determined based on the position closing rules—the stop-gain and stop-loss parameters—decided on in advance. The stop-gain parameter determines at what realized gain the system should close the position. The stop-loss parameter determines the maximum allowable loss for each position and triggers liquidation of the

position whenever the trading strategy hits the stop-loss threshold. For example, a stop gain of 40 pips (0.004) and a stop loss of 20 pips for a long position in EUR/USD exchange rate entered at 1.2950 would result in closing the position should EUR/USD either reach 1.2990 or trip 1.2930. Identifying the trades with the desired characteristics entails separating potential trades into those that encounter the stop gain prior to encountering the stop loss—the successful ones—and those that encounter a stop loss prior to encountering stop gain.

In evaluating the trading opportunities based on regular time intervals (e.g., one hour), care should be taken to ensure that the stop losses are recorded whenever they are triggered, which can happen at times other than when the closing prices are posted for the period. One way to approach this issue is to evaluate the stop losses with the period lows for long positions, and with the period highs for short positions. Thus, a position in EUR/USD that opened with a buy at 1.2950 should be considered stop-lossed whenever a low during any hour drops to 1.2930 or below.

The output of this step is shown in Table 15.2, where 1 indicates a trade that hit the stop gain prior to tripping the stop loss.

Out of eight hours of profitability assessment in the example shown in Table 15.2, three were entry points for profitable buy-initiated trades, and one was an entry point for profitable sell-initiated trade for given levels of stop-gain and stop-loss parameters. Based on these eight hours of assessment, profitable buy trades existed 3/8 or 37.5 percent of time, and profitable sell trades existed 1/8 or 12.5 percent of time.

Eight trades are hardly enough of a sample for meaningful characterization of trade opportunities, as the sample may not converge to a statistically significant description of potential trade population. Just as with back

TABLE 15.2 Trade Profitability Characterization

Date	Time	Profitable Buy Trade?	Profitable Sell Trade?
March 9, 2009	6:00 A.M.	1	0
March 9, 2009	7:00 A.M.	1	0
March 9, 2009	8:00 A.M.	0	0
March 9, 2009	9:00 A.M.	0	0
March 9, 2009	10:00 A.M.	1	0
March 9, 2009	11:00 A.M.	0	1
March 9, 2009	12:00 P.M.	0	0
March 9, 2009	1:00 P.M.	0	0

tests, it is desirable to produce analysis on data of the desired frequency spanning two years or more.

Figures 15.1–15.4 show the results of the potential profitability analysis of EUR/USD on the hourly data sample ranging from January 2001 through December 2008. Figures 15.1 and 15.2 show probabilities of successful buy and sell trades in hourly EUR/USD data with values of stop-gain and stop-loss parameters ranging from 25 to 300 pips (0.0025 to 0.03) in 25-pip intervals. Figures 15.3 and 15.4 show the surface of the average gain per trade for hourly EUR/USD buy and sell decisions for various stop-gain and stop-loss values.

As Figures 15.1 and 15.2 show, the higher the absolute value of the stop-loss parameter relative to the stop-gain, the higher the probability of hitting a successful trade. However, high probabilities of a gain do not necessarily turn into high average gain values per trade, as Figures 15.3 and 15.4 illustrate. Over the 2001–2008 sample period, long EUR/USD trades with high stop-gain and stop-loss parameters achieved higher average gains than short EUR/USD trades, the observation being due to the underlying appreciation of EUR/USD over the period.

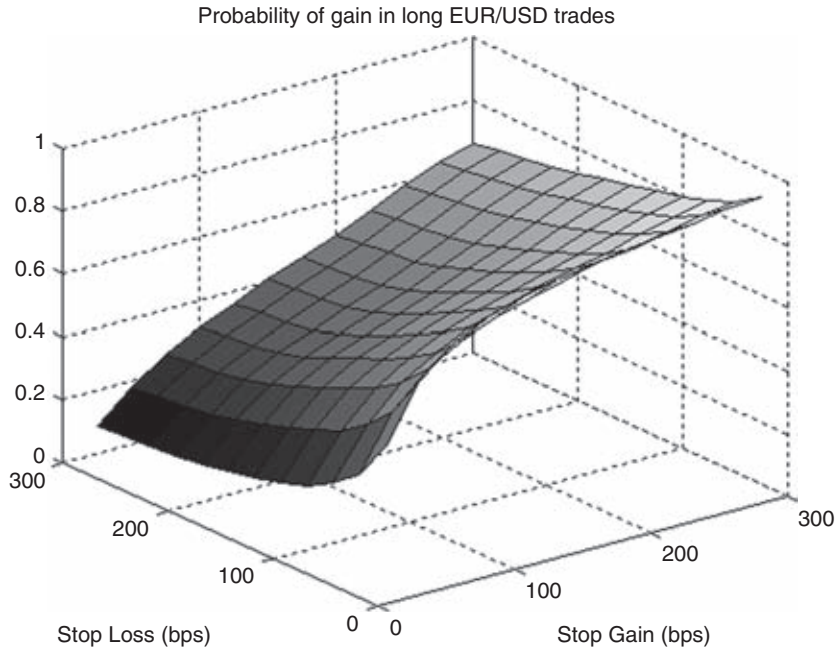


FIGURE 15.1 Probability of successful buy-initiated trades in hourly EUR/USD data for different levels of stop-gain and stop-loss parameters.

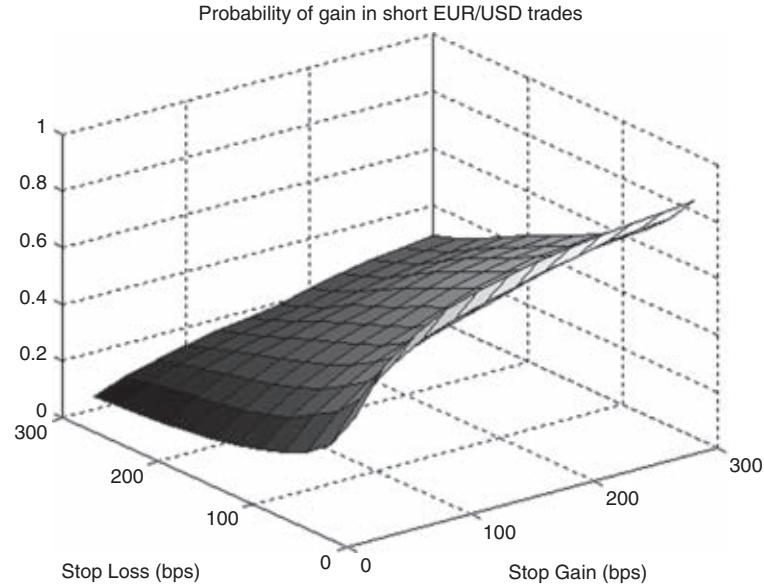


FIGURE 15.2 Probability of successful sell-initiated trades in hourly EUR/USD data for different levels of stop-gain and stop-loss parameters.

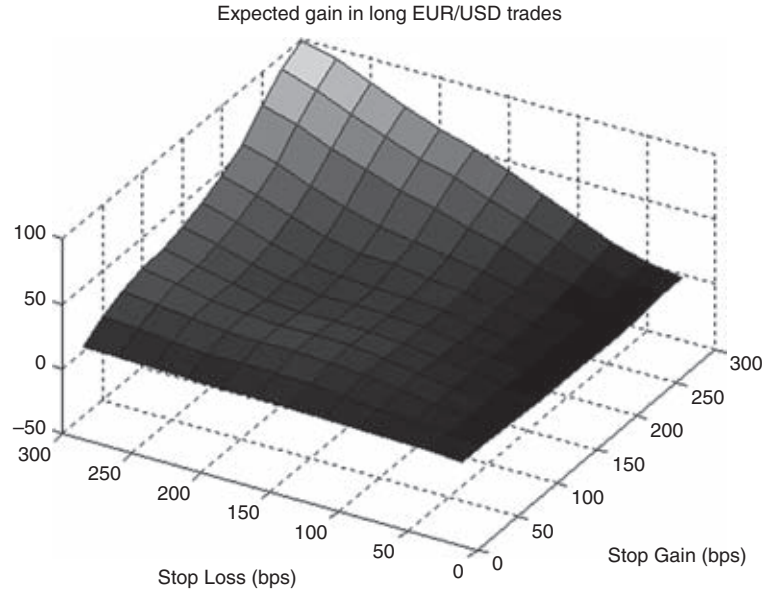


FIGURE 15.3 Average gain per buy-initiated trade in hourly EUR/USD data for different levels of stop-gain and stop-loss parameters.

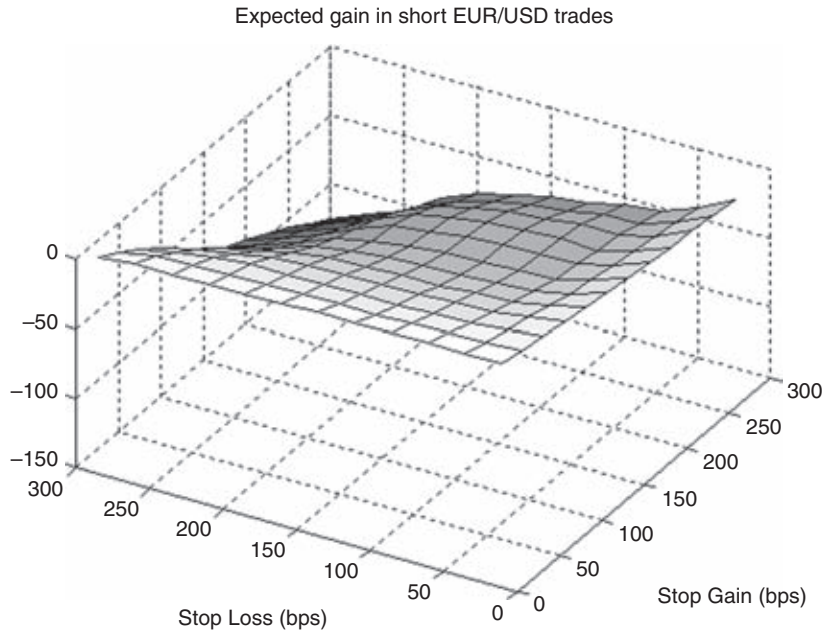


FIGURE 15.4 Average gain per sell-initiated trade in hourly EUR/USD data for different levels of stop-gain and stop-loss parameters.

Computation of Marginal Probabilities

The next step involves matching the results of Steps 1 and 2 in the preceding list to determine the percentage of trade signals that resulted in positive gains as well as the percentage of positive gains that remained undetected by the system. In its most basic approach, this task can be accomplished as follows:

1. Compute the "hit ratio," the percentage of trade signals that resulted in the positive gain. To compute the hit ratio, sum up the number of buy trades with positive outcomes determined in Step 2, the times of which corresponded to the times of buy trades determined by the model presented in Step 1 in the preceding list. Divide the matched number of buy trades with positive outcomes by the total number of buy trades generated by the model in Step 1. Repeat the process for sell trades.
2. Compute the "miss ratio," the percentage of positive outcomes determined in Step 2 that were *not* matched by trades in Step 1.

Although the hit and miss ratio statistics alone are indicative of the relative capabilities of the trading model to exploit market conditions, a graphical representation of trading strategy accuracy generates even stronger and more intuitive comparative insights.

Accuracy Curves

Accuracy curves, also known as Lorenz, Power, or Gini curves, provide a way to graphically compare the accuracy of probabilistic forecasts of trade signals. An accuracy curve plots probabilistic hit rates of different forecasting models versus the ideal (100 percent accurate) forecast.

The trading strategy accuracy (TSA) curves plot the cumulative distribution of the "hits" of the trading models versus the "miss" signals. A "hit" is an outcome whereby the trade signal that generated the outcome is a profitable trade. For example, if following a buy signal on EUR/USD, the currency pair appreciates, allowing us to capture a predetermined gain, the forecast was a "hit." A "miss" outcome is the opposite situation; it is a trade signal that led to a loss. The determination of whether the forecast was a hit or a miss is carried out after the trade has been completed and the trade profitability is fully observable.

Figure 15.5 shows sample accuracy curves. The thick black line extending from (0, 0), first vertically and then horizontally to (100, 100), is the plot of the ideal forecast—that is, the forecast that would ex-ante identify all the hits as hits and all the misses as misses.

The line bisecting the chart at the 45-degree angle corresponds to a completely random forecast—a forecast that is equally likely to be a hit

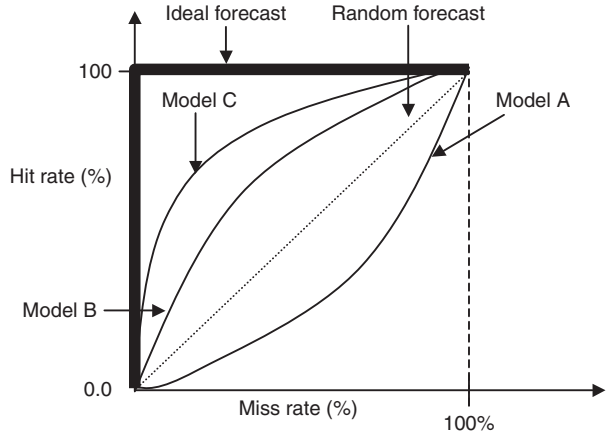


FIGURE 15.5 Trade model evaluation using trading strategy accuracy (TSA) curves.

and a miss. All the other models are then evaluated by locations of their TSA curves relative to the ideal and random forecasts. Model A, for example, is worse than the random forecast. Model B is better than the random forecast, and model C is even better (closer to the ideal) than model B.

A TSA curve is a plot of the cumulative distribution of correctly forecasted losses followed by one of correctly forecasted wins. An ideal model will have a 100 percent hit ratio in all of its forecasts; all the gains will be ex-ante forecasted as gains, and all the losses will be ex-ante forecasted as losses. The model with the TSA curve closest to the ideal curve is the best model. A TSA curve is generated as follows:

1. Gather information on all trade outcomes and their ex-ante forecasts. A winning trade can be identified as a "1," while a losing trade can be identified as a "0." Similarly, a hit ex-ante forecast that predicted a win and resulted in a win can be identified as a "1," and so can an ex-ante hit predicting a loss and resulting in a loss. An ex-ante miss of either the forecasted win resulting in a loss or a forecasted loss resulting in a win can be identified as "0." Table 15.3 shows the resulting data structure for several sample trades.
2. Calculate the total number of hits, H, and misses, M, among all trade outcomes. In our example, there were two hits and three misses. Next, define N as the maximum of the two numbers: $N = \max(H, M)$. In our example, $N = 3$.
3. Compute cumulative hit and miss rates for each trade. The cumulative hit rate for the ith trade, H_i is determined as follows:

$$H_i = \begin{cases} H_{i-1} + 1/N & if\ ith\ trade\ is\ a\ hit \\ H_{i-1} & otherwise \end{cases}$$

$$M_i = \begin{cases} M_{i-1} + 1/N & if\ ith\ trade\ is\ a\ miss \\ M_{i-1} & otherwise \end{cases}$$

TABLE 15.3 Assessing Trade Outcomes and Forecast Hits and Misses

Trade ID	Date	Forecast/ Trade Open Time	Ex-Ante Forecast	Trade Realization	Gain (Loss) MM	Trade Outcome	Hit or Miss
1	3/9/2009	6:00 A.M. ET	Win	Win	59	1	1
2	3/9/2009	7:00 A.M. ET	Loss	Win	70	1	0
3	3/9/2009	8:00 A.M. ET	Win	Loss	(25)	0	0
4	3/9/2009	9:00 A.M. ET	Loss	Loss	(66)	0	1
5	3/9/2009	10:00 A.M. ET	Loss	Win	30	1	0

TABLE 15.4 Cumulative Hit and Miss Rates

Trade ID	Trade Outcome	Hit or Miss	Cumulative Hit Rate	Cumulative Miss Rate
1	1	1	33.3 percent	0 percent
2	1	0	33.3 percent	33.3 percent
3	0	0	33.3 percent	66.7 percent
4	0	1	66.7 percent	66.7 percent
5	1	0	66.7 percent	100 percent

Table 15.4 shows the cumulative hit and miss trades for our example. Trade characteristics have been omitted to save space.

4. We are now ready to plot our sample TSA curve. On the chart, the cumulative miss rate for each trade is plotted on the vertical axis, and the cumulative hit rate is plotted on the vertical axis. Starting at the lower-left corner, at point (0, 0), we now draw the line through the points characterizing the hit and miss rate pairs in our example and then continue the line to the upper-right corner (100 percent, 100 percent) point. Figure 15.6 illustrates the outcome.

The accuracy of the forecast is determined as the total area under the TSA curve. For our example, the area under the curve (shaded region) amounts to 44.4 percent of the total area of the box, indicating a

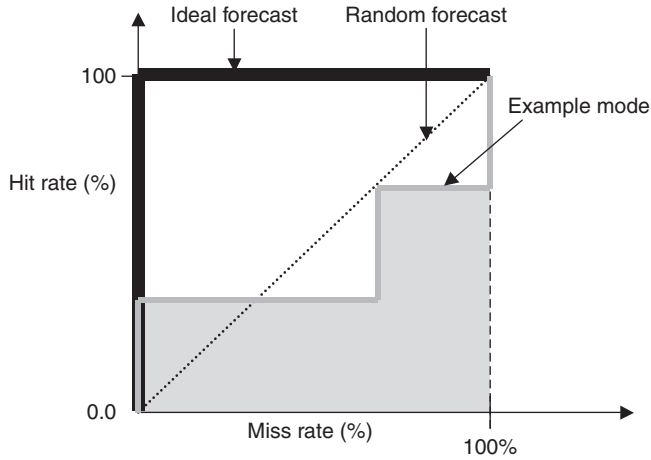


FIGURE 15.6 Trading strategy accuracy (TSA) curve for the foregoing sample trades.

44.4 percent accuracy of our forecasts. Our sample forecasting model performs worse than the random forecasting model, the diagonal. The random forecasting model has an accuracy of 50 percent. Note that in small samples like our example, the accuracy of the estimation will depend on the order of hits and misses within the sample. Depending on the order of hits and misses, the accuracy will vary around its true value.

The TSA curve described here is of the simplest kind; it illustrates the hit ratios without any consideration for the actual profitability of winning versus losing trades. A more advanced version of the TSA curve remedies the situation by splitting all gains into two or more buckets of profitability and splitting all losses into comparable buckets of loss values.

In addition to comparisons of accuracy among different trading models, analyzing potential outcomes of trading systems helps to strengthen and calibrate existing models as well as to evaluate the performance of a combination of different models with mutually exclusive signals. Aldridge (2009a) develops a quantitative methodology of applying hit and miss ratio analyses to enhance the accuracy of predictions of trading models.

CONCLUSION

Various back-test procedures illuminate different aspects of strategy performance on historical data and are performed before the trading strategy is applied to live capital. Observing parameters of strategy performance in back tests allows high-frequency managers to identify the best strategies to include in their portfolio. The same parameters allow modelers to tweak their strategies to obtain even more robust models. Care should be taken to avoid "overfitting"—using the same data sample in repeated testing of the model.

CHAPTER 16

Implementing High-Frequency Trading Systems

Once high-frequency trading models have been identified, the models are back-tested to ensure their viability. The back-testing software should be a "paper"-based prototype of the eventual live system. The same code should be used in both, and the back-testing engine should run on tick-by-tick data to reenact past market conditions. The main functionality code from the back-testing modules should then be reused in the live system.

To ensure statistically significant inferences, the model "training" period T should be sufficiently large; according to the central limit theorem (CLT), 30 observations is the bare minimum for any statistical significance, and 200 observations is considered a reasonable number. Given strong seasonality in intra-day data (recurrent price and volatility changes at specific times throughout the day), benchmark high-frequency models are back-tested on several years of tick-by-tick data.

The main difference between the live trading model and the back-test model should be the origin of the quote data; the back-test system includes a historical quote-streaming module that reads historical tick data from archives and feeds it sequentially to the module that has the main functionality. In the live trading system, a different quote module receives real-time tick data originating at the broker-dealers.

Except for differences in receiving quotes, both live and back-test systems should be identical; they can be built simultaneously and, ideally, can use the same code samples for core functionality. This chapter reviews

the systems implementation process under the assumption that both backtesting and live engines are built and tested in parallel.

MODEL DEVELOPMENT LIFE CYCLE

High-frequency trading systems, by their nature, require rapid hesitation-free decision making and execution. Properly programmed computer systems typically outperform human traders in these "mission-critical" trading tasks, particularly under treacherous market conditions—see Aldridge (2009), for example. As a result, computer trading systems are rapidly replacing traditional human traders on trading desks around the world.

The development of a fully automated trading system follows a path similar to that of the standard software development process. The typical life cycle of a development process is illustrated in Figure 16.1.

A sound development process normally consists of the following five phases:

1. Planning
2. Analysis
3. Design
4. Implementation
5. Maintenance

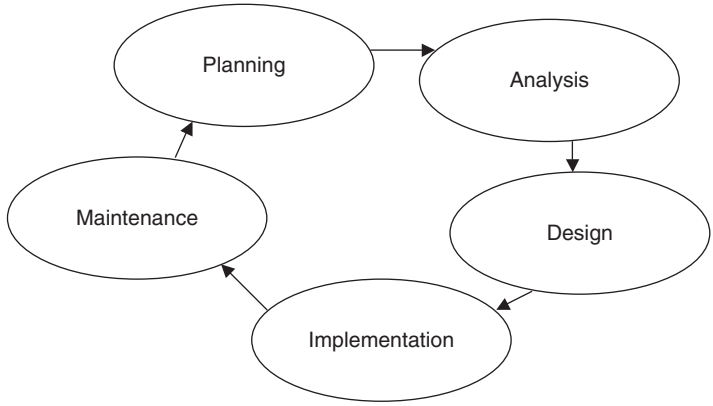


FIGURE 16.1 Typical development cycle of a trading system.

The circular nature of the process illustrates the continuous quality of system development. When a version of the system appears to be complete, new issues demand advanced modifications and enhancements that lead to a new development cycle.

The purpose of the planning phase is to determine the goals of the project as well as to generate a high-level view of what the completed project may look like. The planning is accompanied by a feasibility study that evaluates the project in terms of its economics, operating model, and technical requirements. The economical considerations explore whether the project has a sufficient profit and loss (P&L) potential, whereas operational and technical issues address the feasibility of the project from the compliance, human resources, and other day-to-day points of view. The outputs of the planning phase include concrete goals and targets set for the project, established schedules, and estimated budgets for the entire system.

During the analysis stage of the process, the team aggregates requirements for system functionality, determines the scope of the project (which features are in and which features are out of the current release), and solicits initial feedback from users and management. The analysis stage is arguably the most critical stage in the development process, because it is here that stakeholders have the ultimate ability to shape the functionality of the system given the allocated budget.

The design phase incorporates detailed specifications of functionality, including process diagrams, business rules, and screenshots, along with other output formats such as those of daily reports and other documents. An objective of the design stage is to separate the whole project into discrete components subsequently assigned to teams of software developers; the discrete components will have well-specified interfaces that can lock in seamlessly with other components designed by different teams of software developers. Such early specification of software packaging of internal computer modules streamlines future communication among different software development teams and enables smooth operation of the project going forward. The design phase also outlines test cases—that is, the functionality paths that are later used as blueprints to verify the correctness of the completed code.

The implementation phase, finally, involves actual programming; the software teams or individual programmers develop software modules according to the specifications defined in the design stage. The individual modules are then tested by the development teams themselves against the predefined test cases. When the project management is satisfied that the individual modules have been developed according to the specifications, the project integration work begins. Integration, as its name implies, refers to putting together the individual modules to create a functional system.

While successfully planned projects encounter little variance or problems in the integration stage, some work still remains. Scripts may have to be written to ensure proper communication among various system components, installation wrappers may have to be developed, and, most importantly, the system has to be comprehensively tested to ensure proper operation. The test process usually involves dedicated personnel other than the people who developed the code. The test staff diligently monitors the execution of each functionality according to testing procedures defined in the design stage. The test personnel then documents any "bugs"—that is, discrepancies between the prespecified test case performance and observed performance. The bugs are then sent back over to the development team for resolution and are subsequently returned to the testing teams.

Successful implementation is followed by the deployment and subsequent maintenance phase of the system. The maintenance phase addresses system-wide deviations from planned performance, such as troubleshooting newly discovered bugs.

SYSTEM IMPLEMENTATION

Key Steps in Implementation of High-Frequency Systems

Most systematic trading platforms are organized as shown in Figure 16.2. One or several run-time processors contain the core logic of the trading mechanism and perform the following functions:

- Receive, evaluate, and archive incoming quotes
- Perform run-time econometric analysis
- Implement run-time portfolio management
- Initiate and transmit buy and sell trading signals
- Listen for and receive confirmation of execution
- Calculate run-time P&L
- Dynamically manage risk based on current portfolio allocations and market conditions

A successful high-frequency trading system adapts itself easily to contemporary market conditions. As a result, most high-frequency systems accept, process, and archive volumes of quotes and other market data delivered at real-time frequency. Some systems may convert streaming real-time data into equally spaced data intervals, such as seconds or minutes, for use in their internal econometric analyses. Other systems may run on the raw, irregularly spaced quotes. The decision whether to convert the data should be based on the requirements of the run-time econometric analysis.

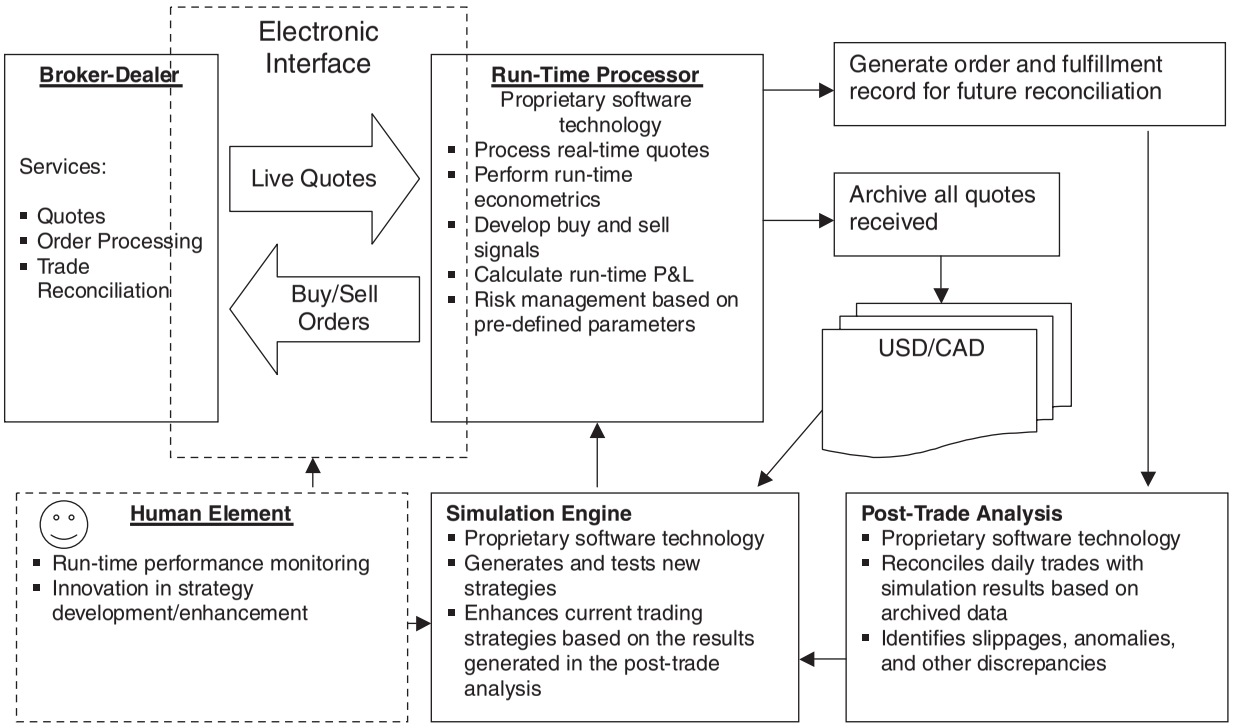


FIGURE 16.2 Typical high-frequency process.

The run-time econometric analysis is a computer program that performs the following three functions:

1. Accepts quotes and order acknowledgments
2. Uses the quotes as input to the core analysis engine
3. Outputs trading signals

The core analysis engine is typically based on the historical analysis identified to generate consistent positive returns over a significant period of time during the simulation and back-testing process. The development of the core engine usually proceeds as follows. First, a quantitative analyst identifies a mispriced security, a market inefficiency, or a persistent deviation from equilibrium. This first modeling step is often done using the MatLab or R programming languages, which are designed to facilitate mathematical operations.

Next, the quantitative analyst, usually in conjunction with technical specialists, back-tests the model on several years of data. A back test of several years (two at the very minimum) should produce a sample distribution of returns that is numerous enough to be close to the true distribution of returns characterizing both past and future performance. If the model delivers consistently positive results in the back test over several years, the model is then programmed into its production state.

Most of the high-frequency production-bound systems are written in C++, although some hedge funds and other investment management firms are known to use Java. C++ is often considered to be "lighter" and "faster" than Java, meaning that C++ programs do not have the processing power overhead required by Java; as a result, C++ systems often work faster than Java-based systems. C++ programmers, however, must be careful in their utilization of the system's run-time memory, whereas Java is designed to take care of all run-time memory issues whether or not the programmer remembers to do so.

The design and implementation of run-time portfolio management reflects the core econometric engine. In addition to the raw quote inputs, the portfolio management framework incorporates inputs from the econometric model, current position sizes, and other information relevant to portfolio diversification and maximization of portfolio returns, while minimizing portfolio risk.

The core engine and the portfolio management framework then initiate and transmit orders to the broker-dealer. Upon receiving and executing an order, the broker-dealer sends back the order status and order-filling price and size to the client. The system then calculates the P&L and assesses risk management parameters that feed back into the portfolio management piece.

Incoming quotes, along with outgoing orders and any other communication between a broker-dealer and a client or an exchange, are most often transmitted via a Financial Information eXchange (FIX) protocol specifically designed for transmission of real-time financial information. According to the FIX industry website (http://www.fixprotocol.org), FIX emerged in 1992 as a bilateral communications framework for equity trading between Fidelity Investments and Salomon Brothers. It has since become the dominant communication method among various broker-dealers, exchanges, and transacting customers. In fact, according to a survey conducted by fixprotocol.org, FIX was used for systematic trading by 75 percent of buy-side firms, 80 percent of sell-side firms, and over 75 percent of exchanges in 2006.

FIX is best described as a programming language that is overseen by a global steering committee, consisting of representatives from banks, broker-dealers, exchanges, industry utilities and associations, institutional investors, and information technology providers from around the world. Its standard is open and free. Implementation of communication process via FIX, however, requires careful planning and dedicated resources and may demand significant expense, much like any other system development process.

A typical FIX message is composed of a header, a body, and a trailer. The header always contains the following three fields: a string identifying the beginning of a message (FIX field # 8), the number of characters in the body of the message to follow the message header (FIX field # 9), and the type of the message (FIX field # 35). Among many message types are quotation and order execution directives and acknowledgments as well as housekeeping messages designed to ensure that the system remains up and running.

For example, MsgType = 0 is the "Heartbeat" message—a message is sent to the other communication party to ensure that the communication connection remains operational and has not been lost as a result of any unforeseen technical problems. The heartbeat message is typically sent after a prespecified number of seconds of inactivity. If either communication party has not received a heartbeat message from the other party, it sends a TestRequest message (MsgType = 1) to "poll" the other communication party. If no heartbeat message is received following a TestRequest message, the connection is considered lost and steps are taken to restart it.

MsgType = 6 is known as "Indication of Interest." Exchanges and broker-dealers use Indication of Interest messages to transmit their interest in either buying or selling in either a proprietary or an agency capacity. MsgType = R indicates a "Quote Request" message with which a client of a broker-dealer requests a quote stream. Under normal circumstances, the broker-dealer responds to the Quote Request message with a continuous

stream of Quote messages (MsgType = S) that carry actual quote information, such as bid or ask prices.

Other message types include orders such as single-name orders, list orders, day limit orders, multiday orders, various cancellation requests, and acknowledgments. All fields in the body are included in the following format:

```
[Field #] = [data]
```

For example, to communicate that the message carries the status of an order, the following sequence is used:

```
35 = 8|
```

All field sequences are terminated with a special character that has a computer value of 0x01. The character looks like "|" when seen on-screen.

The body of the message contains the details of the message, whether it is a quote request, a quote itself, or order and trade information. The message body further specifies the exchange of interest, a timestamp that includes milliseconds, a security symbol, and other necessary transaction data. Like the header, all fields in the body are included in the following format:

```
[Field #] = [data]
```

and each field sequence is terminated by a special computer character 0x01.

Finally, at the end of the body of every message is the "checksum"—a sum of digital values of all the characters in the message included as a verification of whether the message has arrived in full.

For example, a message carrying a quote for USD/CAD at 15:25:20 GMT on July 31, 2007 looked like this:

```
    8=FIX.4.2 | 9=309 | 35=S | 49=ML-FIX-FX |
56=ECHO2-QTS-TEST | 34=5015 | 52=20070731-15:25:20 |
131=1185895365 | 117=ECHO2-QTS-
TEST.00043690C8A8D6B9.00043690D14044C6 | 301=0 |
55=USD/CAD | 167=FOR | 15=USD | 132=1.065450 |
133=1.065850 | 134=5000000.0 | 135=5000000.0 |
647=2000001.0 | 648=2000001.0 | 188=1.06545 |
190=1.06585 | 60=20070731-15:25:20 | 40=H | 64=20070801
| 10=178
```

Dissecting the message, we note the following fields:

`8=FIX.4.2`: Version of the FIX protocol used.
`9=309`: The body of the message is 309 characters long.
`35=S`: This message is carrying a quote.

`49=ML-FIX-FX`: internal identification of the sender of the message; in this case, the sender is Merrill Lynch FX desk.

`56=ECHO2-QTS-TEST`: internal identification of the message recipient.

`34=5015`: sequential message number; this number is used to track all the messages sent. Message sequencing makes it easy for the recipient of the message to identify whether the recipient has received all of the messages and whether they were received in order. Message sequencing may help pinpoint problems with the communication link or message transmission and reception.

`52=20070731-15:25:20`: timestamp corresponding to the time transmission originated. The timestamp consists of the date (yyyymmdd) and time (hh:mm:dd). The time is usually quoted in GMT.

`131=1185895365`: unique identifier corresponding to a message containing an original quote request for a given security.

`117=ECHO2-QTS-TEST.00043690C8A8D6B9.00043690D14044C6`: unique identifier for the quote. Note that the identifier contains the recipient's identification, making it possible for broker-dealers to stream different quotes to clients with different profiles. For example, the broker-dealer may increase spreads for a persistently successful client.

`301=0`: level of response requested from recipient of the quote message; valid responses are 0 = No Acknowledgment (default), 1 = Acknowledge only negative or erroneous quotes, and 2 = Acknowledge each quote message. In our example, Merrill Lynch does not expect any acknowledgment upon receipt of the quote.

`55=USD/CAD`: the ticker symbol of the quoted instrument.

`167=FOR`: the type of the security quoted. Valid values include ABS = Asset-backed Securities, BN = Bank Notes, FUT = Future, and OPT = Option, among many others.

`15=USD`: based currency used for price.

`132=1.065450`: bid price.

`133=1.065850`: offer or ask price.

`134=5000000.0`: bid quantity.

`135=5000000.0`: offer quantity.

`647=2000001.0`: minimum quantity for a bid.

`648=2000001.0`: minimum quantity for an offer.

`188=1.06545`: bid FX spot rate.

`190=1.06585`: offer FX spot rate.

`60=20070731-15:25:20`: timestamp of the quote creation.

`40=H`: available order types.

Order types may assume one of the following values: 1 = Market, 2 = Limit, 3 = Stop/Stop Loss, 4 = Stop Limit, 5 = Market On Close (No

longer used), 6 = With Or Without, 7 = Limit Or Better, 8 = Limit With Or Without, 9 = On Basis, A = On Close (No longer used), B = Limit On Close (No longer used), C = Forex Market (No longer used), D = Previously Quoted, E = Previously Indicated, F = Forex Limit (No longer used), G = Forex Swap, H = Forex Previously Quoted (No longer used), I = Funari (Limit day order with unexecuted portion handles as Market On Close, e.g., Japan), J = Market If Touched (MIT), K = Market With Left Over as Limit (market order with unexecuted quantity becoming limit order at last price), L = Previous Fund Valuation Point (Historic pricing; for CIV), M = Next Fund Valuation Point (Forward pricing; for CIV), P = Pegged, Q = Counter-order selection.

`64=20070801`: trade settlement date. If the order were to be executed on July 31, 2007, the trade would be settled on 8/1/2007.

`10=178`: checksum, a sum of computer codes of all characters in the message. The checksum is used to verify that the message arrived intact.

The best high-frequency trading systems do not stop there. A post-trade analysis engine reconciles production results with simulation results run with the same code on the same data and updates distributions of returns, trading costs, and risk management parameters to be fed back into the main processing engine, portfolio optimization, and risk management components.

The simulation engine is an independent module that tests new trading ideas on past and run-time data without actually executing the trades. Unlike ideas that are still in early stages of development that are often coded in MatLab or Excel, ideas tested in the simulation engine are typically coded in the production language (C++ or Java). Once coded for production setup, the simulation engine is first run on a long sample of historical data in a process known as back-testing. At this point, the simulation engine can be refined to incorporate any system tweaks and bug fixes. Once the back test performs satisfactorily, the system is switched to run on real-time data, the same data that feeds into the production system. At this point, however, the system is still in the testing phase and the system's ability to send production orders is disabled. Instead, all orders that would be sent to the broker-dealer are recorded in a text file. This testing phase of the system on the real-time data is referred to as "paper-trading."

Once paper-trading performance is satisfactory and comparable to that of the back test, paper-trading is moved into production. Continuous human supervision of the system is required to ensure that the system does not fall victim to some malicious activity such as a computer virus or a market event unaccounted for in the model itself. The role of the human trader, however, should normally be limited to making sure that the performance

of the system falls within specific bounds. Once the bounds are breached, the human trader should have the authority to shut down trading for the day or until the conditions causing the breach have been resolved.

Common Pitfalls in Systems Implementation

Time Distortion The simulation runs in its own time using quotes collected and stored during a run-time of another process. The frequency of the quotes recorded by the process that collected the data that is now historical data can vary greatly, mostly because of the following two factors:

1. The number of financial instruments for which the original process collected quotes
2. The speed of the computer system on which the original process ran

Their impact is due to the nature of the quote process and its realization in most trading systems. Most systems comprise a client (the quote collecting and/or trading application) that is geared to receive quotes and the server (a broker-dealer application supplying the quotes). The client is most often a "local" application that runs "locally": on computer hardware over which the trader has full control. The broker-dealer server is almost always a remote application, meaning that the client has to communicate with the server over a remote connection, such as the Internet. To receive quotes, the client application usually has to perform the following communication with the server process:

1. The client sends the server a message or a series of messages with the following information:
   a. Client identification (given to the client by the broker-dealer that houses the server)
   b. Names of financial securities for which the quotes are requested
2. The server will respond, acknowledging the client's message. The server's response will also indicate whether the client is not allowed to receive any of the quotes requested for any reason.
3. The server will begin to stream the quotes to the client. The quotes are typically streamed in an "asynchronous" manner—that is, the server will send a quote to the client as soon as a new quote becomes available. Some securities have higher-frequency quotes than others. For example, during high-volatility times surrounding economic announcements, it is not unusual for the EUR/USD exchange rate to be accompanied by as many as 30 quotes per second. At the same time, some obscure stock may generate only one quote per trading day. It is important to keep in mind the expected frequency of quotes while designing the quote-receiving part of the application.

4. Quote distortion often happens next. It is the responsibility of the client to collect and process all the quotes as soon as they arrive at the client's computer. Here, several issues can occur. On the client's machine, all incoming quotes are placed into a queue in the order of their arrival, with the earliest quotes located closest to the processor. This queue can be thought of as a line for airport check-in. Unlike the airport line, however, the queue often has a finite length or capacity; therefore, any quote arrivals that find the queue full are discarded. Hence the first issue: Quote time series may vary from client to client if the client systems have queues of varying lengths, all other system characteristics being equal.

   Once the quotes are in the queue, the system picks the earliest quote arrival from the queue for processing; then all the quotes in the queue are shifted closer to the processing engine. As noted previously, the quotes may arrive faster than the client is able to process them, filling up the queue and leading the system to discard new quote arrivals until the older quotes are processed. Even a seemingly simple operation such as copying a quote to a file or a database stored on the computer system takes computer time. While the quote-storing time may be a tiny fraction of a second and thus negligible by human time standards, the time can be significant by computer clock, and slow down the processing of incoming quotes.

   A client system may assign the quote an arrival time on taking the quote from its arrival queue. The timestamp may therefore differ from the timestamp given to the quote by the server. Depending on the number of securities for which the quotes are collected and the market's volatility at any given time of day, the timestamp distortion may differ significantly as a result of the quote-processing delay alone. If the quotes are further mathematically manipulated to generate trading signals, the distortions in timestamps may be even more considerable.
5. Naturally, systems running on computers with slower processing power will encounter more timestamp distortion than systems running on faster machines. Faster machines are quicker at processing sequential quotes and drop fewer quotes as a result. Even the slightest differences in system power can result in different quote streams that in turn may produce different trading signals.

The reliability of quote delivery can be improved in the following four ways:

1. Timestamping quotes immediately when each quote arrives before putting the quote into the queue
2. Increasing the size of the quote queue

3. Increasing system memory to the largest size feasible given a cost/benefit analysis
4. Reducing the number of securities for which the quotes are collected on any given client

These four steps toward establishing greater quote reliability are fairly easy to implement when the client application is designed and built from scratch, and in particular when using the FIX protocol for quote delivery. On the other hand, many off-the shelf clients, including those distributed by executing brokers, may be difficult or impossible to customize. For firms planning to use an off-the-shelf client, it may be prudent to ask the software manufacturer how the preceding issues can be addressed in the client.

Speed of Execution Duration of execution can make or break high-frequency trading models. Most strategies for arbitraging temporary market mispricings, for example, depend on the ability to get the orders posted with lightning speed. Whoever detects the mispricing and gets his order posted on the exchange first is likely to generate the most profit.

Speed of execution is controlled by the following components of trading platforms:

- The speed of applications generating trading signals
- The proximity of applications generating trading signals to the executing broker
- The speed of the executing broker's platform in routing execution requests
- The proximity of the executing broker to the exchange
- The speed of the exchange in processing the execution orders

Figure 16.3 illustrates the time-dependent flow of execution process.

To alleviate delays due to the physical transmission of trading signals between clients and the broker and, again, between the broker and the exchange, clients dependent on the speed of execution often choose to locate their systems as close to the broker and the exchange as possible. This practice of placing client computer systems next to the broker and the exchange is known as "co-location." Co-location does not require clients to move their offices; instead, co-location can be achieved through a set of production machines managed in a secure warehouse by an experienced third-party administrator, with the client having a full remote, or "virtual," access to the production machines. Co-location services typically employ systems administration staff that is capable of providing recovery services in case of systems or power failure, making sure that the client applications work at least 99.9 percent of time.

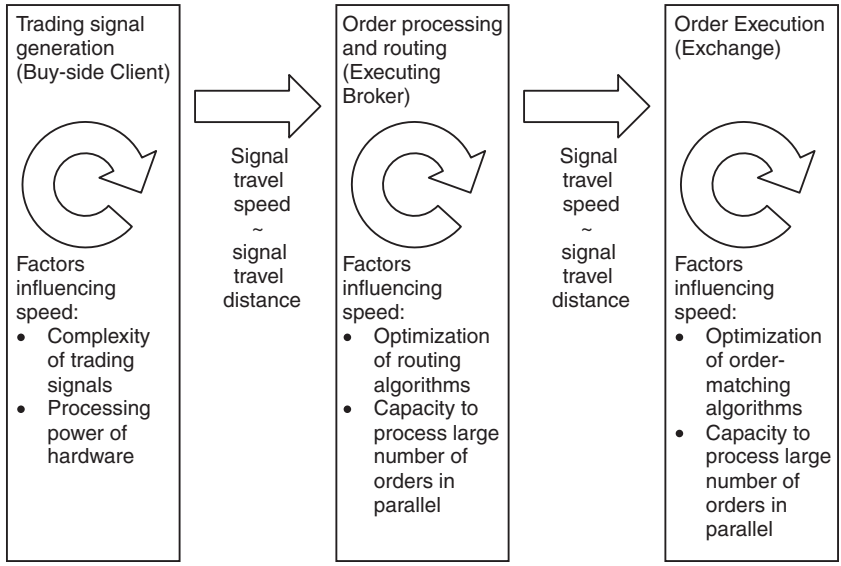


FIGURE 16.3 Execution process.

TESTING TRADING SYSTEMS

The costs of rolling out a system that contains programmatic errors, or bugs, can be substantial. Thorough testing of the system, therefore, is essential prior to wide roll-out of the model. Testing has the following stages:

- Data set testing
- Unit testing
- Integration testing
- System testing
- Regression testing
- Use case testing

Data Set Testing

Data set testing refers to testing the validity of the data, whether historical data used in a back test or real-time data obtained from a streaming data provider. The objective of data testing is to ascertain that the system minimizes undesirable influences and distortions in the data and to ensure that the run-time analysis and trading signal generation work smoothly.

Data set testing is built on the premise that all data received for a particular security should fall into a statistical distribution that is

consistent throughout time. The data should also exhibit consistent distributional properties when sampled at different frequencies: 1-minute data for USD/CAD, for example, should be consistent with historical 1-minute data distribution for USD/CAD observed for the past year. Naturally, data set testing should allow for distributions to change with time, but the observed changes should not be drastic, unless they are caused by a large-scale market disruption.

A popular procedure for testing data is based on testing for consistency of autocorrelations. It is implemented as follows:

1. A data set is sampled at a given frequency—say, 10-second intervals.
2. Autocorrelations are estimated for a moving window of 30 to 1,000 observations.
3. The obtained autocorrelations are then mapped into a distribution; outliers are identified, and their origin is examined. The distributional properties can be analyzed further to answer the following questions:
   - Have the properties of the distribution changed during the past month, quarter, or year?
   - Are these changes due to the version of the code or to the addition or removal of programs on the production box?

The testing should be repeated at different sampling frequencies to ensure that no systemic deviations occur.

Unit Testing

Unit testing verifies that each individual software component of the system works properly. A unit is a testable part of an application; the definition of a unit can range from the code for the lowest function or method to the functionality of a medium-level component—for example, a latency measurement component of the post-trade analysis engine. Testing code in small blocks from the ground up ensures that any errors are caught early in the integration process, avoiding expensive system disruptions at later stages.

Integration Testing

Integration testing follows unit testing. As its name implies, integration testing is a test of the interoperability of code components; the test is administered to increasingly larger aggregates of code as the system is being built up from modular pieces to its completed state. Testing modular interoperability once again ensures that any code defects are caught and fixed early.

System Testing

System testing is a post-integration test of the system as a whole. The system testing incorporates several testing processes described as follows.

Graphical user interface (GUI) software testing ensures that the human interface of the system enables the user (e.g., the person responsible for monitoring trading activity) to perform her tasks. GUI testing typically ensures that all the buttons and displays that appear on screen are connected with the proper functionality according to the specifications developed during the design phase of the development process.

Usability and performance testing is similar in nature to GUI testing but is not limited to graphical user interfaces and may include such concerns as the speed of a particular functionality. For example, how long does the system take to process a "system shutdown" request? Is the timing acceptable from a risk management perspective?

Stress testing is a critical component of the testing of high-frequency trading systems. A stress-testing process attempts to document and, subsequently, quantify the impact of extreme hypothetical scenarios on the system's performance. For example, how does the system react if the price of a particular security drops 10 percent within a very short time? What if an act of God occurs that shuts down the exchange, leaving the system holding its positions? What other worst-case scenarios are there and how will they affect the performance of the system and the subsequent P&L?

Security testing is another indispensable component of the testing process that is often overlooked by organizations. Security testing is designed to identify possible security breaches and to either provide a software solution for overcoming the breaches or create a breach-detection mechanism and a contingency plan in the event a breach occurs. High-frequency trading systems can be vulnerable to security threats coming from the Internet, where unscrupulous users may attempt to hijack account numbers, passwords, and other confidential information in an attempt to steal trading capital. However, intra-organizational threats should not be underestimated; employees with malicious intent or disgruntled workers having improper access to the trading system can wreak considerable and costly havoc. All such possibilities must be tested and taken into account.

Scalability testing refers to testing the capacity of the system. How many securities can the system profitably process at the same time without incurring significant performance impact? The answer to this question may appear trivial, but the matter is anything but trivial in reality. Every incremental security measure added to the system requires an allocation of computer power and Internet bandwidth. A large number of securities processed simultaneously on the same machine may considerably slow down system performance, distorting quotes, trading signals, and the P&L as a

result. A determination of the maximum permissible number of securities will be based on the characteristics of each trading platform, including available computing power.

Reliability testing determines the probable rate of failure of the system. Reliability testing seeks to answer the following questions: What are the conditions under which the system fails? How often can we expect these conditions to occur? The failure conditions may include unexpected system crashes, shutdowns due to insufficient memory space, and anything else that leads the system to stop operating. The failure rate for any well-designed high-frequency trading system should not exceed 0.01 percent (i.e., the system should be guaranteed to remain operational 99.99 percent of the time).

Recovery testing refers to verification that in an adverse event, whether an act of God or a system crash, the documented recovery process ensures that the system's integrity is restored and it is operational within a prespecified time. The recovery testing also ensures that data integrity is maintained through unexpected terminations of the system. Recovery testing should include the following scenarios: When the application is running and the computer system is suddenly restarted, the application should have valid data upon restart. Similarly, the application should continue operating normally if the network cable should be unexpectedly unplugged and then plugged back in.

Use Case Testing

The term *use case testing* refers to the process of testing the system according to the system performance guidelines defined during the design stage of the system development. In use case testing, a dedicated tester follows the steps of using the system and documents any discrepancies between the observed behavior and the behavior that is supposed to occur. Use case testing ensures that the system is operating within its parameters.

CONCLUSION

Implementation of high-frequency systems is a critical process, one in which mistakes can be very costly. Outsourcing noncritical components of the system may be a prudent strategy. However, code that implements proprietary econometric models should be developed internally to ensure maximum strategy capacity.

CHAPTER 17

Risk Management

Effective risk management in a trading operation is as important as the signals that motivate the trades. A well-designed and executed risk management function is key to sustainable profitability in all organizations. This chapter presents the leading approaches for managing risk in high-frequency trading operations that are compliant with Basel II risk management standards.[1]

As with any business decision, the process of building a risk management system involves several distinct steps:

1. First, the overall organization-wide goals of risk management should be clearly defined.
2. Next, potential risk exposure should be measured for each proposed trading strategy and the overall portfolio of the trading operation.
3. Based on the goals and risk parameters determined in the two preceding steps, a risk management system is put in place to detect abnormal risk levels and to dynamically manage risk exposure.

The following sections in turn address each of these steps.

[1]Basel II is the set of recommendations on risk management in financial services issued by the Basel Committee on Banking Supervision in June 2004 with the goal of promoting economic stability.

DETERMINING RISK MANAGEMENT GOALS

The primary objective of risk management is to limit potential losses. Competent and thorough risk management in a high-frequency setting is especially important, given that large-scale losses can mount quickly at the slightest shift in behavior of trading strategies. The losses may be due to a wide range of events, such as unforeseen trading model shortcomings, market disruptions, acts of God (earthquakes, fire, etc.), compliance breaches, and similar adverse conditions.

Determining organizational goals for risk management is hardly a trivial endeavor. To effectively manage risk, an organization first needs to create clear and effective processes for measuring risk. The risk management goals, therefore, should set concrete risk measurement methodologies and quantitative benchmarks for risk tolerance associated with different trading strategies as well as with the organization as a whole. Expressing the maximum allowable risk in numbers is difficult, and obtaining organization-wide agreement on the subject is even more challenging, but the process pays off over time through quick and efficient daily decisions and the resulting low risk.

A thorough goal-setting exercise should achieve senior management consensus with respect to the following questions:

- What are the sources of risk the organization faces?
- What is the extent of risk the organization is willing to undertake? What risk/reward ratio should the organization target? What is the minimum acceptable risk/reward ratio?
- What procedures should be followed if the acceptable risk thresholds are breached?

The sources of risk should include the risk of trading losses, as well as credit and counterparty risk, liquidity risk, operational risk, and legal risk. The risk of trading losses, known as market risk, is the risk induced by price movements of all market securities; credit and counterparty risk addresses the ability and intent of trading counterparties to uphold their obligations; liquidity risk measures the ability of the trading operation to quickly unwind positions; operational risk enumerates possible financial losses embedded in daily trading operations; and legal risk refers to all types of contract frustration. A successful risk management practice identifies risks pertaining to each of these risk categories.

Every introductory finance textbook notes that higher returns, on average, are obtained with higher risk. Yet, while riskier returns are on average higher across the entire investing population, some operations with risky exposures obtain high gains and others suffer severe losses. A successful

risk management process should establish the risk budget that the operation is willing to take in the event that the operation ends up on the losing side of the equation. The risks should be quantified as worst-case scenario losses tolerable per day, week, month, and year and should include operational costs, such as overhead and personnel costs. Examples of the worst-case losses to be tolerated may be 10 percent of organizational equity per month or a hard dollar amount—for example, $150 million per fiscal year.

Once senior management has agreed to the goals of risk management, it becomes necessary to translate the goals into risk processes and organizational structures. Processes include development of a standardized approach for review of individual trading strategies and the trading portfolio as a whole. Structures include a risk committee that meets regularly, reviews trading performance, and discusses the firm's potential exposure to risks from new products and market developments.

The procedures for dealing with breaches of established risk management parameters should clearly document step-by-step actions. Corporate officers should be appointed as designated risk supervisors responsible to follow risk procedures. The procedures should be written for dealing with situations not if, but when a risk breach occurs. Documented step-by-step action guidelines are critical; academic research has shown that the behavior of investment managers becomes even riskier when investment managers are incurring losses. See, for example, Kahneman and Teversky (1979), Kouwenberg and Ziemba (2007), and Carpenter (2000). Previously agreed-on risk management procedures eliminate organizational conflicts in times of crisis, when unified and speedy action is most necessary.

The following sections detail the quantification of risk exposure for different types of risk and document the best practices for ongoing oversight of risk exposure.

MEASURING RISK

While all risk is quantifiable, the methodology for measuring risk depends on the type of risk under consideration. The Basel Committee on Banking Supervision[2], an authority on risk management in financial services, identifies the following types of risk affecting financial securities:

1. Market risk—induced by price movements of market securities
2. Credit and counterparty risk—addresses the ability and intent of trading counterparties to uphold their obligations

[2]More information on the Basel Committee for Banking Supervision can be found by visiting http://www.bis.org/bcbs/ on the Internet.

3. Liquidity risk—the ability of the trading operation to quickly unwind positions
4. Operational risk—the risk of financial losses embedded in daily trading operations
5. Legal risk—the risk of litigation expenses

All current risk measurement approaches fall into four categories:

- Statistical models
- Scalar models
- Scenario analysis
- Causal modeling

Statistical models generate predictions about worst-case future conditions based on past information. The Value-at-Risk (VaR) methodology is the most common statistical risk measurement tool, discussed in detail in the sections that focus on market and liquidity risk estimation. Statistical models are the preferred methodology of risk estimation whenever statistical modeling is feasible.

Scalar models establish the maximum foreseeable loss levels as percentages of business parameters, such as revenues, operating costs, and the like. The parameters can be computed as averages of several days, weeks, months, or even years of a particular business variable, depending on the time frame most suitable for each parameter. Scalar models are frequently used to estimate operational risk.

Scenario analysis determines the base, best, and worst cases for the key risk indicators (KRIs). The values of KRIs for each scenario are determined as hard dollar quantities and are used to quantify all types of risk. Scenario analysis is often referred to as the "stress test."

Causal modeling involves identification of causes and effects of potential losses. A dynamic simulation model incorporating relevant causal drivers is developed based on expert opinions. The simulation model can then be used to measure and manage credit and counterparty risk, as well as operational and legal risks. The following sections discuss the measurement of different types of risk.

Measuring Market Risk

Market risk refers to the probability of and the expected value of a decrease in market value due to market movements. Market risk is present in every trading system and must be competently and thoroughly estimated.

Market risk is the risk of loss of capital due to an adverse price movement in any securities—equities, interest rates, or foreign exchange. Many

securities can be affected by changes in prices of other, seemingly unrelated, securities. The capital invested in equity futures, for example, will be affected by price changes in equities underlying the futures as well as by the changes in interest rates used to value the time component of the futures price. If the capital originates and is settled in EUR, but the investment is placed into equity futures in the U.S. market, then EUR/USD price changes will also affect the value of the portfolio.

To accurately estimate the risk of a given trading system, it is necessary to have a reasonably complete idea of the returns generated by the trading system. The returns are normally described in terms of distributions. The preferred distributions of returns are obtained from running the system on live capital. The back test obtained from running the model over at least two years of tick data can also be used as a sample distribution of trade returns. However, the back-test distribution alone may be misleading, because it may fail to account for all the extreme returns and hidden costs that occur when the system is trading live. Once the return distributions have been obtained, the risk metrics are most often estimated using statistical models and VaR in particular.

The concept of Value-at-Risk (VaR) has by now emerged as the dominant metric in market risk management estimation. The VaR framework spans two principal measures—VaR itself and the expected shortfall (ES). VaR is the value of loss in case a negative scenario with the specified probability should occur. The probability of the scenario is determined as a percentile of the distribution of historical scenarios that can be strategy or portfolio returns. For example, if the scenarios are returns from a particular strategy and all the returns are arranged by their realized value in ascending order from the worst to the best, then the 95 percent VaR corresponds to the cutoff return at the lowest fifth percentile. In other words, if 100 sample observations are arranged from the lowest to the highest, then VaR corresponds to the value of the fifth lowest observation.

The expected shortfall (ES) measure determines the average worst-case scenario among all scenarios at or below the prespecified threshold. For example, a 95 percent ES is the average return among all returns at the 5 percent or lower percentile. If 100 sample observations are arranged from the lowest to the highest, the ES is the average of observations 1 through 5. Figure 17.1 illustrates the concepts of VaR and ES.

An analytical approximation to true VaR can be found by parameterizing the sample distribution. The parametric VaR assumes that the observations are distributed in a normal fashion. Specifically, the parametric VaR assumes that the 5 percent in the left tail of the observations fall at $\mu - 1.65\sigma$ of the distribution, where μ and σ represent the mean and standard deviation of the observations, respectively. The 95 percent parametric VaR is then computed as $\mu - 1.65\sigma$, while the 95 percent parametric ES is

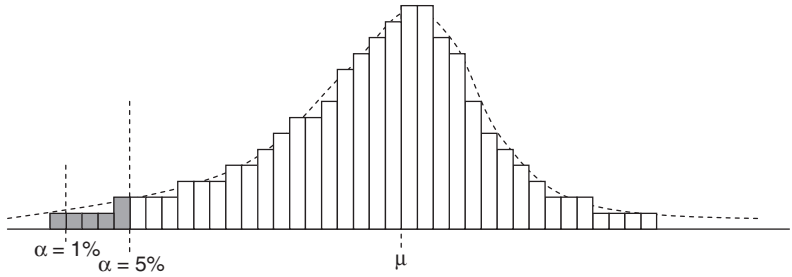


FIGURE 17.1 The 99 percent VaR ($\alpha = 1$ percent) and 95 percent VaR ($\alpha = 5$ percent) computed on the sample return population.

computed as the average of all distribution values from $-\infty$ to $\mu - 1.65\sigma$. The average can be computed as an integral of the distribution function. Similarly, the 99 percent parametric VaR is computed as $\mu - 2.33\sigma$, while the 99 percent parametric ES is computed as the average of all distribution values from $-\infty$ to $\mu - 1.65\sigma$. The parametric VaR is an approximation of the true VaR; the applicability of the parametric VaR depends on how close the sample distribution resembles the normal distribution. Figure 17.2 illustrates this idea.

While the VaR and ES metrics summarize the location and the average of many worst-case scenarios, neither measure indicates the absolute worst scenario that can destroy entire trading operations, banks, and markets. Most financial return distributions have fat tails, meaning that the very extreme events lie beyond normal distribution bounds and can be truly catastrophic.

The limitations of VaR methodology have hardly been a secret. In a *New York Times* article published on January 2, 2009, David Einhorn, the

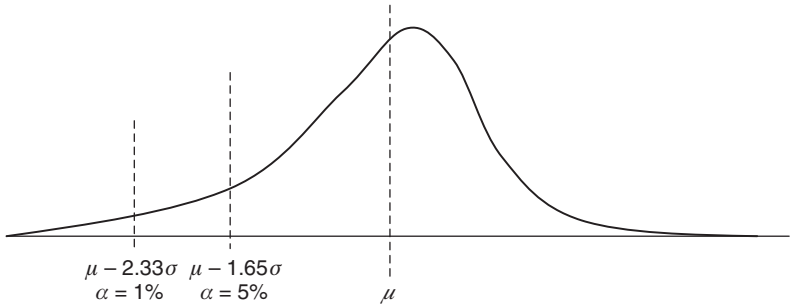


FIGURE 17.2 The 95 percent parametric VaR corresponds to $\mu - 1.65\sigma$ of the distribution, while the 99 percent parametric VaR corresponds to $\mu - 2.33\sigma$ of the distribution.

founder of the hedge fund Greenlight Capital, stated that VaR was "relatively useless as a risk-management tool and potentially catastrophic when its use creates a false sense of security among senior managers and watchdogs. This is like an air bag that works all the time, except when you have a car accident." The article also quoted Nassim Nicholas Taleb, the best-selling author of *The Black Swan*, as calling VaR metrics "a fraud." Jorion (2000) points out that the VaR approach both presents a faulty measure of risk and actively pushes strategists to bet on extreme events. Despite all the criticism, VaR and ES have been mainstays of corporate risk management for years, where they present convenient reporting numbers.

To alleviate the shortcomings of the VaR, many quantitative outfits began to parameterize extreme tail distributions to develop fuller pictures of extreme losses. Once the tail is parameterized based on the available data, the worst-case extreme events can be determined analytically from distributional functions, even though no extreme events of comparable severity were ever observed in the sample data.

The parameterization of the tails is performed using the extreme value theory (EVT). EVT is an umbrella term spanning a range of tail modeling functions. Dacorogna et al. (2001) note that all fat-tailed distributions belong to the family of Pareto distributions. A Pareto distribution family is described as follows:

$$G(x) = \begin{cases} 0 & x \leq 0 \\ \exp(-x^{-\alpha}) & x > 0, \alpha > 0 \end{cases} \tag{17.1}$$

where the tail index α is the parameter that needs to be estimated from the return data. For raw security returns, the tail index varies from financial security to financial security. Even for raw returns of the same financial security, the tail index can vary from one quoting institution to another, especially for really high-frequency estimations.

When the tail index α is determined, we can estimate the magnitude and probability of all the extreme events that may occur, given the extreme events that did occur in the sample. Figure 17.3 illustrates the process of using tail parameterization:

1. Sample return observations obtained from either a back test or live results are arranged in ascending order.
2. The tail index value is estimated on the bottom 5 percentile of the sample return distribution.
3. Using the distribution function obtained with the tail index, the probabilities of observing the extreme events are estimated. According to the tail index distribution function, a –7 percent return would occur with a probability of 0.5 percent, while a return of –11 percent would register with a probability of 0.001 percent.

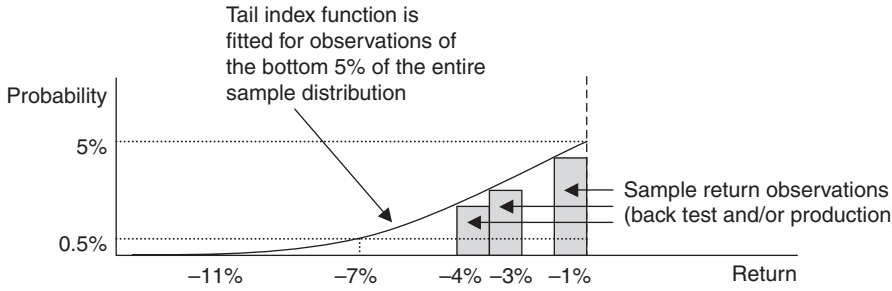


FIGURE 17.3 Using tail index parameterization to predict extreme events.

The tail index approach allows us to deduce the unobserved return distributions from the sample distributions of observed returns. Although the tail index approach is useful, it has its limitations. For one, the tail index approach "fills in" the data for the observed returns with theoretical observations; if the sample tail distribution is sparse (and it usually is), the tail index distribution function may not be representative of the actual extreme returns. In such cases, a procedure known as "parametric bootstrapping" may be applicable.

Parametric bootstrap simulates observations based on the properties of the sample distribution. The technique "fills in" unobserved returns based on observed sample returns. The parametric bootstrap process works as follows:

1. The sample distribution of observed returns delivered by the manager is decomposed into three components using a basic market model:
   - **a.** The manager's skill, or alpha
   - **b.** The manager's return due to the manager's portfolio correlation with the benchmark
   - **c.** The manager's idiosyncratic error

   The decomposition is performed using the standard market model regression:

$$R_{i,t} = \alpha_i + \beta_{i,x} R_{x,t} + \varepsilon_t \tag{17.2}$$

   where $R_{i,t}$ is the manager's raw return in period t, $R_{x,t}$ is the raw return on the chosen benchmark in period t, α_i is the measure of the manager's money management skill or alpha, and $\beta_{i,x}$ is a measure of the dependency of the manager's raw returns on the benchmark returns.

TABLE 17.1 Examples of Generated Bootstrap Components

Observation No.	$R_{i,t}$	$R_{x,t}$	$\hat{\alpha}_i$	$\hat{\beta}_{i,x}R_{x,t}$	$\varepsilon_{i,t}$
1	0.015	–0.001	0.002	0.00005	0.01295
2	0.0062	0.0034	0.002	–0.00017	0.00403

2. Once parameters $\hat{\alpha}_i$ and $\hat{\beta}_{i,x}$ are estimated using equation (17.2), three pools of data are generated: one for $\hat{\alpha}_i$ (constant for given manager, benchmark, and return sample), $\hat{\beta}_{i,x}R_{x,t}$, and $\varepsilon_{i,t}$.[3] For example, if $\hat{\alpha}_i$ and $\hat{\beta}_{i,x}$ were estimated to be 0.002 and –0.05, respectively, then the component pools for a sample of raw returns and benchmarked returns may look as shown in Table 17.1.
3. Next, the data is resampled as follows:
   a. A value $\varepsilon_{i,t}^S$ is drawn at random from the pool of idiosyncratic errors, $\{\varepsilon_{i,t}\}$.
   b. Similarly, a value $\hat{\beta}_{i,x}R_{x,t}^S$ is drawn at random from the pool of $\{\beta_{i,x}R_{x,t}\}$.
   c. A new sample value is created as follows:

$$\hat{R}_{i,t}^S = \hat{\alpha}_i + \hat{\beta}_{i,x}R_{x,t}^S + \varepsilon_t^S \tag{17.3}$$

   d. The sampled variables ε_t^S and $\hat{\beta}_{i,x}R_{x,t}^S$ are returned to their pools (not eliminated from the sample).

   The resampling process outlined in steps a–d above is then repeated a large number of times deemed sufficient to gain a better perspective on the distribution of tails. As a rule of thumb, the resampling process should be repeated at least as many times as there were observations in the original sample. It is not uncommon for the bootstrap process to be repeated thousands of times. The resampled values $\hat{R}_{i,t}^S$ can differ from the observed sample distribution, thus expanding the sample data set with extra observations conforming to the properties of the original sample.
4. The new distribution values obtained through the parametric process are now treated as were other sample values and are incorporated into the tail index, VaR, and other risk management calculations.

[3]The "hat" notation on variables, as in $\hat{\alpha}_i$ and $\hat{\beta}_{i,x}$, denotes that the parameters were estimated from a sample distribution, as opposed to comprising the true distribution values.

The parametric bootstrap relies on the assumption that the raw returns' dependence on a benchmark as well as the manager's alpha remain constant through time. This does not have to be the case. Managers with dynamic strategies spanning different asset classes are likely to have time-varying dependencies on several benchmarks. Despite this shortcoming, the parametric bootstrap allows risk managers to glean a fuller notion of the true distribution of returns given the distribution of returns observed in the sample.

To incorporate portfolio managers' benchmarks into the VaR framework, Suleiman, Shapiro, and Tepla (2005) propose analyzing the "tracking error" of the manager's return in excess of his benchmark. Suleiman, Shapiro, and Tepla (2005) define tracking error as a contemporaneous difference between the manager's return and the return on the manager's benchmark index:

$$TE_t = \ln(R_{i,t}) - \ln(R_{X,t}) \tag{17.4}$$

where $R_{i,t}$ is the manager's return at time t and $R_{x,t}$ is return on the manager's benchmark, also at time t. The VaR parameters are then estimated on the tracking error observations.

In addition to VaR, statistical models may include Monte Carlo simulation–based methods to estimate future market values of capital at risk. The Monte Carlo simulations are often used in determining derivatives exposure. Scenario analyses and causal models can be used to estimate market risk as well. These auxiliary types of market risk estimation, however, rely excessively on qualitative assessment and can, as a result, be misleading in comparison with VaR estimates, which are based on realized historical performance.

Measuring Credit and Counterparty Risk

The credit and counterparty risk reflects the probability of financial loss should one party in the trading equation not live up to its obligations. An example of losses due to a counterparty failure is a situation in which a fund's money is custodied with a broker-dealer, and the broker-dealer goes bankrupt. The collapse of Lehman Brothers in October 2008 was the most spectacular counterparty failure in recent memory. According to Reuters, close to $300 billion was frozen in bankruptcy proceedings as a result of the bank's collapse, pushing many prominent hedge funds to the brink of insolvency. Credit risk is manifest in decisions to extend lines of credit or margins. Credit risk determines the likelihood that creditors will default on their margin calls, should they encounter any. In structured products,

credit risk measures the likelihood and the impact of default of the product underwriter, called the reference entity.

Until recently, the measurement of credit and counterparty risk was delegated to dedicated third-party agencies that used statistical analysis overlaid with scenario and causal modeling. The most prominent of these agencies, Standard & Poor's and Moody's, came under fire during the credit crisis of 2007–2008 because their ratings may have failed to capture the true credit and counterparty risk, and it was revealed that in many instances the rating agencies had cozy relationships with the firms they rated. As credit and counterparty data becomes increasingly available, it may make good sense for firms to statistically rate their counterparties internally. The remainder of this section describes common techniques for measuring credit and counterparty risk.

Entities with publicly traded debt are the easiest counterparties to rank. The lower the creditworthiness of the entity, the lower the market price on the senior debt issued by the entity and the higher the yield the entity has to pay out to attract investors. The spread, or the difference between the yield on the debt of the entity under consideration and the yield on the government debt with comparable maturity, is a solid indicator of the creditworthiness of the counterparty. The higher the spread, the lower the creditworthiness of the counterparty. Because yields and spreads are inversely related to the prices of the bonds, the creditworthiness of a counterparty can also be measured on the basis of the relative bond price of the firm: the lower the bond price, the higher the yield and the lower the creditworthiness. Market prices of corporate debt provide objective information about the issuer's creditworthiness. The prices are determined by numerous market participants analyzing the firms' strategies and financial prospects and arriving at their respective valuations.

Table 17.2 shows senior bond prices for selected firms and their relative creditworthiness rankings on May 15, 2009. A creditworthiness of 100 indicates solid ability to repay the debt, while a creditworthiness of 0 indicates the imminence of default. From the perspective of a fund deciding on May 15, 2009 whether to use Morgan Stanley or Wells Fargo & Co. as its prime broker, for example, Morgan Stanley may be a better choice in that the firm shows higher creditworthiness and lower counterparty risk. As discussed in the section on implementation of risk management frameworks that follows, a diversification of counterparties is the best way to protect the operation from credit and counterparty risk.

The creditworthiness of private entities with unobservable market values of obligations can be approximated as that of a public firm with matching factors. The matching factors should include the industry, geographic location, annual earnings of the firms to proxy for the firms' sizes, and various accounting ratios, such as the quick ratio to assess short-term

TABLE 17.2 Senior Bond Prices for Selected Firms and Their Relative Creditworthiness Rank on May 15, 2009

Firm	Bond Ticker Symbol	Bond Price at Close on 5/15/2009	Relative Creditworthiness Rank
Coca Cola Enterprises, Inc.	191219AP9	131.07	80
Morgan Stanley	617446HD4	103.25	55
Wells Fargo & Co.	949746FS5	95.83	40
Marriott Int'l, Inc. (New)	571903AF0	90.43	30
American General Fin. Corp.	02635PRT2	47.18	5

solvency. Once a close match with publicly traded debt is found for the private entity under evaluation, the spread on the senior debt of the publicly traded firm is used in place of that for the evaluated entity.

In addition to the relative creditworthiness score, the firms may need to obtain a VaR-like number to measure credit and counterparty risk. This number is obtained as an average of exposure to each counterparty weighted by the counterparty's relative probability of default:

$$\text{CCExposure} = \frac{1}{N}\sum_{i=1}^{N} \text{Exposure}_i \times \text{PD}_i \tag{17.5}$$

where CCExposure is the total credit and counterparty exposure of the organization, N is the total number of counterparties of the organization, Exposure_i is the dollar exposure of the ith counterparty, and PD_i is the probability of default of the ith counterparty:

$$\text{PD}_i = \frac{100 - (\text{Creditworthiness Rank})_i}{100}\% \tag{17.6}$$

The total credit and counterparty exposure is then normalized by the capital of the firm and added to the aggregate VaR number.

Measuring Liquidity Risk

Liquidity risk measures the firm's potential inability to unwind or hedge positions in a timely manner at current market prices. The inability to close out positions is normally due to low levels of market liquidity relative to the position size. The lower the market liquidity available for a specific instrument, the higher the liquidity risk associated with that instrument. Levels of liquidity vary from instrument to instrument and depend on the number of market participants willing to transact in the instrument under

consideration. Bervas (2006) further suggests the distinction between the trading liquidity risk and the balance sheet liquidity risk, the latter being the inability to finance the shortfall in the balance sheet either through liquidation or borrowing.

In mild cases, liquidity risk can result in minor price slippages due to the delay in trade execution and can cause collapses of market systems in its extreme. For example, the collapse of Long-Term Capital Management (LTCM) in 1998 can be attributed to the firm's inability to promptly offload its holdings. The credit crisis of 2008 was another vivid example of liquidity risk; as the credit crisis spread, seemingly high-quality debt instruments such as high-grade CDOs lost most of their value when the markets for these securities vanished. Many firms holding long positions in these securities suffered severe losses. Another, simpler, example of liquidity risk is provided by out-of-the-money options nearing expiration; the markets for out-of-the-money options about to become worthless disappear entirely.

The number of transacting parties usually depends on the potential profitability and degree of regulation in the trades of interest. No one is inclined to buy worthless options just before the options expire. In the case of CDOs in the fall of 2008, the absence of markets was largely due to regulation FAS 133 enacted in 2007. FAS 133 mandates that all securities be marked to their market prices. In the case of CDOs, for example, the market price is the last trade price recorded for the security by the firm holding CDOs in its portfolio. As a result of this regulation, the firms that held CDOs at 100 percent of their face value on the books refused to sell a portion of their CDOs at a lower price. The sale would result in devaluation of their remaining CDOs at the lower market price, which would trigger devaluation of the fund as a whole and would, in turn, result in increased investor redemptions. At the same time, potential buyers of the CDOs faced a similar problem: those already holding CDOs on their books at 100 percent of face value would face sharp devaluations of their entire funds if they chose to purchase new CDOs at significantly reduced prices. The recently proposed taxation scheme of charging transaction costs on trading as a tax may similarly destroy the liquidity of currently liquid instruments.

To properly assess the liquidity risk exposure of a portfolio, it is necessary to take into account all potential portfolio liquidation costs, including the opportunity costs associated with any delays in execution. While liquidation costs are stable and are easy to estimate during periods with little volatility, the liquidation costs can vary wildly during high-volatility regimes. Bangia et al. (1999), for example, document that liquidity risk accounted for 17 percent of the market risk in long USD/THB positions in May 1997, and Le Saout (2002) estimates that liquidity risk can reach over 50 percent of total risk on selected securities in CAC40 stocks.

Bervas (2006) proposes the following liquidity-adjusted VaR measure:

$$VaR^L = VaR + Liquidity\ Adjustment = VaR - \left(\mu^S + z_\alpha \sigma^S\right) \tag{17.7}$$

where VaR is the market risk value-at-risk discussed previously in this chapter, μ^S is the mean expected bid-ask spread, σ^S is the standard deviation of the bid-ask spread, and z_α is the confidence coefficient corresponding to the desired $\alpha-$ percent of the VaR estimation. Both μ^S and σ^S can be estimated either from raw spread data or from the Roll (1984) model.

Using Kyle's λ measure, the VaR liquidity adjustment can be similarly computed through estimation of the mean and standard deviation of the trade volume:

$$\begin{aligned} VaR^L &= VaR + Liquidity\ Adjustment \\ &= VaR - \left(\hat{\alpha} + \hat{\lambda}(\mu^{NVOL} + z_\alpha \sigma^{NVOL})\right) \end{aligned} \tag{17.8}$$

where $\hat{\alpha}$ and $\hat{\lambda}$ are estimated using OLS regression following Kyle (1985):

$$\Delta P_t = \alpha + \lambda NVOL_t + \varepsilon_t \tag{17.9}$$

ΔP_t is the change in market price due to market impact of orders, and $NVOL_t$ is the difference between the buy and sell market depths in period t.

Hasbrouck (2005) finds that the Amihud (2002) illiquidity measure best indicates the impact of volume on prices. Similar to Kyle's λ adjustment to VaR, the Amihud (2002) adjustment can be applied as follows:

$$VaR^L = VaR + Liquidity\ Adjustment = VaR - (\mu^\gamma + z_\alpha \sigma^\gamma) \tag{17.10}$$

where μ^γ and σ^γ are the mean and standard deviation of the Amihud (2002) illiquidity measure γ, $\gamma_t = \frac{1}{D_t} \sum_{d=1}^{D_t} \frac{|r_{d,t}|}{v_{d,t}}$, D_t is the number of trades executed during time period t, $r_{d,t}$ is the relative price change following trade d during trade period t, and $v_{d,t}$ is the trade quantity executed within trade d.

Measuring Operational Risk

Operational risk is the risk of financial losses resulting from one or more of the following situations:

- Inadequate or failed internal controls, policies, or procedures
- Failure to comply with government regulations
- Systems failures
- Fraud
- Human error
- External catastrophes

Operational risk can affect the firm in many ways. For example, a risk of fraud can taint the reputation of the firm and will therefore become a "reputation risk." Systems failures may result in disrupted trading activity and lost opportunity costs for capital allocation.

The Basel Committee for Bank Supervision has issued the following examples of different types of operational risk:

- **Internal fraud**—misappropriation of assets, tax evasion, intentional mismarking of positions, and bribery
- **External fraud**—theft of information, hacking damage, third-party theft, and forgery
- **Employment practices and workplace safety**—discrimination, workers' compensation, employee health and safety
- **Clients, products, and business practice**—market manipulation, antitrust, improper trade, product defects, fiduciary breaches, account churning
- **Damage to physical assets**—natural disasters, terrorism, vandalism
- **Business disruption and systems failures**—utility disruptions, software failures, hardware failures
- **Execution, delivery, and process management**—data entry errors, accounting errors, failed mandatory reporting, negligent loss of client assets

Few statistical frameworks have been developed for measurement of operational risk; the risk is estimated using a combination of scalar and scenario analyses. Quantification of operational risk begins with the development of hypothetical scenarios of what can go wrong in the operation. Each scenario is then quantified in terms of the dollar impact the scenario will produce on the operation in the base, best, and worst cases. To align the results of scenario analysis with the VaR results obtained from estimates of other types of risk, the estimated worst-case dollar impact on operations is then normalized by the capitalization of the trading operation and added to the market VaR estimates.

Measuring Legal Risk

Legal risk measures the risk of breach of contractual obligations. Legal risk addresses all kinds of potential contract frustration, including contract formation, seniority of contractual agreements, and the like. An example of legal risk might be two banks transacting foreign exchange between the two of them, with one bank deciding that under its local laws, the signed contract is void.

The estimation of legal risk is conducted by a legal expert affiliated with the firm, primarily using a causal framework. The causal analysis identifies the key risk indicators embedded in the current legal contracts of the firm and then works to quantify possible outcomes caused by changes in the key risk indicators. As with other types of risk, the output of legal risk analysis is a VaR number, a legal loss that has the potential to occur with just a 5 percent probability for a 95 percent VaR estimate.

MANAGING RISK

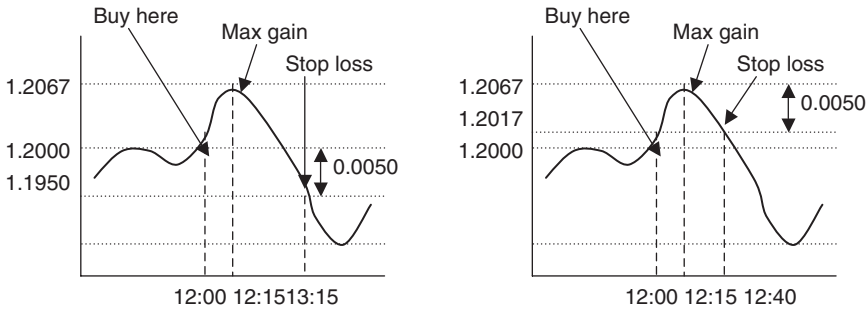
Once market risk has been estimated, a market risk management framework can be established to minimize the adverse impact of the market risk on the trading operation. Most risk management systems work in the following two ways:

1. Stop losses—stop current transaction(s) to prevent further losses
2. Hedging—hedge risk exposure with complementary financial instruments

Stop Losses

A stop loss is the crudest and most indispensable risk management technique to manage the risk of unexpected losses. In the case of market risk, a stop loss is a threshold price of a given security, which, if crossed by the market price, triggers liquidation of the current position. In credit and counterparty risk, a stop loss is a level of counterparty creditworthiness below which the trading operation makes a conscious decision to stop dealing with the deteriorating counterparty. In liquidity risk, the stop loss is the minimum level of liquidity that warrants opened positions in a given security. In operations risk, the stop loss is a set of conditions according to which a particular operational aspect is reviewed and terminated, if necessary. For example, compromised Internet security may mandate a complete shutdown of trading operations until the issue is resolved. Finally, a stop loss in legal risk can be a settlement when otherwise incurred legal expenses are on track to exceed the predetermined stop-loss level.

In market risk management, a simple stop loss defines a fixed level of the threshold price. For example, if at 12:00 P.M. EST we bought USD/CAD at 1.2000 and set a simple stop loss at 50 bps, the position will be liquidated whenever the level of USD/CAD drops below 1.1950, provided the position is not closed sooner for some other reason. Figure 17.4, panel (a) illustrates the idea. A simple stop loss does not take into account any price



Panel (a). Simple (fixed) stop loss. Panel (b). Trailing stop loss.

FIGURE 17.4 The difference between simple (fixed) and trailing stop-loss thresholds.

movement from the time the position was open until the time the stop loss was triggered, resulting in a realized loss of 50 bps.

A trailing stop, on the other hand, takes into account the movements of the security's market price from the time the trading position was opened. As its name implies, the trailing stop "trails" the security's market price. Unlike the simple stop that defines a fixed price level at which to trigger a stop loss, the trailing stop defines a fixed stop-loss differential relative to the maximum gain attained in the position. For example, suppose we again bought USD/CAD at 12:00 P.M. EST at 1.2000 and set a trailing stop loss at 50 bps. Suppose further that by 12:15 P.M. EST the market price for USD/CAD rose to 1.2067, but by 13:30 P.M. EST the market price dropped down to 1.1940. The trailing stop loss would be triggered at 50 bps less the market price corresponding to the highest local maximum of the gain function. In our example, the local maximum of gain appeared at 1.2067 when the position gain was $1.2067 - 1.2000 = 0.0067$. The corresponding trailing stop loss would be hit as soon as the market price for USD/CAD dipped below 1.2067 –50 bps = 1.2017, resulting in a realized profit of 17 bps, a big improvement over performance with a simple stop loss. Figure 17.4, panel (b) shows the process.

How does one determine the appropriate level of the stop-loss threshold? If the stop-loss threshold is too narrow, the position may be closed due to short-term variations in prices or even due to variation in the bid-ask spread. If the stop-loss threshold is too wide, the position may be closed too late, resulting in severe drawdowns. As a result, many trading practitioners calibrate the stop-loss thresholds to the intrinsic volatility of the traded security. For example, if a position is opened during high-volatility conditions with price bouncing wildly, a trader will set wide stop losses. At

the same time, for positions opened during low-volatility conditions, narrow stop thresholds are required.

The actual determination of the stop-loss threshold based on market volatility of the traded security is typically calibrated with the following two factors in mind:

1. Average gain of the trading system without stop losses in place, $E[G]$
2. Average loss of the trading system without stop losses in place, $E[L]$

The probabilities of a particular position turning out positive also play a role in determining the optimal stop-loss threshold, but their role is a much smaller one than that of the averages. The main reason for the relative insignificance of probabilities of relative occurrence of gains and losses is that per the Gambler's Ruin Problem, the probability of a gain must always exceed the probability of a loss on any given trade; otherwise, the system faces the certainty of bankruptcy. Please refer to Chapter 10 for details.

The information on average upside and downside is typically determined from the system's back test. The back test normally spans at least two years of data and produces a sample return distribution with a number of observations sufficient to draw unbiased inferences about the distribution of the return population with the use of the techniques such as VaR, tail parameterization, or benchmarked performance measurement, discussed subsequently. Armed with distributional information about returns of the trading system, we can estimate the maximum (trailing) loss allowed that would keep our system consistently positive. The maximum trailing stop loss, L_M, has to satisfy the following three conditions:

1. In absolute terms, the maximum loss is always less than the average gain, $|L_M| < E[G]$; otherwise, the system can produce negative cumulative results.
2. Also in absolute terms, the maximum loss is always less than the average loss, $|L_M| \leq |E[L]|$; otherwise, the system can deliver almost identical results with no stop losses.
3. After the introduction of stop losses, the probability of a gain still exceeds the probability of a loss in a back test.

Once the maximum stop loss is determined, the stop loss can be further refined to tighten dynamically in response to different volatility conditions. Dynamic calibration of stop losses to market volatility is more art than science. Dacorogna et al. (2001), for example, describe a moving average–based model with stop losses of low-volatility and high-volatility

regimes. Dacorogna et al. (2001) use the absolute value of the gain or loss as a proxy for "volatility" and consider "volatility" to be low if the absolute gain or loss is less than 0.5 percent (50 bps). The model thresholds change in accordance with the volatility conditions. In Dacorogna et al. (2001), for example, the thresholds increase 10 times their low-volatility value when the "volatility" defined previously exceeds 0.5 percent. The low-volatility parameter is calibrated in the back test on the historical data.

The stop-loss thresholds in other types of risk are similarly determined based on the expected market gain and total maximum loss considerations presented previously.

Hedging Portfolio Exposure

Hedging is closely related to portfolio optimization: the objective of any hedging operation is to create a portfolio that maximizes returns while minimizing risk—downside risk in particular. Hedging can also be thought of as a successful payoff matching: the negative payoffs of one security "neutralized" by positive payoffs of another.

Hedging can be passive or dynamic. Passive risk hedging is most akin to insurance. The manager enters into a position in a financial security with the risk characteristics that offset the long-term negative returns of the operation. For example, a manager whose main trading strategy involves finding fortuitous times for being long in USD/CAD may want to go short in the USD/CAD futures contract to offset his exposure to USD/CAD. As always, detailed analysis of the risk characteristics of the two securities is required to make such a decision.

Dynamic hedging is most often done through a series of short-term, potentially overlapping, insurance-like contracts. The objective of the short-term insurance contracts is to manage the short-term characteristics of trading returns. In the case of market risk hedging, dynamic hedging may be developed for a particular set of recurring market conditions, when behaviors of the trading systems may repeat themselves. It may be possible to find a set of financial securities or trading strategies the returns of which would offset the downside of the primary trading strategy during these particular market conditions. For example, during a U.S. Fed announcement about the level of interest rates, the USD/CAD exchange rate is likely to rise following a rise in the U.S. interest rates, while U.S. bond prices are likely to fall following the same announcement. Depending upon return distributions for USD/CAD and U.S. bonds, it may make sense to trade the two together during the U.S. interest rate announcements in order to offset the negative tail risk in either. Mapping out extensive distributions of returns as described previously in this chapter would help in determining the details of such a dynamic hedging operation.

As with mean-variance portfolio construction, a successful hedging strategy solves the following optimization problem:

$$\begin{aligned} &\max\ x\,E[R] - A\,x'Vx \\ &s.t.\ \sum x_i = 1 \end{aligned} \tag{17.11}$$

where x_i is the portfolio weight of security i, $i \in [1, \ldots, I]$, $E[R]$ is a vector of expected returns of I securities, V is an $I \times I$ variance-covariance matrix of returns, and A is the coefficient reflecting the risk aversion of the trading operation. A is commonly assumed to be 0.5 to simplify the solution. A dynamic state-dependent hedging would repeat the process outlined in equation (17.11), but only for returns pertaining to a specific market state.

Like market risk, other types of risk can be diversified through portfolio risk management. The counterparty risk, for example, can be diversified through establishment of several ongoing broker-dealer relationships. Citi prime brokerage even markets itself as a secondary broker-dealer for funds already transacting, or "priming," with another broker-dealer.

Similarly, liquidity risk can be diversified away through using several liquidity providers. The American Academy of Actuaries (2000) provided the following guidance for companies seeking to diversify their liquidity exposure: "While a company is in good financial shape, it may wish to establish durable, ever-green (i.e., always available) liquidity lines of credit. The credit issuer should have an appropriately high credit rating to increase the chances that the resources will be there when needed." According to Bhaduri, Meissner, and Youn (2007), five derivative instruments can be specifically used for hedging liquidity risk:

- Withdrawal option is a put on the illiquid underlying asset.
- Bermudan-style return put option is a right to sell the underlying asset at a specified strike on specific dates.
- Return swap allows swapping the return on the underlying asset for LIBOR.
- Return swaption is an option to enter into the return swap.
- Liquidity option is a "knock-in" barrier option, where the barrier is a liquidity metric.

Regular process reviews ensure that the operations are running within predetermined guidelines on track to set goals, minimizing the probability of failure of oversight, regulatory breaches, and other internal functions. For example, the addition of new trading strategies into the trading portfolio should undergo rigid product review processes that analyze the return and risk profiles and other profitability and risk factors of proposed capital allocations, as described in Chapter 5. In addition to detailed process

guidelines, operational risk can be hedged with insurance contracts offered by specialty insurance firms and by entirely outsourcing noncritical business processes to third-party vendors.

Legal risk is most effectively managed via the causal analysis used for its measurement. The key risk indicators are continuously monitored, and the effect of their changes is assessed according to the causal framework developed in the estimation of legal risk.

CONCLUSION

This chapter has examined the best practices in risk management followed by successful high-frequency operations. While the process of identification, measurement, and management of risk can consume time and effort, the process pays off by delivering business longevity and stability.

CHAPTER 18

Executing and Monitoring High-Frequency Trading

Once a high-frequency trading system is designed and back-tested, it is applied to live capital (i.e., executed). The execution process can be complex, particularly as the capital allocated to the strategy grows and the adverse cost of market impact begins to take effect. To maximize trading performance and minimize costs, the best high-frequency trading systems are executed through optimization algorithms. To ensure that all algorithms of the trading system work as intended, a strict monitoring process is deployed.

This chapter discusses the best contemporary practices in the execution and monitoring of high-frequency trading systems.

Execution optimization algorithms tackle the following questions:

- Should a particular order issued by the trading strategy be executed in full or in smaller lots?
- Should the order be optimally processed as a market or a limit order?
- Is there an order-timing execution strategy that delivers a better-than-expected order fill price, given current market conditions?

The optimization algorithms can be developed internally or purchased off the shelf. Off-the-shelf algorithms are often cheaper, but they are less transparent than internally developed platforms. Both external and internal execution optimization systems, however advanced, may possess unexpected defects and other skews in performance and result in costly execution blunders.

To detect undesirable shifts in costs and other trading parameters during execution, all execution processes must be closely monitored. Even the

most miniscule problems in execution may have fast and dramatic effects on performance; timely identification of potential issues is a nonnegotiable necessity in high-frequency operations.

EXECUTING HIGH-FREQUENCY TRADING SYSTEMS

Overview of Execution Algorithms

Optimization of execution is becoming an increasingly important topic in the modern high-frequency environment. Before the introduction of computer-enabled trading optimization algorithms, investors desiring to trade large blocks of equity shares or other financial instruments may have hired a broker-dealer to find a counterparty for the entire order. Subsequently, broker-dealers developed "best execution" services that split up the order to gradually process it with limited impact on the price. The advent of algorithmic trading allowed institutional traders to optimize trading on their own, minimizing the dominance of broker-dealers and capturing a greater profit margin as a result.

Optimization algorithms take into account a variety of current market conditions as well as characteristics of the orders to be processed: order type, size, and frequency. Bertsimas and Lo (1998) developed optimization strategies to take advantage of contemporary price changes. Engle and Ferstenberg (2007) examined the risks embedded in execution. Almgren and Chriss (2000) and Alam and Tkatch (2007), among others, studied the effects of "slicing" up orders into batches of smaller size. Obizhaeva and Wang (2005) optimize execution, assuming that post-trade liquidity is not replenished immediately. Kissell and Malamut (2006) adapt the speed of order processing to traders' current beliefs about the impending direction of market prices.

In addition to algorithms optimizing the total execution cost of trading, algorithms have been developed to optimize liquidity supply, hedge positions, and even to optimize the effort extended in monitoring position changes in the marketplace. See Foucault, Roell and Sandas (2003) for an example of the latter. In this chapter, we consider three common forms of executing optimization algorithms:

1. Trading-aggressiveness selection algorithms, designed to choose between market and limit orders for optimal execution
2. Price-scaling strategies, designed to select the best execution price according to the prespecified trading benchmarks

3. Size-optimization algorithms that determine the optimal ways to break down large trading lots into smaller parcels to minimize adverse costs (e.g., the cost of market impact)

Market-Aggressiveness Selection

Aggressive execution refers to high trading frequency and to short trading intervals that may lead to high market impact. Aggressive execution most often skews toward the heavy use of market orders. Passive trading, on the other hand, is lower frequency, depends more on limit orders, but may be subject to non-execution risk should the market move adversely. To balance passive and aggressive trading, Almgren and Chriss (1999) propose the following optimization:

$$\min_{\alpha} \ \text{Cost}(\alpha) + \lambda \ \text{Risk}(\alpha) \tag{18.1}$$

where α is the trading rate often calculated as a percentage of volume (POV) or liquidity that the strategy absorbs during the trading period and λ is the coefficient of risk aversion of the investor. Plotting cost/risk profiles for various algorithms identifies efficient trading frontiers that are well-suited for comparisons of algorithm efficiencies and for determining the suitability of a particular algorithm to the trading needs of a particular investor.

According to Kissell and Malamut (2005), market aggressiveness (POV or α) can be varied using a combination of market and limit orders. Market orders tend to increase the POV or α, whereas limit orders decrease market aggressiveness.

The cost and risk functions used in the optimization equation (18.1) are defined as follows:

$$\text{Cost}(\alpha) = E_0 \left[P(\alpha) - P_b \right] \tag{18.2}$$

$$\text{Risk}(\alpha) = \sigma(\varepsilon(\alpha)) \tag{18.3}$$

$$\text{P}(\alpha) = P + f(X, \alpha) + g(X) + \varepsilon(\alpha) \tag{18.4}$$

where E_0 denotes the ex-ante expectation at the start of the trading period,
P_b is the benchmark execution price,
$\text{P}(\alpha)$ is the realized execution price defined in equation (18.4),
$\varepsilon(\alpha)$ is a random deviation of the trading outcome, $E[\varepsilon(\alpha)] = 0$, $\text{Var}[\varepsilon(\alpha)] = \sigma^2(\alpha)$.

P is the market price at the time of order entry,
$f(X, \alpha)$ is a temporary market impact due to the liquidity demand of trading, and
$g(X)$ is the permanent price impact due to information leakage during order execution.

Price-Scaling Strategies

The main objective of so-called price-scaling execution algorithms is to obtain the best price for the strategy. The best price can be attained relative to a benchmark—for example, the average daily price for a given security. The best price can also be attained given the utility function of the end investor or a target Sharpe ratio of a portfolio manager.

The algorithm that minimizes the cost of execution relative to a benchmark is known as a Strike algorithm. The Strike is designed to capture gains in periods of favorable prices; the algorithm is aggressive (executes at market prices) in times of favorable prices and passive (places limit orders) in times of unfavorable prices. The Strike strategy dynamically adjusts the percent of volume rate α used to process market orders of the strategy to minimize the quadratic execution cost of the strategy:

$$\min_{\alpha_t} E_t \left[P_{t+1}(\alpha_t) - P_{b,t} \right]^2 \tag{18.5}$$

where $P_{t+1}(\alpha_t)$ is the realized price obtained using the trading aggressiveness level α_t decided upon at time t, and $P_{b,t}$ is the benchmark price at time t used to compare the trading performance.

The Plus algorithm maximizes the probability of outperforming a specified benchmark while minimizing risk. To do so, the algorithm maximizes the following Sharpe ratio–like specification:

$$\max_{\alpha_t} \frac{E_t \left[P_{t+1}(\alpha_t) - P_{b,t} \right]}{\left(V(P_{t+1}(\alpha_t) - P_{b,t}) \right)^{1/2}} \tag{18.6}$$

where, as before, $P_{b,t}$ is the benchmark price at time t used to compare the trading performance, and $P_{t+1}(\alpha_t)$ is the realized price obtained using the trading aggressiveness level α_t decided upon at time t.

Finally, the Wealth algorithm maximizes investor wealth in the presence of uncertainty. The Wealth algorithm is passive during periods of favorable prices, but acts aggressively during periods of unfavorable prices with the goal of preserving the investor's wealth in adverse conditions. The Wealth strategy is obtained by optimizing the following expression:

$$\max_{\alpha_t} \log E_t \left[U(P_{t+1}(\alpha_t)) \right] \tag{18.7}$$

where $U(.)$ is a utility function approximating the risk-return preferences of the investor. The utility function may be the one shown in equation (18.8):

$$U(x) = E[x] - \lambda V[x] \tag{18.8}$$

where x is the realized payoff and λ is the risk aversion coefficient of the investor. The risk-aversion coefficient λ is 0 for a risk-neutral investor, or an investor insensitive to risk. A risk-averse investor will have λ greater than 0; λ of 0.5 and above would characterize a highly risk-averse investor.

The profitability of execution algorithms depends on concurrent market conditions. Kissell and Malamut (2005) compared the three execution strategies in detail and found that all three strategies consistently outperform random, nonsystematic execution. Among the algorithms, the Strike method delivers a lower average cost but ignores participation in favorable price conditions. The Plus strategy also delivers a low average cost, but increases the risk of unfavorable prices. Finally, the Wealth strategy is able to capture a greater proportion of favorable price conditions but at the expense of higher average prices.

Slicing Large Orders

Kyle (1985) and Admati and Pfleiderer (1988) were the first to suggest that for informed investors to profit from their information, they need to trade in a fashion that precludes other market participants from recognizing the informed investors' order flow. Should other investors recognize the order flow of informed investors, they could front-run the informed parties, diluting their profitability. Barclay and Warner (1993) argue that for institutions to trade with their positions undetected, their large order packets need to be broken up into parcels of medium size—not too big and not too small—in order to minimize other trading participants' ability to distinguish these orders from other, "noise," orders. Chakravarty (2001) studies the impact of stealth trading—that is, trading by breaking down large trading blocks into small order parcels with the intent of causing the least market impact. Chakravarty (2001) finds that, consistent with the hypotheses of Barclay and Warner (1993), medium-sized orders indeed are followed by disproportionally large price changes, relative to all price and overall proportion of trades and volume.

Alam and Tkatch (2007) analyzed data from the Tel-Aviv Stock Exchange to study the performance of institutional investors who slice their orders into blocks of equal size in order to avoid being detected and picked off by other traders. Alam and Tkatch (2007) detect these orders as groups of equally sized, equally priced same-direction orders placed within two

minutes of each other. Alam and Tkatch (2007) report that sliced orders have a median of four "slices" or consecutively streaming components.

Out of all the slice orders submitted, about 79 percent are executed and 20 percent are canceled by the trader prior to execution. The execution rate of slice orders compares favorably with the execution rate of all orders; only 63 percent of all orders, including sliced and non-sliced orders, are executed.

Another metric of slice efficiency is order fill rate. The order fill rate measures the proportion of the order that was "hit" or executed. Completely executed orders have a fill rate of 100 percent; the order that failed to execute has a fill rate of 0 percent. Regular, non-sliced, orders may encounter a partial fill, depending on the order size. Alam and Tkatch (2007) show that non-sliced orders have a fill rate of 40 percent, while sliced orders have a fill rate of 48 percent. Slicing particularly improves the fill rate of limit orders; regular limit orders have a fill rate of 42 percent, while sliced limit orders have a fill rate of 77 percent.

Sliced orders are executed more quickly. Alam and Tkatch (2007) report that the mean execution time for a fully filled sliced order is 3 minutes and 29 seconds, while the mean execution time for a regular order is 11 minutes and 54 seconds.

Execution is costly not only in terms of the average transaction costs but in terms of risks associated with execution. The risk embedded in execution comprises primarily two types of risk: (1) the uncertainty of the price at which market orders are executed and (2) the uncertainty in the timing of the execution of limit orders and the associated opportunity cost. Extreme examples of such costs include the possible failure to execute a limit order and an insufficient market depth at a reasonable range of prices for market order execution.

Execution risk creates an additional dimension for portfolio risk/return optimization and has to be taken into account. Engle and Ferstenberg (2006) propose that the study of possible execution risks is necessary to determine the following aspects of portfolio management:

- Is risk-neutral portfolio management optimal in the presence of execution risks?
- Is execution risk diversifiable in a portfolio of several financial instruments?
- Can execution risk be hedged?

Instead of executing the total order size at the same time, institutions employ strategies to minimize market impact by, for example, splitting the total order size into discrete blocks executed over time, often several days. The identification of impending trading periods with extensive liquidity, therefore, becomes an important problem for optimization of

execution. Several recent studies have characterized properties of liquidity that may assist managers in forecasting liquidity going forward; specifically, liquidity has been shown to be time varying, yet persistent from one period to the next. These studies include those of Chordia, Roll, and Subrahmanyam (2001, 2002); Hasbrouck and Seppi (2001); and Huberman and Halka (2001).

Obizhaeva and Wang (2005) analytically derive optimal execution sizes depending on the execution horizon of the trade and the "speed of recovery" of the limit order book for a given security. The speed of recovery is a measure of how fast the limit order book absorbs the market impact generated by the previous lot in the execution sequence. Obizhaeva and Wang (2005) find that for securities with a reasonable speed of limit order book recovery, the optimal trading strategy is to process large lots at the beginning and at the end of the execution period with small lots spaced in between. The spacing of smaller lots depends on whether the speed of recovery for the traded security is uniform throughout the day. If the speed of recovery is not uniform throughout the day, larger lots should be processed at times with higher speeds of recovery.

Nevmyvaka, Kearns, Papandreou, and Sycara (2006) have developed an algorithm for optimizing execution through a dynamic combination of market and limit orders. The optimization is focused on a specific task: to acquire V shares of a particular financial security within T seconds of the order. The authors compare the following three market and limit order execution scenarios to obtain a certain number of shares, V:

1. Submit a market order for V shares immediately at the beginning of the trading period, time 0. This approach guarantees execution, but the liquidity required to fulfill the order may be costly; the trader may need to explore the depth of the book at suboptimal prices and wide bid-ask spreads.
2. Wait until the end of the trading period and submit a market order for V shares at time T. This strategy may improve upon the obtained price, but it is also subject to market volatility risks. Full bid-ask spread is present.
3. Submit a limit order for V shares at the beginning of the trading period (time 0) and a market order for the unexecuted shares (if any) at the end of the trading period (time T). This strategy avoids paying bid-ask spread if the limit order is executed. The worst-case outcome of this strategy is that presented in case 2.

In all three scenarios, the trading period ends with the same number of shares, V. In each scenario, however, the V shares can potentially be obtained at a different cost.

Nevmyvaka, Kearns, Papandreou, and Sycara (2006) found that the best strategy is strategy 3, with limit orders placed at the beginning of the trading period and besting the market price by one tick size. For example, if we want to buy 500 shares of IBM within 300 seconds, the current market bid and offer prices are $93.63 and $93.67, and the minimum tick size is $0.01, the optimal strategy will be to submit a limit buy order at $93.64, one tick better than the best limit buy currently available on the market. The unfilled portion of the order is then executed at market at the end of the 300-second period.

MONITORING HIGH-FREQUENCY EXECUTION

Monitoring high-frequency execution involves a two-part process:

- First, allowable ranges of trading and other run-time parameters are identified through pre-trade analysis.
- Next, the run-time performance is continuously compared to the pre-trade estimates; the decisions to shut down the system are made in cases when the run-time parameters breach pre-trade guidelines.

The sections that follow detail the key considerations in pre-trade analysis and run-time monitoring.

Pre-Trade Analysis

Pre-trade analysis is designed to accomplish the following objectives:

- Estimate expected execution costs given current market conditions.
- Estimate expected execution risks:
  - The risk of non-execution at a desired price
  - The risk of non-execution due to insufficient liquidity
  - The risk of non-execution due to system breakdown

The estimates are then included in the determination of run-time stop-gain and stop-loss parameters.

Solid high-frequency systems specify and monitor the following micro-level deviations:

- Allowable versus realized deviations in price of the traded instrument
- Allowable versus realized deviations in market volume or security volume
- Maximum allowable versus realized trade duration

Monitoring Run-Time Performance

High-frequency trading is particularly vulnerable to deviations of trading behavior from the expected norm. Even the smallest deviations in trading costs, for example, can destroy the profitability of high-frequency trading strategies capturing minute bursts of price movements. As a result, run-time monitoring of trading conditions is critical to successful implementation of high-frequency strategies.

Monitoring trading performance can be delegated to a designated human trader armed with an array of dynamically updated performance gauges. Kissell and Malamut (2005) list the following metrics of trading performance as desirable tools for performance monitoring:

1. Allowable deviation in the price of the traded instrument from the target execution price ensures that the execution is suspended whenever the market impact costs become too high for the strategy to remain profitable. For example, a strategy with an average net per-trade gain of 5 bps or pips can sustain the maximum market impact costs of 4 bps or pips. A market impact cost of 5 bps or more renders the strategy unprofitable.
2. Processing market orders in high-volume conditions limits the market impact of the strategy and increases profitability. Specifying the minimum level of volume allowable to run the strategy caps market impact costs.
3. The longer the limit orders have been outstanding, the higher is the probability that the market price has moved away from the limit order prices, increasing the risk of non-execution. Specifying the maximum allowable duration of orders reduces the risk of non-execution: if a limit order is not executed within the prespecified time period, the order is either canceled or executed at market.

CONCLUSION

Successful execution is key to ensuring profitability of high-frequency strategies. Various algorithms have been developed to optimize execution. Furthermore, a human trader tasked with observing the trading parameters should have strict directions for termination of outstanding positions. Such oversight ensures smooth operation and swift reaction to disruptive and potentially costly events.

CHAPTER 19

Post-Trade Profitability Analysis

Trading costs can make and break the profitability of a high-frequency trading strategy. Transaction costs that may be negligible for long-term strategies are amplified dramatically in a high-frequency setting.

If market movements are compared to ocean wave patterns, long-term investment strategies can be thought of as surfers riding the trough to crest waves. High-frequency strategies are like pebbles thrown parallel to the ocean floor and grazing small ripples near the shore. Small changes in the wave pattern do not make a significant difference in the surfer's ability to tame large waves. On the other hand, a minute change in the wave structure can alter the pebble's trajectory. The smaller the pebble, the higher the influence of the wave shape, size, and speed. Transaction costs can be thought of as the market wave properties barely perceivable to the low-frequency strategies seeking to ride large market movements. At the same time, transaction costs substantially affect the profitability of high-frequency trades, seeking to capture the smallest market ripples. This chapter focuses on the transparent and latent costs that impact high-frequency trading. The roles of inventory and liquidity on the structure of a market and on realized execution are discussed, as are order slicing and other trading-optimization techniques that allow traders to obtain the best price. In addition to identification and management of trading costs, the chapter also reviews common approaches to analyzing post-trade performance.

Post-trade analysis has two parts:

1. **Cost analysis**—realized execution costs for all live trading strategies
2. **Performance analysis**—execution performance relative to a benchmark.

Post-trade analyses can be run after each trade, as well as at the end of each trading day. The analyses are often programmed to start and run automatically and to generate consistent daily reports. The reports are generated for each trading strategy and are studied by every portfolio manager or strategist and every trader, if any are involved in the execution process. Cost analysis and benchmarking analysis are discussed in the sections that follow.

POST-TRADE COST ANALYSIS

Analysis of execution costs begins with identification and estimation of costs by type and as they are incurred in an individual trade, in a trading strategy, by a portfolio manager, or by an execution trader. Execution costs are the trading fees or commissions paid by either the buyer or the seller but not received by the buyer or the seller. A novice may assume that trading costs comprise only the broker commissions and exchange fees. In reality, most trades incur at least nine types of cost, most of which are not observable directly and require a rigorous estimation process. The most common execution costs are the following:

- Transparent execution costs:
  - Broker commissions—fixed and variable components
  - Exchange fees
  - Taxes
- Latent execution costs:
  - Bid-ask spread
  - Investment delay
  - Price appreciation
  - Market impact
  - Timing risk
  - Opportunity cost

Costs known prior to trading activity are referred to as "transparent" or "explicit," and costs that have to be estimated are known as "latent" or "implicit." According to Kissell and Glantz (2003), while the transparent

costs are known with certainty prior to trading, latent costs can only be estimated from the costs' historical distribution inferred from the data of past trades. The goal of estimating latent cost values is to remove the pre-trade uncertainty about these costs during execution. Once all applicable execution costs have been identified and estimated, cost information is relayed to the trading team to find ways to deliver better, more cost-efficient execution. At a minimum, the cost analysis should produce cost estimates in the format shown in Table 19.1. The mechanics of identification and estimation of each type of execution cost are described in the following sections.

Transparent Execution Costs

Broker Commissions Brokers charge fees and commissions to cover the costs of their businesses, which provide connectivity to different exchanges and inter-dealer networks. Broker commissions can have both fixed and variable components. The fixed component can be a flat commission per month or a flat charge per trade, often with a per-trade minimum. The variable component is typically proportional to the size of each trade, with higher trade sizes incurring lower costs.

Brokers set custom price schedules to differentiate their businesses. The differences in cost estimates from one executing broker to another can be significant, because some brokers may quote lower fees for a particular set of securities while charging premium rates on other trading instruments.

Broker commissions may also depend on the total business the broker receives from a given firm, as well as on the extent of "soft-dollar" transactions that the broker provides in addition to direct execution services. Brokers' commissions typically cover the following services:

- Trade commissions
- Interest and financing fees
- Market data and news charges
- Research
- Other miscellaneous fees

Some broker-dealers may charge their customers additional fees for access to streaming market data and other premium information, such as proprietary research. Others may charge separately for a host of incremental miscellaneous fees.

Broker commissions generally come in two forms—bundled and unbundled. Bundled commissions are fixed all-in prices per contract and may include the fees of the exchanges through which equity, futures, or commodity trades are executed. For example, a fixed bundled fee can be USD

TABLE 19.1 A Sample Cost Reporting Worksheet

Metric	Financial Security	Strategy/ Portfolio Manager	Executing Broker	Characteristics			
				Mean	Std Dev	Skewness	Kurtosis
Broker Fees and Commissions							
Exchange Fees							
Taxes							
Bid-Ask Spread							
Investment Delay							
Price Appreciation							
Market Impact							
Timing Risk							
Opportunity Cost							

0.10 per stock share. The unbundled fees account for exchange fees and broker commissions separately. Since exchanges charge different rates, the unbundled fee structures allow investors to minimize the commissions they pay. Equity brokers charge USD 0.0001 to USD 0.003 commissions per share of stock traded through them in addition to the exchange fees, discussed in the following section. Similarly, in foreign exchange, some broker-dealers offer "no commission" platforms by pricing all costs in the increased bid-ask spreads. Others go to the opposite extreme and price all trades according to the "unbundled" list of minute trade features.

Broker-dealers also differ on the interest they pay their clients on cash accounts as well as on the financing fees they charge their clients for services such as margin financing and other forms of leverage. The cash account is the portion of the total capital that is not deployed by the trading strategy. For example, if the total size of the account a firm custodies with a broker-dealer is $100,000,000, and out of this amount one actively trades only $20,000,000, the remaining $80,000,000 remains "in cash" in the account. Brokers typically use this cash to advance loans to other customers. Brokers pay the cash account owners interest on the passive cash balance; the interest is often the benchmark rate less a fraction of a percent. The benchmark rate is typically the Fed Funds rate for the USD-denominated cash accounts and the central-bank equivalents for deposits in other currencies. A sample rate may be quoted as LIBOR minus 0.1 percent, for example. Brokers usually charge the benchmark rate plus a spread (0.05 percent – 1 percent) for financing borrowing investors' leverage and generate income on the spread between their borrowing and lending activities. The spread ideally reflects the creditworthiness of the borrower.

Broker commissions are negotiated well in advance of execution. Detailed understanding of broker commission costs allows optimization of per-order cost structures by bundling orders for several strategies together or by disaggregating orders into smaller chunks.

Exchange Fees Exchanges match orders from different broker-dealers or electronic communication networks (ECNs) and charge fees for their services. The core product of every exchange is the inventory of open buy and sell interest that traders are looking to transact on the exchange. To attract liquidity, exchanges charge higher fees for orders consuming liquidity than for orders supplying liquidity. In an effort to attract liquidity, some exchanges go as far as paying traders that supply liquidity, while charging only the traders that consume liquidity.

Liquidity is created by open limit orders; limit buy orders placed at prices below the current ask provide liquidity, as do limit sell orders placed at prices above the current bid. Market orders, on the other hand, are matched immediately with the best limit orders available on the exchange,

consuming liquidity. Limit orders can also consume liquidity; a limit buy placed at or above the market ask price will be immediately matched with the best available limit sell, thus removing the sell order from the exchange. Similarly, a limit sell placed at or below the market bid price will be immediately matched with the best available bid, as a market sell would.

Like broker commissions, exchange fees are negotiated in advance of execution.

Taxes According to Benjamin Franklin, "In this world nothing can be said to be certain, except death and taxes." Taxes are charged from the net profits of the trading operation by the appropriate jurisdiction in which the operation is domiciled. High-frequency trading generates short-term profits that are usually subject to the full tax rate, unlike investments of one year or more, which fall under the reduced-tax capital gains umbrella in most jurisdictions. A local certified or chartered accountant should be able to provide a wealth of knowledge pertaining to proper taxation rates. Appropriate tax rates can be determined in advance of trading activity.

Latent Execution Costs

Bid-Ask Spreads A bid-ask spread is the price differential between the market bid (the highest price at which market participants are willing to buy a given security) and the market ask (the lowest price at which the market participants agree to sell the security). Most commonly, the bid-ask spread compensates the market participants for the risk of serving as counterparties and cushions the impact of adverse market moves. A full discussion of the bid-ask spread is presented in detail in Chapters 6, 9, and 10.

Bid-ask spreads are not known in advance. Instead, they are stochastic or random variables that are best characterized by the shape of the distribution of their historical values. The objective of the cost analysis, therefore, is to estimate the distributions of the bid-ask spreads that can be used to increase the accuracy of bid-ask spread forecasts in future simulations and live trading activity.

To understand the parameters of a bid-ask distribution, the trader reviews key characteristics of bid-ask spreads, such as their mean and standard deviation. Approximate locations of the spreads based on historical realizations are made by computing statistical characteristics of spreads grouped by time of day, market conditions, and other factors potentially affecting the value of the spread.

Investment Delay Costs The cost of investment delay, also referred to as the latency cost, is the adverse change in the market price of the

traded security that occurs from the time an investment decision is made until the time the trade is executed. The following example illustrates the concept of the investment delay cost. The trading strategy identifies a stock (e.g., IBM) to be a buy at $56.50, but by the time the market buy order is executed, the market price moves up to $58.00. In this case, the $1.50 differential between the desired price and the price obtained on execution is the cost of the investment delay.

In systematic high-frequency trading environments, investment delay costs are generated by the following circumstances:

1. Interruptions in network communications may disrupt timely execution and can delay transmission of orders.
2. The clearing counterparty may experience an overload of simultaneous orders, resulting in an order-processing backlog and subsequent delay in execution. Such situations most often occur in high-volatility environments. In the absence of large-scale disruptions, delays due to high trading volume can last for up to a few seconds.

The cost of investment delays can range from a few basis points in less volatile markets to tens of basis points in very liquid and volatile securities such as the EUR/USD exchange rate. The investment delay costs are random and cannot be known with precision in advance of a trade. Distribution of investment delay costs inferred from past trades, however, can produce the expected cost value to be used within the trading strategy development process.

While the investment delay costs cannot be fully eliminated, even with current technology, the costs can be minimized. Backup communication systems and continuous human supervision of trading activity can detect network problems and route orders to their destinations along alternative backup channels, ensuring a continuous transmission of trading information.

Price Appreciation Costs The price appreciation cost refers to the loss of investment value during the execution of a large position. A position of considerable size may not be immediately absorbed by the market and may need to be "sliced" into smaller blocks.[1] The smaller blocks are then executed one block at a time over a certain time period. During execution,

[1]Chan and Lakonishok (1995), for example, show that if a typical institutional trade size were executed all at once, it would account for about 60 percent of the daily trading volume, making simultaneous execution of the order expensive and difficult, if not impossible.

the price of the traded security may appreciate or depreciate as a result of natural market movements, potentially causing an incremental loss in value. Such loss in value is known as price appreciation cost and can be estimated using information on past trades. The price appreciation cost is different from the market impact cost, or the adverse change in price generated by the trading activity itself, discussed subsequently.

For an example of the price appreciation cost, consider the following EUR/USD trade. Suppose that a trading strategy determines that EUR/USD is undervalued at 1.3560, and a buy order of $100 million EUR/USD is placed that must be executed over the next three minutes. The forecast turns out to be correct, and EUR/USD appreciates to 1.3660 over the following two minutes. The price appreciation cost is therefore 50 bps per minute. Note that the price appreciation cost is due to the fundamental appreciation of price, not the trading activity in EUR/USD.

Market Impact Costs Market impact cost measures the adverse change in the market price due to the execution of a market order. More precisely, the cost of market impact is the loss of investment value caused by the reduction in liquidity following market order–driven trades.

Every market order reduces available liquidity and causes a change in the price of the traded security. A market buy order reduces the available supply of the security and causes an instantaneous appreciation in the price of the security. A market sell order decreases the demand for the security and causes an instantaneous depreciation in the price of the security.

The market impact may be due to the imbalances in inventories created by the order, to the order pressures on the supply or demand, or to the informational content of the trades signaling an undervalued security to other market participants. Market impact is most pronounced when large orders are executed. Breaking orders into smaller, standard-size "clips" or "rounds" has been shown to alleviate the market impact. The properties of market impact can be described as follows:

1. When the limit order book is not observable, ex-ante expectations of market impact are the same for buy and sell orders in normal trading conditions. In other words, in the absence of information it can with reasonable accuracy be assumed that the number of limit buys outstanding in the market equals the number of limit sells. However, if the limit order book can be observed, market impact can be calculated precisely based on the limit orders present in the order book by "walking" the order through the order book.
2. Market impact is proportional to the size of the trade relative to the overall market volume at the time the trade is placed.

3. Market impact due to inventory effects is transient. In other words, if any price appreciation following a buy order is due to our executing broker's "digestion" of the order and not to market news, the price is likely to revert to its normal levels after the executing broker has finished "digesting" the order. Whether the market impact cost is transient or permanent depends on the beliefs and actions of other market participants.
4. Market impact accompanies market orders only; limit orders do not incur market impact costs.
5. The informational content of market impact is canceled out by opposing orders.

In ideal market conditions, the market impact cost is measured as the difference in the market price of the security between two states of the market:

State 1—the order was executed; execution was initiated at time t_0, and the execution was completed at time t_1.
State 2—the order was not executed (the market was left undisturbed by the order from t_0 to t_1).

In real-life conditions, simultaneous observations of both the undisturbed market and the effects of the trade execution on the market are hardly feasible, and the true value of the market impact may not be readily available. Instead, according to Kissell and Glantz (2003), the market impact is estimated as the difference between the market price at t_0 and the average execution price from t_0 to t_1:

$$MI = P_0 - \frac{1}{N}\sum_{N} P_{\tau,n} \tag{19.1}$$

where MI stands for "market impact," P_0 is the market price immediately prior to execution at time t_0, N is the total number of trades required to process the entire position size from t_0 to t_1, and $P_{\tau,n}$ is the price at which the nth trade was executed at time τ, $\tau \in [t_0, t_1]$.

While the costs of market impact are difficult to measure both pre- and post-trade, market impact costs can be estimated as a percentage of the total market liquidity for a given security. The higher the percentage of market liquidity the strategy consumes, the higher the adverse price movement following the trades, and the higher the market impact cost incurred by subsequent trades in the same direction.

Consumed liquidity can be approximated as a percentage of the observed market volume that is directly due to market-order execution. Since

market orders are processed at the latest market prices, market orders consume available liquidity and create market impact costs that may make subsequent trades in the same direction more expensive. Limit orders, on the other hand, supply liquidity, are executed only when "crossed" by a market order, and generate little market impact at the time the order is executed. Limit orders, however, may fail to execute and present a significant risk in case the markets move adversely.

A combination of market and limit orders can help balance the costs of market impact with the risks of non-execution. The optimal proportion of market and limit orders may depend on the risk-aversion coefficient of the trading strategy: Almgren and Chriss (1999), for example, specify the market versus limit optimization problem as follows:

$$\min_{\alpha} MICost(\alpha) + \lambda Risk(\alpha) \tag{19.2}$$

where α is the trading rate calculated as a percentage of market volume due to market orders placed by the strategy, λ is the coefficient of risk aversion of the strategy, and *MICost* stands for the market impact cost function. As usual, a risk aversion of 0.5 corresponds to a strategy for a conservative wealth-preserving investor, while a risk aversion of 0 corresponds to a risk-neutral strategy that is designed to maximize returns with little consideration for risk. The optimization of equation (19.2) can be solved by plotting *MICost*/Risk profiles for various strategies; the resulting efficient trading frontier identifies the best execution strategies.

According to Kissell and Malamut (2005), market impact costs can also be optimized using dynamic benchmarking, often referred to as "price-scaling." For example, a "strike" price–scaling strategy dictates that there is an increase in the proportion of market orders whenever prices are better than the benchmark and a decrease in market orders whenever prices are worse than the benchmark. A feasible alternative strategy, known as the "wealth" strategy, posts limit orders during favorable prices and market orders during adverse market conditions to minimize exposure to the adverse changes in the traded security. A "plus" strategy maximizes the probability of outperforming a benchmark within a risk/return framework. Each of the price-scaling strategies is discussed in detail in Chapter 18.

In dark pools of liquidity and similar trading environments where the extent of the order book cannot be observed directly, Kissell and Glantz (2003) propose to estimate the cost of market impact using the following expression:

$$k(x) = \frac{I}{X} \sum_{j} \frac{x_j^2}{x_j + 0.5 v_j} \tag{19.3}$$

where I is the instantaneous market impact cost for security i, X is the order size for security i, x_j is the order size of the parcel of security i traded at time j (assuming that the total order was broken down into smaller parcels), and v_j is the expected volume for security i at time j. Equation (19.3) accounts for the trade size relative to the total inventory of the security; the smaller the size of an individual parcel order relative to the total market volume of the security, the smaller the realized market impact of the order. The 0.5 coefficient preceding the market volume at time j, v_j, reflects the naïve expectation of a balanced order book in the absence of better order book details; a guess of an equal number of buy and sell orders results in half the book being relevant for each trade parcel.

To estimate the ex-ante risk of the market impact for a portfolio of several securities due to be executed simultaneously, Kissell and Glantz (2003) compute liquidity risk as variance of the potential market impact as follows:

$$\sigma^2(k(x)) = \sum_i \left(\frac{I_i}{X_i}\right)^2 \sum_j \frac{x_{ij}^4 \sigma^2(v_{ij})}{4(x_{ij} + 0.5v_{ij})^4} \tag{19.4}$$

The term $\sigma^2(v_{ij})$ in equation (19.4) refers to expected variance in volume of security i at time j.

Other approaches, such as proposed by Lee and Ready (1991), are available for estimation of potential market depth and the corresponding market impact when the true market depth and market breadth values are not observable.

Timing Risk Costs Timing risk costs are due to random, unforecasted price movements of the traded security that occur while the execution strategy is waiting to pinpoint or "hit" the optimal execution price. The cost of timing risk describes by how much, on average, the price of the traded security can randomly appreciate or depreciate within 1 second, 10 seconds, 1 minute and so on from the time an investment decision is made until the market order is executed. The timing risk cost applies to active market timing activity, usually executed using market orders. The timing risk cost does not apply to limit orders. Timing risk captures several sources of execution uncertainty:

- Price volatility of the traded asset
- Volatility of liquidity of the asset
- Uncertainty surrounding the potential market impact of the order

Like other costs that are due to the price movements of the underlying security, timing risk costs can be estimated from historical trade data.

While the timing risk costs tend to average to zero, the costs nevertheless impact the risk profile of trading strategies with their volatility. The timing risk is modeled as a distribution, with the worst-case scenarios being estimated using the value-at-risk (VaR) framework.

Opportunity Costs The opportunity cost is the cost associated with inability to complete an order. Most often, opportunity cost accompanies limit order–based strategies, but it can also be present in market-order execution. The inability to fulfill an order can be due to one of several factors:

- The market price never crossed the limit price.
- The market did not have the liquidity (demand or supply) sufficient to fulfill the order at the desired price.
- The price moved away so quickly that fulfilling the order would render the transaction unprofitable, and the transaction was canceled as a result.
- The opportunity cost is measured as the profit expected to be generated had the order been executed.

Cost Variance Analysis

Cost variance analysis summarizes deviations of realized costs from the cost averages. The latest realized costs are compared against population distributions of previously recorded costs with matching transaction properties—same financial security, same strategy or portfolio manager, and same executing broker. Over time, cost variance analysis gives portfolio managers a thorough understanding of the cost process and improves the system's ability to manage trading costs during strategy run-time.

Suppose that a particular trade in USD/CAD driven by strategy i and executed by broker j generated cost ς_{ij}, and that the population mean and standard deviation for costs of all USD/CAD trades on record generated by the same strategy i and executed by the same broker j is represented by $\bar{\varsigma}_{ij}$ and $\sigma_{\varsigma,ij}$, respectively. Then the deviation of the realized cost from its population mean is $\Delta\varsigma_{ij} = \varsigma_{ij} - \bar{\varsigma}_{ij}$. Whenever the deviation of the realized cost from population mean falls outside one standard deviation,

$$\Delta\varsigma_{ij} \notin [\bar{\varsigma}_{ij} - \sigma_{\varsigma,ij}, \bar{\varsigma}_{ij} + \sigma_{\varsigma,ij}]$$

the reason for the deviation should be investigated and noted.

Often deviations can be due to unusual market conditions, such as an unexpected interest rate cut that prompts exceptional market volatility. High-cost conditions that occur independently of unusual market events may signal issues at the broker-dealer's and should be paid close attention.

Cost Analysis Summary

While transparent costs are readily measurable and easy to incorporate into trading models, it is the costs that are latent and unobservable directly that have the greatest impact on trading profitability, according to Chan and Lakonishok (1995) and Keim and Madhavan (1995, 1996, and 1998), among others. Understanding the full cost profile accompanying execution of each security improves the ability to successfully model trading opportunities, leading to enhanced profitability of trading strategies.

POST-TRADE PERFORMANCE ANALYSIS

Efficient Trading Frontier

Transaction costs may vary from strategy to strategy, portfolio manager to portfolio manager, and executing broker to executing broker. Some strategies may be designed to execute in calm market conditions when slippage is minimal. Other strategies may work in volatile markets, when latency impact is palpable and makes up the bulk of transaction costs.

Performance can be further compared to differentiate value added and non–value added execution. Almgren and Chriss (2000) propose that the evaluation of execution be based on the "efficient trading frontier" methodology. Reminiscent of the efficient frontier of Markowitz (1952) used in portfolio optimization, the efficient trading frontier (ETF) identifies the lowest execution cost per level of market risk at the time the order was executed. The ETF is computed for each security, strategy, executing broker, and cost type. Figure 19.1 illustrates this idea.

The efficient frontier is traced across all executed transactions in a given security; it can be broken down by type of the transaction cost, strategy, executing broker, and so forth. The goal of the exercise is to use execution with the most optimal trading frontier going forward. Depending on the scope of the analysis, the transaction cost can be measured as the implementation shortfall (IS) (discussed further along in this chapter) or as an individual cost component as shown in Table 19.1. The market risk at the time of execution can be measured as the volatility of an aggregate market index, such as the S&P 500. Alternatively, the market risk at the time of execution can be specific to each security traded and can be measured in the following ways: as a historical volatility of the mid price over a prespecified number of seconds or minutes, or as a size in bid-ask spread during the time of execution, among other methods. The bid-ask spread, while easy to estimate from the historical data, may be a biased measure specific to the executing broker (some brokers have higher

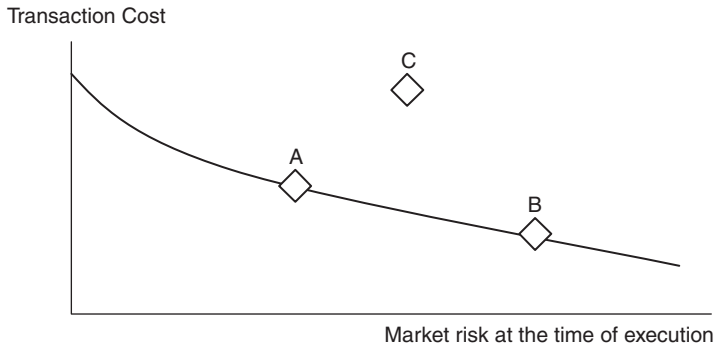


FIGURE 19.1 Efficient trading frontier (ETF). Trades A and B span the efficient frontier. Trade C is inefficient.

spreads than do other brokers throughout the entire spectrum of market conditions).

In Figure 19.1, the efficient trading frontier is traced by trades A and B. Trade C is not efficient, and the causes of the deviation of trade C from the efficient trading frontier should be investigated. If trades A, B, and C are recorded for the same security and strategy but different executing brokers, the costs of the broker responsible for trade C should be addressed, or the bulk of trading should be moved from the broker that traded C to the brokers that traded A and B.

Benchmarked Analysis

In the benchmarked analysis, the dollar value of the executed position is compared to the dollar value of the position executed at a certain price, known as the benchmark price. The benchmarks typically fall into one of the following categories:

- Pre-trade
- Post-trade
- Intra-trade

The pre-trade benchmarks are known at the time the trading begins and are usually the market prices at the outset of the trading period—or, for lower trading frequencies, the daily opening prices. Pre-trade benchmarks may also be based on the trade decision price, the price at which the trading system makes the decision to execute the trade. Benchmarking to the trade decision prices is often referred to as "implementation shortfall," and is discussed in detail later in this chapter.

The post-trading benchmarks can be any prices recorded after the trading period. A market price at the end of an intra-day trading period can be a post-trading benchmark, as can be the daily close. Perold (1988) points out that to the extent that the trading system places trades correctly—buys a security that rises through the remainder of the trading period, for example—comparing execution price with the closing price for the trading period will make execution look exceptionally, but unjustifiably, good.

Intra-trading benchmarks include various weighted price averages. The most popular benchmarks are the volume-weighted average price (VWAP, pronounced "vee-wop") and the time-weighted average price (TWAP, pronounced "tee-wop"). Other benchmarks include averages of the open, high, low, and close prices (OHLC) within the given trading interval that are designed to proxy for the intra-period range of price movement and measure the algorithm's capability to navigate volatility.

Both the VWAP and the TWAP benchmarks can be based on daily, hourly, or even higher-frequency price data surrounding the trade. The VWAP for a particular security i on day T is computed as follows:

$$VWAP_i = \frac{\sum_t v_{it} p_{it}}{\sum_t v_{it}}, \ \{t\} \in T \tag{19.5}$$

where v_{it} is the volume of security i traded at time t, and p_{it} is the market price of security i at time t.

VWAP is often thought to be a good indicator of market price throughout the period under consideration (a minute, an hour, a day, etc.). Execution geared to outperform VWAP typically succeeds at minimizing market impact, and VWAP-based performance measures reflect the success of cost minimization strategies. On the other hand, VWAP-based performance metrics do not assess the performance of strategies trying to minimize risk or other variables other than market cost.

TWAP benchmarking measures the ability of the execution algorithm to time the market. TWAP benchmark price computes the price that would be obtained if the order were split into equal-sized parcels and traded one parcel at a time at equally spaced time intervals within the designated trading time period:

$$TWAP_i = \frac{1}{T} \sum_{t=1}^{T} p_{it}, \ \{t\} \in T \tag{19.6}$$

where p_{it} is the market price of security i at time t.

Finally, the OHLC benchmark is a simple average of the open, high, low, and close prices recorded during the trading period of interest:

$$OHLC_i = \frac{1}{4}\left(p_{it}^O + p_{it}^H + p_{it}^L + p_{it}^C\right), \ \{t\} \in T \tag{19.7}$$

where p_{it}^O, p_{it}^H, p_{it}^L and p_{it}^C are the market open, high, low, and close prices of security i during the time interval t. The OHLC benchmark incorporates the intra-period price volatility by including the high and low price values. The OHLC benchmark does not, however, account for volume or liquidity available on the market.

Kissell and Malamut (2005) point out that different investors may have natural preferences for different benchmarks. Value investors may want to execute at their decision price or better, mutual fund managers may need to execute at the daily closing prices to facilitate the fund's accounting, and others may prefer VWAP, the below-average price for the pre-specified trading period. It is informative to compare performance of an algorithm against all benchmarks.

Overall, Kissell and Glantz (2003) caution that the benchmarked evaluation of execution performance may not be thoroughly useful for the following reasons:

1. Benchmarked assessment does not lend itself to execution comparisons across asset classes, a comparison that may be desirable in assessing performance of different executing brokers.
2. Benchmarked assessments are geared to minimization of execution prices; other execution-related performance characteristics may be plausible optimization candidates.
3. Furthermore, according to Kissell and Glantz (2003), benchmarked assessment strategies can be manipulated to show better performance than is warranted by the actual execution, making the portfolio manager incur higher costs at the same time.

Relative Performance Measurement

To address the flaws of the benchmarked performance measurement, Kissell and Glantz (2003) propose "relative performance measurement" as an alternative to the benchmarked analysis. The relative performance measure is based on either volume or the number of trades and determines the proportion of volume or trades for which the market price throughout the trading time period (a minute, an hour, a day, etc.) was more favorable than the execution price. In other words, the relative performance measure assesses at what percentage of volume or trades throughout the

specified period of time the trade could have been executed on even better terms than it was actually executed. Specifically, relative performance measure (RPM) is computed as follows:

$$\text{RPM(volume)} = \frac{\text{Total volume} \mid \text{price better than execution price}}{\text{Total volume}}$$
$$\text{RPM(trades)} = \frac{\text{Total \# of trades} \mid \text{price better than execution price}}{\text{Total \# of trades}} \tag{19.8}$$

According to Kissell and Glantz (2003), the relative performance measure allows a comparison of execution performance across different financial instruments as well as across time. Unlike the benchmarked approach that produces performance assessments in dollars and cents, the relative performance measure outputs results in percentages ready for cross-sectional comparisons. For example, suppose we would like to compare performance of execution of two stocks, IBM and AAPL, within a given hour. Suppose further that the benchmarked approach tells us that IBM outperformed its VWAP by 0.04, whereas AAPL outperformed its VWAP by 0.01. The two measures are not comparable, as neither one takes into account the relative prices of the securities traded. The relative performance measure, on the other hand, produces the following numbers—50 percent for IBM and 5 percent for AAPL—and allows us to objectively deduce that AAPL execution maximized its market advantage during the trading window, while execution of IBM can be improved further.

Implementation Shortfall

The implementation shortfall (IS) measure due to Perold (1988) measures the efficiency of executing investment decisions. The IS is computed as the difference between the realized trades and the trades recorded in paper trading. The paper trading process usually runs in parallel with the live process and records all the trades as if they were executed at desirable price at optimal times.

The paper-trading system of Perold (1988) executes all trades at the mid-point between the market bid and ask quotes, ignoring all transaction costs (spreads, commissions, etc.). The paper-trading system also assumes that unlimited volume can be processed at any point in time at the market price, ignoring the market depth or liquidity constraints and the associated slippage and market impact. The IS metric then measures the cost of running the trading system in real market conditions as compared to the costs incurred in the idealized paper-trading environment.

As Perold (1988) notes, the observed IS can be due to several factors:

- Liquidity constraints
- Price movement due to information imputed in market prices
- Random price oscillations
- Latency in execution
- Market impact
- Commissions
- Fees
- Spreads
- Taxes

The IS delivers a bundled estimate of the component costs and the estimate is difficult to disaggregate into individual cost centers. As a result, the IS methodology of Perold (1988) has been subject to criticism. To measure the costs of execution with greater precision, Wagner and Banks (1992) and Wagner and Edwards (1993) adjust IS by known transaction costs.

Furthermore, the implementation shortfall analysis can help in calculating the cost of market impact. A paper-trading system run concurrently with production can note two types of orders in parallel: (1) market orders at the market prices when the order decisions are made and (2) limit orders when the market price crosses the limit price. Such analysis will help assess the probability of hitting a limit order for a particular strategy, as shown by equation (19.9):

$$\Pr(\text{Limit Execution}) = \frac{\#\text{ of Executed Limit Orders}}{\#\text{ of Orders Placed}} \tag{19.9}$$

For example, if out of 75 orders placed as limit orders, only 25 were executed, the probability of executing a limit order for a given strategy is 33 percent.

The analysis will also help describe the opportunity cost of missing profits for limit orders that are never hit, as shown in equation (19.10):

$$\text{Opp Cost per Limit Order} = -\frac{\sum \text{Gain}_{\text{Market Orders}} - \sum \text{Gain}_{\text{Limit Orders}}}{\#\text{ of Orders Placed}} \tag{19.10}$$

Both the probability and the opportunity costs accompanying limit orders are useful tools in designing and updating future trading systems. The opportunity cost associated with a limit order failing to execute should be taken into account when deciding whether to send a particular order as a

limit or as a market order. As usual, the decision should be based on the expected gain of the limit order, computed as shown in equation (19.11):

$$\begin{aligned} E[\text{Gain}_{\text{Limit Orders}}] = &(\text{Opp Cost per Limit Order}) \times \\ &\Pr(\text{Limit Execution}) + (1 - \Pr(\text{Limit Execution})) \times \\ &\frac{\sum \text{Gain}_{\text{Limit Orders}}}{\#\text{ of Executed Limit Orders}} \end{aligned} \tag{19.11}$$

An order should be executed on limit instead of on market if the expected gain associated with limit orders is positive.

For example, suppose that a particular trading system on average executes two limit orders for every three limit orders placed. In this case, $\Pr(LimitExecution) = 66.7$ percent. In addition, suppose that every executed limit order on average gains 15 bps and every executed market order gains 12 bps. Then the opportunity cost on 100 orders placed can be computed as follows:

$$\text{Opp Cost per Limit Order} = -\frac{12 \text{ bps} \times 100 \text{ orders} - 15 \text{ bps} \times 100 \text{ orders} \times 66.7\%}{100 \text{ orders}} = -2 \text{ bps}$$

$$E[\text{Gain}_{\text{Limit Orders}}] = (-2 \text{ bps}) \times 66.7\% + (1 - 66.7\%) \times 15 \text{ bps} = 3.67 \text{ bps}$$

The limit orders will continue being placed as limit orders (as opposed to market orders) for as long as $E[\text{Gain}_{\text{Limit Orders}}]$ remains positive.

Performance Analysis Summary

Both cost and performance analyses, performed post-trade, generate insights critical to understanding the real-life trading behavior of trading models. The results of the analyses provide key feedback ideas for improving existing trading methodologies.

CONCLUSION

Post-trade analysis is an important component of high-frequency trading. At low trading frequencies, where the objective is to capture large gains over extended periods of time, transaction costs and variations in execution prices are negligible in comparison with the target trade gain. High-frequency trading, however, is much more sensitive to increases in costs and decreases in performance. At high frequencies, costs and

underperformance accumulate rapidly throughout the day, denting or outright eliminating trading profitability. Understanding, measuring, and managing incurred costs and potential performance shortcomings become paramount in the high-frequency setting.

The issues of costs and execution-related performance are bound to become more pronounced as the field of high-frequency trading expands. With multiple parties competing to ride the same short-term price oscillations, traders with the most efficient cost and performance structures will realize the biggest gains.

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About the Web Site

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- An intraday equity arbitrage strategy, presented in Chapter 13
- A market-neutral arbitrage strategy, also from Chapter 13
- A classic portfolio-optimization algorithm of Markowitz (1952), explained in Chapter 14
- The Strike execution algorithm from Chapter 18

In addition to the programming code, the web site provides tick data samples on selected instruments, well suited for testing the algorithms and for developing new trading models.

About the Author

Irene Aldridge is a managing partner and quantitative portfolio manager at ABLE Alpha Trading, LTD, a proprietary trading vehicle specializing in high-frequency systematic trading strategies. She is also a founder of AbleMarkets.com, an online resource making the latest high-frequency research accessible to institutional and retail investors.

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